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Model and Algorithm for Traffic Network Design by the User Equilibrium Allocation

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Abstract

Starts from the analysis of the different levels of service constraints impact to the network design, this paper combines the indicators in the levels of network service and road service, and then builds the bi-level programming model of road network design based on the level of service. And the paper solves the bi-level programming model based on the simulated annealing to verify that the saturation constraint values of the level of link service can be relaxed under the premise of ensuring the overall level of network service, by which road resources are be used fully with little impact of the road network.

Key words: *traffic network design, level of service, bi-level programming model, equilibrium allocation, the simulated annealing*

1. Introduction

In view of the actual environment of traffic demand model and the difference of traveler's route choice behavior, this paper combines the application scope of bi-level programming model with the constraints of service level index, and gives a description of corresponding objective function and constraints in order to make the model more close to the actual situation[1-3].

Traffic demand model in different road network and traveler's route choice behavior exist difference, this paper investigates such a kind of traffic network. The travelers of this traffic network usually have fixed travel demand, and they know from experience that the crowded degree and the needed walking time of every line in the traffic network [4-5].

The small and medium cities of no less than 500,000 can be divided into this type of traffic network, because of their relatively small size and less turnover of people, the traveler's fixed traffic demand and the specific path selection are easier to be satisfied [6-8].

2. Modeling Assumption and Symbol Definition

According to the traditional modeling method，we assume that the traveler's path selection criteria is the shortest actual travel time [9-10]，and all travelers' selection criteria is consistent. Here, travel behavior difference which caused by the factors such as travel purpose and income level are not considered.

The symbols referred to in this paper are defined as follows：

$G(N, A)$ ——traffic network；

A ——road set；

N ——nodes set

$G_a(y_a)$ ——investment cost function；

$t_a(x_a, y_a)$ ——travel time function of road a；

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f_p^{rs} ——traffic volume of p section of the road rs to OD；

P^{rs} ——road set of rs to OD；

$\delta_{a,p}^{rs}$ ——road/path associated variables (0-1 variables)；

δ_{G1} ——road section saturation；

δ_{G2} ——traffic network saturation；

x_a ——traffic flow on the road a；

l_a ——the length of the road a；

c_a ——he maximum capacity of the road a；

ϕ ——the coefficient match the investment cost and the system total impedance unit；

μ_1, μ_2 ——constraint value of the maximum saturation；

R ——origin set；

S ——destination set.

3. Model for Traffic Network Design by the User Equilibrium Allocation

Suppose a small city's traffic network: it has a population less than 500,000, its urban area is not big and city land development is relatively balanced, their travelers have a fixed travel demand. The needed walking time of every line in the traffic network because the less turnover of people in this traffic network [11-12].

Considering the resident trip of small city is more regular and travelers usually have specific travel purpose, that is to say, most travelers' starting point and destination are relatively fixed so the proportion of the traffic volume between origin-destination of every road to the total traffic is determinate [13-14].

When the existing roads of this kind traffic network can't satisfy travelers' traffic demand with the growth of travel demand, at this time we need to the study the network design problem in order to derive the optimal strategy.

Considering such traffic network is relatively stable and its traffic demand is relatively fixed and travelers know very well from experience that the crowded degree and the needed walking time of every line in the traffic network, therefore, we can approximately think that people completely know the travel time of each section in this traffic network [15-16]. At the same time, this paper doesn't consider the difference caused by factors such as traveler's income level and so on, it assumes that the criterion of all travelers' route choice behavior is their shortest actual travel time. So this traffic network can be thought to approximately correspond with the two preconditions of UE (User Equilibrium), the lower model takes the UE model to describe traveler's route choice behavior. When choosing the objective function of the upper resource allocation, considering the objective function of minimum system travel cost is the most commonly used, we intend to apply this objective function. The premise of application of this objective function is that the traffic demand is fixed between origin-destination of the road. We put the investment cost function as weighing ϕ in the objective function of the upper model because of without restricting maximum rating of investment cost. ϕ is the proportionality coefficient that synthesizes information about weighing of 2 purposes (minimum system travel cost and minimum investment cost function) and matching of measurement unit of travel cost and investment cost. The higher the value of ϕ , the smaller the investment budget; the lower the value of ϕ , the larger the investment fund.

Combined with above description，the lower model of this traffic network takes the UE model, the objective function of the upper model is the smallest sum of system travel cost and the investment cost. At the same time, we use the following comprehensive service level evaluation index as the constraint of upper model in order to make road service level to meet expectation of the traffic planners and travelers

$$\delta_{G1} = x_a / c_a \le \mu_1 \qquad (1)$$

$$\delta_{G2} = \sum_{a \in A} x_a l_a / c_a l_a \le \mu_2 \qquad (2)$$

The corresponding relation of the constraint value of the road section saturation μ_1 and the service level should refer to Table 1.

Table 1. Service Standard Level of Intersections and Sections

Service standard level	A	B	C	D	E	F
Saturation (V / C)	< 0.4	0.4~0.6	0.6~0.75	0.75~0.9	0.9~1.0	> 1.0

The bi-level programming model under the given traffic model is shown below:
The upper:

$$\min T = \sum_{a \in A} x_a t_a(x_a, y_a) + \phi \sum_{a \in A} G_a(y_a) \qquad (3)$$

$$\text{s.t.}\ \delta_{G1} = x_a / c_a \le \mu_1 \qquad (4)$$

$$\delta_{G2} = \sum_{a \in A} x_a l_a / c_a l_a \le \mu_2 \qquad (5)$$

The lower:

$$\min Z = \sum_{a \in A} \int_0^{x_a} t_a(w)\, dw \qquad (6)$$

$$\text{s.t.}\ \sum_p f_p^{rs} = q_{rs}, \forall r \in R, \forall s \in S \qquad (7)$$

$$f_p^{rs} \ge 0, \forall r \in R, \forall s \in S, \forall p \in P^{rs} \qquad (8)$$

$$x_a = \sum_r \sum_s \sum_p f_p^{rs} \delta_{a,p}^{rs}, a \in A \qquad (9)$$

The traffic network maximum saturation limit μ_2 is determined by the formula $\mu_2 = \lambda \mu_1$, among which λ is a correction factor of the traffic flow distribution balanced or not. The value of λ in this literature is 0.8~1.0, and λ is 1.0 when the influence of the traffic flow distribution is not considered.

It's should be noted that without budget limit doesn't mean the investment cost can be completely unlimited, it considers the travel time cost and the invest-building cost together, put them in the objective function at a certain weight and calculates minimal value of their sum.

4. The Simulated Annealing Algorithm of the Bi-Level Programming Model

The advantages of using the bi-level programming model to study the network design problem are simple and easy to understand, but the solving of the bi-level programming model is difficult. We must use the algorithm to solve it although the decision variables in this paper are continuous. Heuristic algorithm is often used by previous scholars, such as iterative optimization algorithm and sensitivity analysis.

Iterative optimization algorithm: first given an initial solution of the upper model, bring the initial solution into the lower model to calculate the traffic flow, and then using the traffic flow as the known condition to evaluate the extremum of the upper objective function in order to get the new solution. The iteration is terminated until the convergence condition is satisfied. Iterative optimization algorithm is efficient, but it is may not converge. Sensitivity analysis is similar to iterative optimization algorithm; the difference is that it uses the gradient of the traffic flow to the value added of traffic capacity to search for new solution. The iteration is terminated until the optimal condition is satisfied. Sensitivity analysis is more accurate than iterative optimization algorithm, but its amount of calculation is too large. The difference of the two algorithms above is that the iterative way of the decision variables of the upper model and the lower model, and we can see that heuristic algorithm's aim is to get a more satisfactory solution, but not necessarily the optimal solution.

Most of the objective function of continuous network design problem has multiple low peaks , and heuristic algorithm takes local search approach to search for new solution, so most of its final solutions are local optimums. A minimum of the objective function is obtained by heuristic algorithm. In order to obtain globally optimal solution, some intelligent optimization algorithms have appeared in recent years, such as simulated annealing algorithm, genetic algorithm, particle swarm optimization and so on. These algorithms have already applied in the traffic network design problem by many scholars and get optimal solution or approximate optimal solution of the bi-level programming model problem. Therefore this paper chooses simulated annealing algorithm which is often used to solve the bi-level programming model.

Simulated annealing algorithm derived from the concept of the principle of solid annealing in physics. Solid annealing is that making the solid material melt when heated to a certain temperature, then cooling it slowly. Each temperature reaches a new equilibrium state when the temperature falls, and the energy of the system is also reduced with temperature in the process of cooling. The energy of the system is reduced to the lowest value when the material solidifies into a crystal.

The concept of solid annealing is extended into the algorithm. A lot of feasible solutions are like each temperature reaches a new equilibrium state in the process of cooling, and the objective function of simulated annealing accomplishes optimally to the time when the energy of the system is reduced to the lowest value.

The globally optimal search is the most distinguishing characteristic of simulated annealing algorithm. The superiority of the algorithm is that it is able to accept the solution bigger than the current objective function at a certain probability. So even if it falls into the scope of a certain minimum during the process of solving, it also has a certain probability to jump out the local search limit and find the optimal solution in the global scope. The acceptability probability uses Metropolis criteria and its formula is shown below:

$$P(\Delta E) = \exp\left(-\frac{\Delta Z}{k_s T}\right) \qquad (10)$$

k_s is Boltzmann constant, ΔZ is the difference of the new objective function value and the previous value, T is the temperature parameters of the current equilibrium state.

Simulated annealing algorithm is implemented through the following means. Firstly, give an initial value to the particle of the equilibrium state under a given temperature, and get a value objective function. Then, give a random disturbance to the particle's value and get a new value of the particle, comparing the current objective function value with previous objective function value. Notice new solutions are generated in random directions, not the down direction. This laid a solid foundation for it to jump out the local search limit. At this time, accept the new objective function value if it is smaller than previous; Otherwise, decide whether to accept the new value or not by Metropolis criteria. The particle can reach thermal equilibrium state after the process repeats several times, namely the algorithms under this temperature is finished when all the feasible solutions obey Boltzmann distribution. This process is also called inner loop process, the Boltzmann distribution formula as shown below:

$$P(E_i) = \frac{\exp\left(-\frac{Z_i}{k_B T}\right)}{\sum_j \exp\left(-\frac{Z_i}{k_B T}\right)} \qquad (11)$$

Outer loop process is relatively simple, gives a higher initial temperature firstly, and then implements the inner loop under each temperature until it reaches equilibrium. After that reduces temperature at the given step. It is needed to implement the inner loop and reach equilibrium after lowering the temperature in every time, the process repeats several times until the temperature drops to a minimum.

Step 1: Initialization. Set the initial values of the parameters, and distribute the traffic flow according to the initial value of the traffic capacity increment, obtaining the value of the objective function.

Step 2: Inner loop. Give a random disturbance to the initial value of the traffic capacity increment, and distribute the new value of the traffic capacity increment, and then get a new value of the objective function. If the new objective function value is smaller than the original objective function value, accept the new traffic capacity increment. Otherwise, using Metropolis criteria to judge whether accept the new value or not. Go to step 3 if it reaches the maximum number of the inner loop, otherwise back to step 2.

Step 3: Terminate judgment. The new value of the traffic capacity increment is the solution if the temperature falls to a minimum, and the algorithm is ended, otherwise, gets a new step, go back to step 2.

The formulas referred to the new traffic capacity increment and the new step is listed as follows:

$$\hat{y} = y + \rho U \qquad (12)$$

$$\rho' = \rho\sqrt{\frac{h}{0.11M}} \qquad (13)$$

In the formulas, $\hat{y}$ is the new traffic capacity increment; y is the initial value of the traffic capacity increment; ρ is the step size; U is random vector which obeys the uniform distribution on $\left[-\sqrt{3}, \sqrt{3}\right]$; ρ' is the new step size; M is the iterations of the inner loop; h is a constant greater than 1.

It should be pointed out that step 3 of algorithm describes the range of the decreasing of temperature, the solution may not be the globally optimal solution if the temperature ranges is too large, it is only the locally optimal solution; The amount of calculation increase greatly and the computing speed of the algorithm become slower if the temperature ranges is too small. Based on the above analysis, this paper takes the predecessors' effective cooling scheme for reference. That is slowly first and then faster. In the first 12 times, the new temperature is 80% of the previous temperature, after 12 times, the new temperature is half of the previous temperature.

5. Numerical Example

5.1. The Design Target and Basic Assumptions of the Numerical Example

There are two design targets of the numerical example in the paper. Firstly, through the analysis of research results of previous traffic network design problem and the results in this paper to verify that the traffic network design model added comprehensive service level evaluation index as constraint condition will get better road network transformation scheme. Secondly, correct the parameter values in the comprehensive service level evaluation index, and by comparison of the results before and after correction in order to get the parameter values modified of the better road network transformation scheme.

The design of urban road network generally requires road service level at B or C and the actual saturation of the road G_1 should not below level C, so this paper assumes that the expectation of traffic planner and traveler is level C. Based on this assumption to select the section maximum saturation limit and the road network maximum saturation limit. Two factors are considered when selecting the section maximum saturation limit. One is the traveler service level requirement for peak time is lower than that of the off-peak hours; the temporarily low service level can be accepted during peak hours. The other one is that the road network saturation limit in the comprehensive service level evaluation index can be formed from the whole network, and the difference of section saturation limit may be large. Based on the above consideration, this paper selects the lower limit value of the saturation interval corresponding the C level as the section maximum saturation limit, that is $\mu_1 = 0.75$, and μ_2 is determined by the formula $\mu_2 = \lambda\mu_1$. The value of λ is 0.8 ~ 1.0, and it is determined by the equilibrium degree of the traffic flow distribution, and λ is 1.0 when the influence of the traffic flow distribution is not considered. Let $\lambda = 1$ compare with $\lambda = 0.75$ in this paper, and μ_2 is 0.75 and 0.6375 at this time. In general, it is only able to get one of the improved results of the traffic flow date and traffic capacity date in the past; the section saturation limit can't be calculated. So the paper studies the network design problem with limited capacity. In conclusion, this paper studies several kinds of parameter as follows to verify its idea.

Case 1: The capacity constraints. The road traffic flow is smaller than its capacity; this is the network design problem with limited capacity in the past. Make a comparison of the improved level of service and the two kinds of situation followed to verify that the service level is poor if only considered the capacity constraints, and is reasonably if joining the service level constraints.

Case 2: The constraints without correction. Although the service level constraints are added, but without considering the effects of traffic flow distribution on the network's saturation，that is $\lambda=1$, $\mu_1=\mu_2=0.75$. There is no difference between the section saturation limit and the road network saturation limit. At this time, the values of μ_1 and μ_2 take the lower limiting value of the saturation interval corresponding the C level as the criterion.

Case 3: The constraints with correction. λ Is determined by the actual equilibrium degree of the traffic flow distribution, and λ is 0.85 in the paper. And the values of μ_1 and μ_2 are respectively 0.75 and 0.6375. Case 3 make a comparison of case 2 to verify that the road network maximum saturation limit added service level limit can form operative constraint on the whole, and reach the system optimization.

Use the BPR function as the walking time function in the example, after the assignment of undetermined parameters, it is as follows:

$$t_a(x_a, y_a) = t_a^0\left[1 + 0.15\left(x_a / (c_0 + y_a)\right)^4\right] \quad (15)$$

t_a^0, x_a and y_a respectively as the vehicles' average free walking time, traffic flow and capacity increment on road a.

5.2. The Basic Information of the Traffic Network

Select a common test network is shown in figure 1; it consists of four nodes and five roads. This test traffic network is simple and less constraint, it is applicable to a variety of types of traffic network. So it has been widely used in the traffic network design problem. Many scholars carried out example analysis based on it, Song yifan and Gao ziyou studied the urban traffic network design model and algorithm in 1999, Xu liang and Gao ziyou studied the urban traffic network design based on the reliability of the road capacity in 2006, Huang zhongxiang studied the reliability of the service level in 2007 and Xuli studies the optimization of urban traffic network design problem in her master's thesis.

Considering the test network has been widely applied in the study of network design problem and gained wide acceptance, so, the paper uses this test traffic network to study the traffic network design model and algorithm based on the user equilibrium allocation.

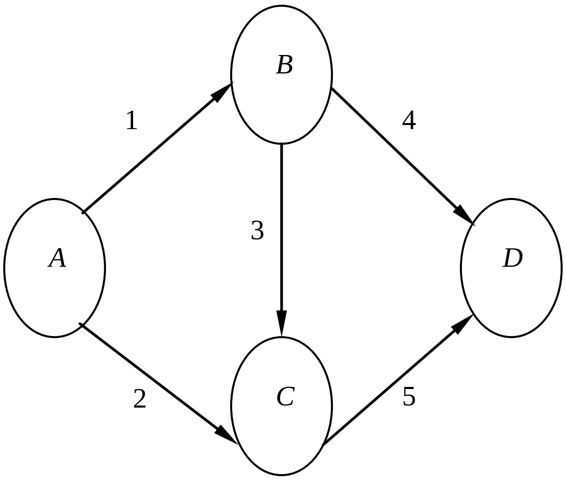

Figure 1. Test Traffic Network

Refer to the previous reference; the vehicles' average free walking time of each road, designed traffic capacity, the road length and the investment cost parameter are in Table 2. The traffic demand $q_{AD} = 6000\, pcu/h$, the traffic demand is greater than the road capacity at this time, so it needs to study the traffic network design problems. Assumes that the investment cost is relatively abundant, and traffic planners and the travelers expect that the service is above C level. The broaden investment costs function is $G_a(y_a) = 1.5 d_a \times \left(\frac{y_a}{60}\right)^2$, and its weight $\phi = 0.8$.

Table 2. Road's Parameters Setting

road	average vehicle's free run time h	design traffic capacity (pcu/h)	road length (km)	investment cost parameter 数 d_a
1	0.02	2200	6	3
2	0.08	2200	6	3
3	0.04	3400	4	1
4	0.03	2200	6	3
5	0.05	2200	6	3

The paper uses the model without budget constraint, through the bi-level programming model which shows in formulas (3) ~ (9) and Mat lab programming calculation to solve the problem. Assign the values of μ_1 and μ_2 according to the above 3 kinds of cases, run the code to calculate, and get the results under the 3 kinds of cases.

5.3. The Results and Analysis of the Example

For the first design goal of the example, the paper compares case 1 with case 2 in order to verify that the model adopts constraint of comprehensive service level evaluation index is better than the one only uses the capacity limit. Improved saturation of the two cases is shown at Table 3:

Table 3. Improved Saturation of the Two Cases

road	Case 1		Case 2	
	Capacity increment (pcu/h)	Improved saturation	Capacity increment (pcu/h)	Improved saturation
1	540.0	0.99	1770.0	0.7498
2	740.0	0.90	1795.6	0.6848
3	0.0	0.09	0.0	0.0939
4	354	0.99	1318.9	0.7499
5	780	0.99	1882.1	0.7499

Table 3 shows that most of the roads have a high saturation in case 1, and the service level of these roads is very low at this time, the traffic is in saturate flow state. Although the road is still in the connected state, but the road traffic condition is not stable and is easy to cause traffic congestion. So although the previous traffic network design problems only considered capacity limit could ensure that the traffic flow is less than the section capacity to solve the problem of optimal investment decision, but the improved roads have a poor service level, and form certain potential risks to the traffic.

In order to solve this problem, the paper joins the comprehensive saturation evaluation index constraint in case 2 to ensure that the road service level is above C, the correction of the traffic network maximum saturation constraint are not considered at this time. The improved saturation of the roads as in table 4, it shows that all the saturation is lower than 0.75 and the service level is above C.

The actual traffic network saturation of the two cases is respectively 0.8623 and 0.6699. Through the above comparison: the model joins the service level constraint can better meet the traffic planner and traveler's expectation of road service level and keep the road service in reasonable level, and get better road network transformation scheme than previous model. But, the correction of the traffic network saturation constraint is not considered in case 2, the actual traffic network saturation is 0.6699, it does not form an active constraint to the service level from the whole aspects. So the following text studies the problem combines with case 3.

For the second design goal of the example, the paper studies the optimal objective function values of the three cases. The model in this paper doesn't limit the biggest quota of investment cost and put the investment cost function as weighing ϕ in the objective function of the upper model. So the upper objective function is made up of two parts, one is the investment cost function, the other one is the travel impedance function. ϕ is the proportionality coefficient that synthesizes information about weighing of 2 purposes (minimum system travel cost and minimum investment cost function) and matching of measurement unit of travel cost and investment cost. The higher the value of ϕ, the smaller the investment budget; the lower the value of ϕ, the larger the investment fund. So let's compare $\phi = 0.002$ with $\phi = 0.8$, and get the objective function values of three kinds of cases are displayed in Table 4.

Table 4. Target Values of the Three Cases

$\phi = 0.002$	general objective function value	investment cost	travel resistance	$\phi = 0.8$	general objective function value	investment cost	travel resistance
case 1	923.4	11.9	911.5	case 1	2271.5	1301.7	969.8
case 2	835.5	32.5	803.0	case 2	8583.9	7759.1	824.8
case 3	834.8	40.1	794.7	case 3	9724.5	8901.5	823.0

Table 4 shows that it gets different values of the optimal system objective function when assign ϕ and μ the different constraint values.

Let's analyze this phenomenon, the function value decreases with the decrease of the maximum saturation constraint value when $\phi = 0.002$, this is because the travel resistance becomes small and the weight of the investment cost is very small, which results the investment cost isn't predominant in the general objective function value. The reason why the travel resistance becomes small is that although the traffic volume makes a little change, the added value of traffic capacity increases with the decrease of the maximum saturation constraint. Similarly, the function value increases with the decrease of the maximum saturation constraint value when $\phi = 0.8$, which because the investment cost increases with the increase of traffic capacity and the weight of investment cost increases in the travel resistance. The above analysis is consistent with actual examples of data changes, thus proving analysis is reasonable.

So the investment cost pressure and traffic planner and traveler's expectation of road service level should be comprehensively considered in practical application. When the investment cost is relatively abundant, using a smaller saturation constraint value can get better system target, this is advantageous to the road to achieve better service level. The results of case 3 is listed in Table 5.

Table 5. Improved Results of Case 3

road	capacity increment (pcu/h)	traffic flow$V$ (pcu/h)	traffic capacity (pcu/h)	degree of saturation (V/C)	service level
1	1785.0	3126.8	4185.0	0.7471	C
2	1870.8	2873.2	4270.8	0.6728	C
3	0	338.0	3600.0	0.0939	A
4	1325.0	2788.7	3725.0	0.7487	C
5	1889.2	3211.3	4289.2	0.7487	C

All the road service level is above C in Table 1, and the actual traffic network saturation is 0.6375. It shows that traffic network saturation in case 3 forms more effective constraint than case 2, and it improves the road service level from the whole aspects. So the data of case3 is also more reasonable than case 2, and ensures better use of the road based on the service level meets traffic planners and people's expectation.

The above analysis verifies the paper’s idea, that is we can get the better travel system impedance when limit the road maximum saturation constraint above the expected level to amend the traffic network saturation constraint. Using this comprehensive service level evaluation index constraint can take into account both the integrity and diversity of road and obtain better network retrofit scheme.

6. Conclusions

Traffic network design problem is a game between traffic planning department and traveler. The paper adopts the bi-level programming model to describe the game progress, and sets up the traffic network design model based on the user equilibrium allocation. The upper resource allocation problem is used to describe the traffic planner’s decision, and it chooses the minimum system impedance as the objective function; the lower traffic flow problem reflects traveler’s response to the decision, and it chooses the UE model to describe the traveler route choice behavior. In the process of modeling it join s comprehensive service level evaluation index constraint on basis of the previous research, makes comparative research on the three cases through different parameter values to verify we can get better traffic network retrofit scheme if the comprehensive service level evaluation index is added as limit condition in the traffic network design model.

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