

A Reconstruction of the Learning Hierarchy of Middle School Probability and Statistics Section: Focusing on 2015 Revision Mathematics Curriculum of Korea

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Abstract

In this study, the researchers analyzed the mathematics curriculum of elementary and middle school in the section of 'probability and statistics'. They also reconstructed the learning hierarchy by using a national curriculum document and the results of the previous studies. Based on the results of the 2015 revision curriculum analysis and the previous studies on the learning hierarchy segmentation, the learning hierarchy of 'probability and statistics' section are reconstructed from elementary school mathematics to middle school mathematics. The researchers extracted the content elements using the revision curriculum [5] and Kim's report [6], and they subdivided and restructured the learning contents of 'probability and statistics' using the results of the learning hierarchy segmentation of Park's report [3]. The segmentation and reconstruction of the learning hierarchy in the section of 'probability and statistics' of school mathematics facilitates the diagnosis of the learning deficits about the statistical concepts because the segmentation of the learning hierarchy can increase the accuracy of the diagnostic tool. Moreover, it enables the customized prescriptions for learners in the section of 'probability and statistics'. Since the diagnosis and prescription functions of learning are essential elements in an AI-based mathematics education system, the result of this study can be used as basic data for building the system.

Keywords: *Learning hierarchy, Probability and statistics, Curriculum, Mathematics*

1. Introduction

Mathematics is a hierarchical study, so if a deficit occurs in a prerequisite learning element, the student cannot understand the mathematical concepts he or she is trying to learn. Therefore, it is essential to subdivide the mathematics learning hierarchy because it is necessary to discover the cause of where students' mathematics learning deficits began. Recently, several studies on the analysis of mathematics learning hierarchy have been conducted at the time of national curriculum revision [1][2][7][8]. A study on segmentation of mathematics learning hierarchy was already conducted by Park et al., [3] (hereinafter, Park's report), but the results of this study were subdivided based on the learning contents of the 2009 revision mathematics curriculum [4]. In Park's report, they presented a method for extracting content elements of school mathematics.

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In addition, a study on extraction of content elements from the mathematics curriculum and textbooks was conducted by Kim et al., [6] (hereinafter, Kim's report). In the Kim's report, they discovered a framework of content system suitable for analyzing mathematics curriculum. They also presented the contents of primary and secondary mathematics curriculum in accordance with the framework of the developed system. Moreover, they drew a system diagram using this. It was based on achievement standards of the 2009 revision mathematics curriculum.

Thus, Park's and Kim's report did not reflect the changes of the 2015 revision curriculum [5]. In the 2015 revision mathematics curriculum, statistics education, use of engineering tools, and information processing capabilities are emphasized. Moreover, in the current mathematics curriculum, the learning hierarchy is presented a big concept size, that is, a unit size of the mathematics textbook. Therefore, it is necessary to analyze and subdivide the learning hierarchy by reflecting the 2015 revision curriculum.

Statistics education is being emphasized in school mathematics due to the influence of the 4th industrial revolution. In the era of 4th industrial revolution, data utilization is important and big data analysis is especially emphasized. For this reason, it is timely to analyze and segment the learning hierarchy in 'probability and statistics' section of school mathematics.

In this study, the researcher analyses the learning elements of 'probability and statistics' section of middle school in the 2015 revision mathematics curriculum and reconstructs the learning hierarchy by using the results of the previous studies with a purpose of subdividing learning elements. Furthermore, it is aimed to establish the foundation for the development of an AI-based mathematics education system.

The research questions of this study are as follows.

First, what is the result of subdividing and restructuring the content elements of the 'probability' part of middle school until elementary level according to the 2015 revision mathematics curriculum?

Second, what is the result of subdividing and restructuring the content elements of the 'statistics' part of middle school until elementary level according to the 2015 revision mathematics curriculum?

2. Methodology

자료와 가능성	평균	① 평균의 의미를 알고, 주어진 자료의 평균을 구할 수 있으며, 이를 활용할 수 있다.	평균	<Extraction factors> average, picture graph, bar graph, circle graph <Segmentation factors> graph using 'O, X, /', table, classifying
	자료의 정리	① 실생활 자료를 그림그래프로 나타내고, 이를 활용할 수 있다.	그림그래프	
		② 주어진 자료를 피그레프와 원그래프로 나타낼 수 있다.	피그레프, 원 그래프	
		③ 자료를 수집, 분류, 정리하여 목적에 맞는 그래프로 나타내고, 그래프를 해석할 수 있다.	그래프 해석	
	가능성	① 실생활에서 가능성과 관련된 상황을 불가능하다, 아닐 것 같다, 반반이다, 일 것 같다, 확실하다 등으로 나타낼 수 있다.	가능성	
		② 가능성을 수나 말로 나타낸 예를 찾아보고, 가능성을 비교할 수 있다.	가능성	
		③ 사건이 일어날 가능성을 수로 표현할 수 있다.	사건	

Figure 1. How to extract from the results of Kim's report according to the 2015 revision mathematics curriculum

First, the learning elements in ‘probability and statistics’ section of elementary and middle school were extracted by analyzing the 2015 revision mathematics curriculum. Moreover, the extracted learning elements were compared to the Kim’s report. The researcher segmented the learning elements of the 2015 revision mathematics curriculum using the above results. An example of the extraction and segmentation is shown below [Figure 1].

Second, the learning hierarchy diagrams of Park’s report were modified to fit the 2015 revision curriculum. Three mathematics education specialists including a researcher reconstructed the learning hierarchy diagrams. For the reconstruction of the diagrams, they conducted an expert council. An example of the reconstruction is shown below [Figure 2].

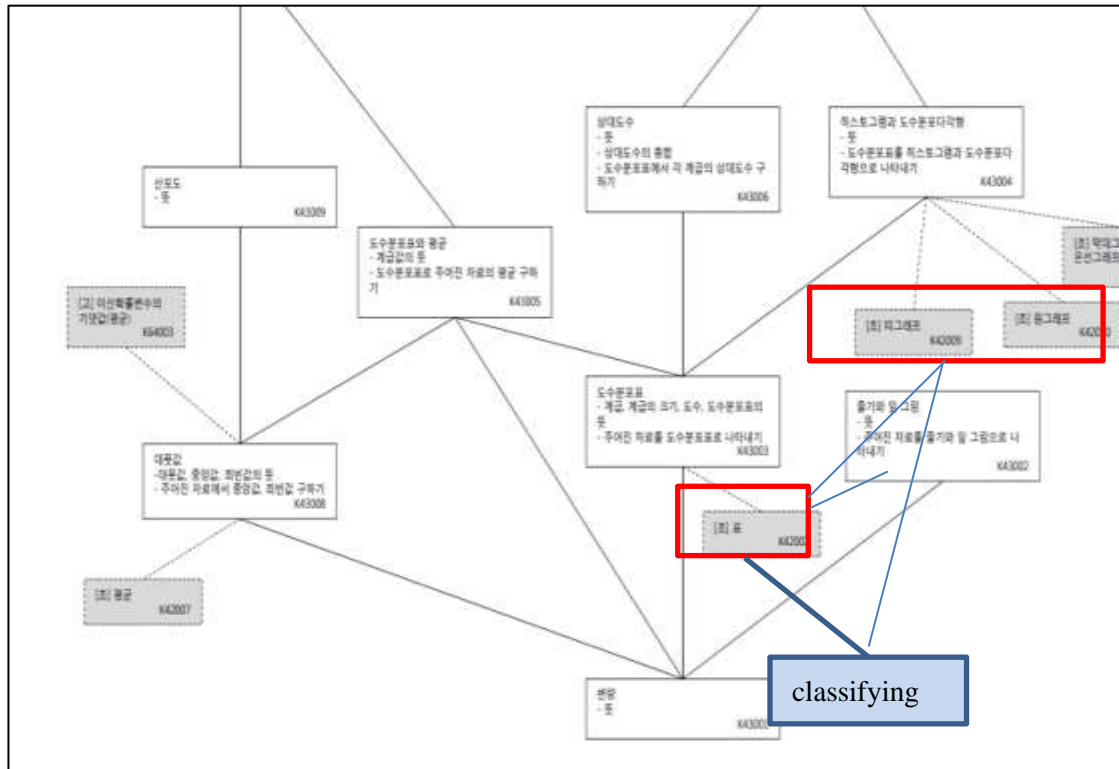


Figure 2. How to reconstruct from the results of Park’s report according to the 2015 revision mathematics curriculum

3. Result

3.1. Analysis of the 2015 revision curriculum: Data process and possibility section of elementary school mathematics

The researcher built the following [Table 1] using the learning elements of the 2015 revision curriculum and they segmented and reconstructed the learning elements in data process and possibility section of elementary school mathematics. Meanwhile, considering the development of AI-based system, ‘data process’ part and ‘possibility part’ were presented separately and the specialist subdivided the learning hierarchy in detail. The [Table 1] divides learning elements of ‘data process and possibility’ section by grade of elementary school.

Table 1. Learning elements of probability section and statistics section by grade in elementary school

Sub-section	Grade	Learning elements
Data process	1~2	<ul style="list-style-type: none"> • Classifying <ul style="list-style-type: none"> - Counting the objects by classifying with a criterion in real world contexts • Table <ul style="list-style-type: none"> - Making a table using the classified objects • Graph using 'O, X, /' <ul style="list-style-type: none"> - Drawing a graph using the classified objects
	3~4	<ul style="list-style-type: none"> • Simple picture graph • Bar graph • Line graph
	5~6	<ul style="list-style-type: none"> • Average • Picture graph • Band graph • Circle graph • Analyzing the graphs
Possibility	1~2	None
	3~4	None
	5~6	<ul style="list-style-type: none"> • Possibility <ul style="list-style-type: none"> - Expressing 'possibility' by using verbs in real world contexts - Expressing 'possibility' by using numbers - Comparing various 'possibilities'

3.2. Analysis of the 2015 revision curriculum: Probability and statistics section of middle school mathematics

The researcher built the following [Table 2] using the learning elements of the 2015 revision curriculum and they segmented and reconstructed the learning elements in probability and statistics section of middle school mathematics. Meanwhile, considering the development of AI-based system, 'probability' part and 'statistics' part was presented separately and the specialist subdivided the learning hierarchy in detail. The [Table 1] divides learning elements of 'probability and statistics' section by grade of middle school.

Table 2. Learning elements of probability section and statistics section by grade of middle school

Sub-section	Grade	Learning elements
Probability	1	None
	2	<ul style="list-style-type: none"> • Number of cases <ul style="list-style-type: none"> - Sum of cases - Product of cases • Definition of probability • Properties of probability <ul style="list-style-type: none"> - Understanding properties of probability - Calculating probability
	3	None
Statistics	1	<ul style="list-style-type: none"> • Organizing data <ul style="list-style-type: none"> - Stem and leaf plot - Frequency distribution table - Histogram - Frequency distribution polygon - Relative frequency - Distribution of relative frequency • Analyzing data
	2	None
	3	<ul style="list-style-type: none"> • Representative value <ul style="list-style-type: none"> - Median - Mode - Mean • Variance and standard deviation • Scatter plot <ul style="list-style-type: none"> - Expressing data as a 'scatter plot' - Explaining 'correlation'

3.3. Reconstruction of the learning hierarchy: Probability and statistics section of middle school mathematics

The researcher reconstructed the learning hierarchy of 'probability and statistics' section of middle school mathematics using the results of the 2015 revision curriculum analysis and the learning hierarchy segmentation such as [Table 1] and [Table 2]. The above [Table 1] and [Table 2] can be used to identify mathematics learning elements in detail.

The [Figure 3] shows a learning hierarchy diagram of 'probability and statistics' section of middle school mathematics according to 2009 revision curriculum.

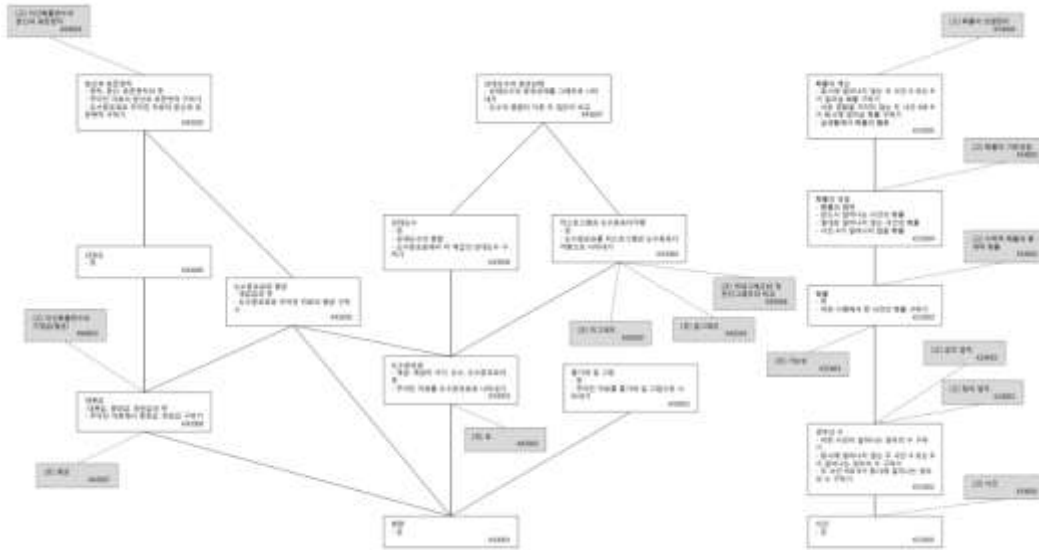


Figure 3. A learning hierarchy diagram of 'probability and statistics' section in Park's report

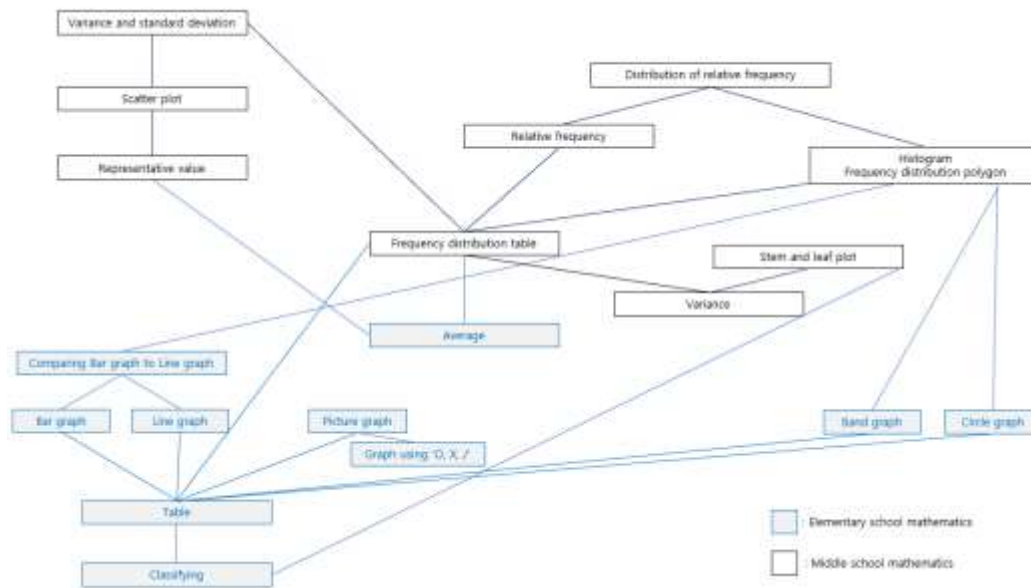


Figure 4. Segmentation and reconstruction of 'statistics' part of middle school until elementary level according to the 2015 revision mathematics curriculum

The researcher drew the below diagrams; [Figure 4] and [Figure 5] by reconstructing a diagram in Park's report. They presented 'statistics' part and 'probability' part separately in consideration of the hierarchical characteristics of learning elements. To make the learning hierarchy easier to check at a glance, a hierarchy was created centering on the main learning elements.

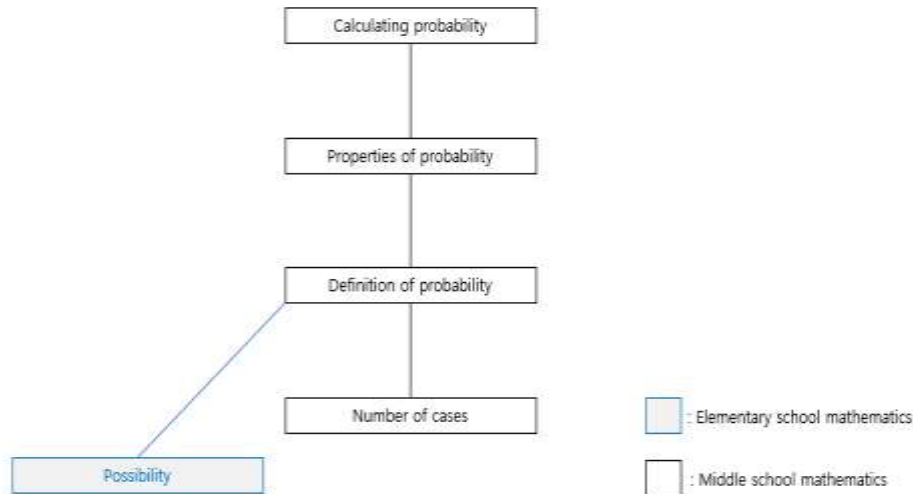


Figure 5. Segmentation and reconstruction of 'probability' part of middle school until elementary level according to the 2015 revision mathematics curriculum

4. Conclusion and Suggestions

In this study, the researchers segmented and reconstructed the learning contents of 'probability and statistics' section of middle school according to the 2015 revision mathematics curriculum, Korea. In the statistics part of middle school mathematics, a 'classifying' factor is added to connect it with 'table' and 'stem and leaf'. Moreover, a 'table' factor is connected with the various graph such as 'bar graph', 'line graph', 'picture graph', 'band graph', and 'circle graph'. In the probability part of middle school mathematics, a 'possibility' factor is added to connect it with 'definition of probability'.

The result of this study can provide the following implications for mathematics education.

First, the segmentation and reconstruction of the mathematics learning hierarchy facilitates the diagnosis of learning deficits in the section of 'probability and statistics' of school mathematics. The process of segmentation and reconstruction of the mathematics learning hierarchy is a major task to be performed before the development of diagnostic tool. In addition, the segmentation of the learning hierarchy can increase the accuracy of diagnostic tools, allowing students to be customized for mathematics learning prescriptions.

Second, it can be the basis for building an AI-based mathematics education system. In the AI mathematics education system, the AI assistant teacher should perform the function of diagnosing and prescribing students' mathematics learning. For this, machine learning technology must be introduced into the system, and the subdivided and restructured data of the mathematics learning hierarchy can be used as basic data for machine learning. Therefore, it is necessary to carry out a follow-up study to develop the mathematics diagnosis tools in the section of 'probability and statistics' of school mathematics.

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