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The Finite Element Method for the Buried Pipes in Groud Source Heat Pump System

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Abstract

Directed at the key problem about the heat transfer between heat pipes and piles found in the ground source heat pump system (GSHP), a new finite element method is adopted in this paper to provide a finite element scheme for piles and U-bent tubes.And then a calculation on the term of heat transfer between the pile and the U-bent tube through the point integration method is made.Then the value of the convective heat transfer coefficient h will be determined through the definition of the Nussellt number. Finally three illustrative examples are given in this paper to verify the rationality of our format. The model proposed in this paper has broken through the restrictions of the traditional analytical solutions, making it able to be applied widely in various projects.

Keywords:*convective heat transfer in porous media, finite element, the term of heat transfer, pile and U-bent tube, Nussellt number*

1. Introduction

Nowadays energy shortage has become one of the major social problems in the world. Due to the economic development and the improvement of people's living standards, energy consumption has been increased year by year in the world, while the fossil energy resource, which is also the main body of energy consumption, is becoming less and less. Since the fossil energy consumption always brings about the environmental problem, it makes people begin to raise the problem about sustainable development.

Geothermal energy is one of the clean and renewable energy resources. Now in some well-developed countries such as Europe, USA and Japan, the GSHP has become very matured and completely industrialized, which however just starts to be studied in our country. As an efficient green and energy saving product, ground source heat pump will play an increasing significant role in the building air-conditioning system in our country. Hence, the research output of the GSHP will have a broad application prospect in our country. The application of such a system will bring about significant economic and social benefits.

Heat transfer problem found in porous media is also the core problem of the GSHP. Many predecessors have proposed a variety of analytical solutions to deepen the research on such problem, for which Lauwerier proposed for the first time the 2D heat transfer modelfor the heat transfer in porous media and deduced the analytical solution for this model [7] . In 1961, Ogata & Banks revised the boundary conditions for Lauwerier problem by setting the overlying and the underlying strata as the adiabatic boundaries[8]. In 2010, on the basis of Lauwerier model, Barends utilized Laplace transformation and inverse transformation to provide an analytical solution [9] for Lauwerier problem when the water-bearing stratum and the heat conduction of the overlays are taken into account. Since the boundary conditions

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required by Ogata & Banks' model are relatively simple, it's rather easy to obtain the solution through this model. Although Lauwerier' model never simplifies the boundary conditions, it fails to reflect the influence of heat conduction in the governing equation to make its solution become problematic in such a heat transfer problem where convection is not dominant. Although the analytical solution given by Barends is a complete solution, the solving process is extremely complex in handling the heat dissipation of the overlying strata with little attention paid to the temperature distribution of the overlying strata in a geothermal project[2].

Therefore limited by the applicability conditions, the analytical solutions are not adapted to solving the complex engineering problems. On account of this, a finite element method is proposed in this paper to study such a problem. A new finite element solution scheme is adopted to solve the problem of heat transfer between heat pipes and piles simultaneously, meanwhile it will avoid taking the heat pipes as a form of heat source,and realize a synchronous solving of the problems and improve accordingly the computational accuracy. Our finite element schemes have broken through the restrictions of an analytical solution in the solving process, making they have broader applications in the projects.

2. Mathematical Model

Considering that the porous media, the concrete and the liquid contained in the soil layer are incompressible, therefore only the permeability of the porous media, instead of the pile is taken into account in this paper. Hence the applicable mathematical models include:

For soil: $\rho c_p \frac{\partial T}{\partial t} + \frac{\partial}{\partial x_j}(v_j(\rho c_p T)) = \frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}) + q$

For pile: $\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}) + q + \int_{\partial\Gamma}\tilde{q}ds$

For U-bent tube： $\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}) + q - \int_{\partial\Gamma}\tilde{q}ds$

Where $\int_{\partial\Gamma}\tilde{q}ds$ is the term of heat transfer between the pile and the U-bent tube, T is the temperature of the medium, ρ refers to the medium density, c_p represents the specific heat capacity of the medium, k is the thermal conductivity of the medium, q is the heat source and $v_j (j = 1,2,3)$ means the heat transfer rate in three directions.

3. The Finite Element Schemes

3.1 Scheme for Piles

Regarding the temperature of $\int_{\partial\Gamma}\tilde{q}ds$ inside the pile, it's necessary to consider the temperature exchange with the U-bent tube. Hence the mathematical model should be:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}) + q + \int_{\partial\Gamma}\tilde{q}ds$$

Multiply simultaneously each side of the equation by a virtual displacement δT and do the integral in the integral domain to obtain:

$$(\rho c_p \frac{\partial T}{\partial t}, \delta T)_\Omega = (\frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}), \delta T)_\Omega + (q, \delta T)_\Omega + \int_{\partial\Gamma} \tilde{q}\,\delta T ds$$

Do the integration by parts to get :

$$(\rho c_p \frac{\partial T}{\partial t}, \delta T)_\Omega = -(k\frac{\partial T}{\partial x_j}, \frac{\partial \delta T}{\partial x_j})_\Omega + (k\frac{\partial T}{\partial x_j}, \delta T)_\Omega + (q, \delta T)_\Omega + \int_{\partial\Gamma} \tilde{q}\,\delta T ds$$

Where $(k\frac{\partial T}{\partial x_j}, \delta T)_{\partial\Omega}$ is the boundary term, which means the heat exchange on the boundary or the boundary for thermal radiation, while $\int_{\partial\Gamma} \tilde{q}\,\delta T ds$ is the term of heat transfer between the U-bent tube and the pile.

3.2 Scheme for U-bent Tubes

Regardless of the compressibility of the fluid in the pipe, when only the heat exchange between the U-bent tube and the pile is considered, it can be simplified as a linear unit since the size of the U-bent tube would be extremely small compared with the pile and the stratum. Hence the mathematical model should be:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}) + q - \int_{\partial\Gamma} \tilde{q} ds$$

Multiply simultaneously each side of the equation by a virtual displacement δT and do the integral in the integral domain to obtain:

$$\begin{aligned}&(\rho c_p \frac{\partial T}{\partial t}, \delta T)_{\partial\Gamma} + (\rho c_p v_j \frac{\partial T}{\partial x_j}, \delta T)_{\partial\Gamma}\\ &= (\frac{\partial}{\partial x_j}(k\frac{\partial T}{\partial x_j}), \delta T)_{\partial\Gamma} + (q, \delta T)_{\partial\Gamma} - \int_{\partial\Gamma} \tilde{q}\,\delta T ds\end{aligned}$$

Do the integration by parts to get :

$$\begin{aligned}&(\rho c_p \frac{\partial T}{\partial t}, \delta T)_{\partial\Gamma} + (\rho c_p v_j \frac{\partial T}{\partial x_j}, \delta T)_{\partial\Gamma}\\ &= -(k\frac{\partial T}{\partial x_j}, \frac{\partial \delta T}{\partial x_j})_{\partial\Gamma} + (k\frac{\partial T}{\partial x_j}, \delta T)_{\partial\Gamma} + (q, \delta T)_{\partial\Gamma} - \int_{\partial\Gamma} \tilde{q}\,\delta T ds\end{aligned}$$

Where $(k\frac{\partial T}{\partial x_j}, \delta T)$ is the boundary term, which means the heat exchange on the boundary or the boundary for thermal radiation, while $\int_{\partial\Gamma} \tilde{q}\,\delta T ds$ is the term of heat transfer between the U-bent tube and the pile.

3.3 Special Processing of the Term of Heat Exchange

$\int_{\partial\Gamma}(k\frac{\partial T}{\partial x_j})ds$ is the term of heat transfer between the U-bent tube and the pile, then make a calculation on it through the point integration method.

Assume that $\int_{\partial\Gamma}(k\frac{\partial T}{\partial x_j})ds=\int_{\partial\Gamma}\tilde{q}ds$, then process the U-bent tube as a linear unit $\partial\Gamma$, device the boundary into n segments, $\partial\Gamma=\sum_{i=1}^{n}\partial\Gamma_i$. Number the units by 1,2,……,n , and the nodes by 1,2,……,n+1,

$$\int_{\partial\Gamma}(k\frac{\partial T}{\partial x_j})\delta T ds=\int_{\partial\Gamma}\tilde{q}\,\delta T ds=\sum_{i=1}^{n}\int_{\partial\Gamma_i}\tilde{q}\,\delta T ds$$
$$=\sum_{i=1}^{n}\int_{\partial\Gamma_i}\frac{\tilde{q}_i+\tilde{q}_j}{2}\delta T_i ds=\sum_{i=1}^{n}\frac{\tilde{q}_i+\tilde{q}_j}{2}\delta T_i\int_{\partial\Gamma_i}ds$$

Where $\int_{\partial\Gamma_i}ds=l_i$ and l_i means the length of the ith linear unit. Then process the above formula to obtain:

$$\int_{\partial\Gamma}(k\frac{\partial T}{\partial x_j})\delta T ds=\int_{\partial\Gamma}\tilde{q}\,\delta T ds=\sum_{i=1}^{n}\int_{\partial\Gamma_i}\tilde{q}\,\delta T ds$$
$$=\sum_{i=1}^{n}\int_{\partial\Gamma_i}\frac{\tilde{q}_i+\tilde{q}_j}{2}\delta T_i ds=\sum_{i=1}^{n}\frac{\tilde{q}_i+\tilde{q}_j}{2}\delta T_i l_i$$

3.4 Process of the Convective Heat Transfer Coefficient

In the term of heat transfer $\int_{\partial\Gamma}\tilde{q}ds$, $\tilde{q}=h\frac{T-T_{ref}}{b}$ where h is the convective heat transfer coefficient , b is the radius of the U-bent tube.

Determine the value of the convective heat transfer coefficient h by defining the Nussellt number [6]:

$$h=Nu\frac{\lambda}{d},\ Nu=N_1+N_2\,\mathrm{Re}^{N_3}\,\mathrm{Pr}^{N_4}$$
$$\mathrm{Re}=\frac{ud}{v},\ \mathrm{Pr}=\frac{v}{D}=\frac{v\rho c}{\lambda}$$

Nu (Nussellt, Nusselt number) number reflecting the strength of the convective heat transfer ,and it is the dimensionless temperature gradient of the heat exchange surface.Prandtl number and Reynolds number are the general parameters in the fluid mechanics , Re number used to differentiate laminar flow from turbulent fluid , related to u, the flow velocity of the fluid, d (the pipe diameter) and v (the kinematic viscosity). Pr number indicating the relative ratio of the momentum diffusion and the heat diffusion by the molecules in the fluid, related to D ,the thermal diffusion coefficient of the fluid and, the kinematic viscosity.

In the experiment, $Re = 0.177 \times 0.04 \times 1000 / 4.7 \times 10^{-4} = 15064 > 4000$ representing the turbulent fluid ,it is a forced convection heat transfer problem, which can be expressed through Dittus-Boelter formula [2]: $Nu = 0.023 Re^{0.8} Pr^{0.3}$,Dittus-Boelter formula can be applied in the scope where the temperature difference between water and the environment is within 30℃. Since the temperature difference between water and the soil mass in our example is around 40℃, it would be feasible to increase the Nu number moderately. Finally the values of the Nu number are separately N1=0, N2=0.03, N3=0.8 and N4=0.33.

4. Illustrative Examples

4.1 Example 1

In consideration of the experiment to calculate the thermal response of a single pile in the soil layer, the soil layer has a radius of 1.2m at a depth of 35m, the pile is calculated with a radius of 400mm at a height of 30 meters, the heat pipe within the pile has a radius of 20mm with the height around 27 meters and the flow velocity of the fluid inside the heat pipe is 0.177m/s with the inlet temperature at 60℃ and the ambient temperature at 20℃.

The thermophysical parameters of the soil layer, the pile and the fluid within the pipe are given in the table as below.

Table 1. Calculation of the Thermophysical Parameters

	Density (kg/m^3)	Specific heat capacity (J/kg•K)	Thermal conductivity (W/m•K)
Fluid (Water)	1000	4200	0.65
Concrete (Pile)	2400	840	1.54
Soil layer	2000	2000	2

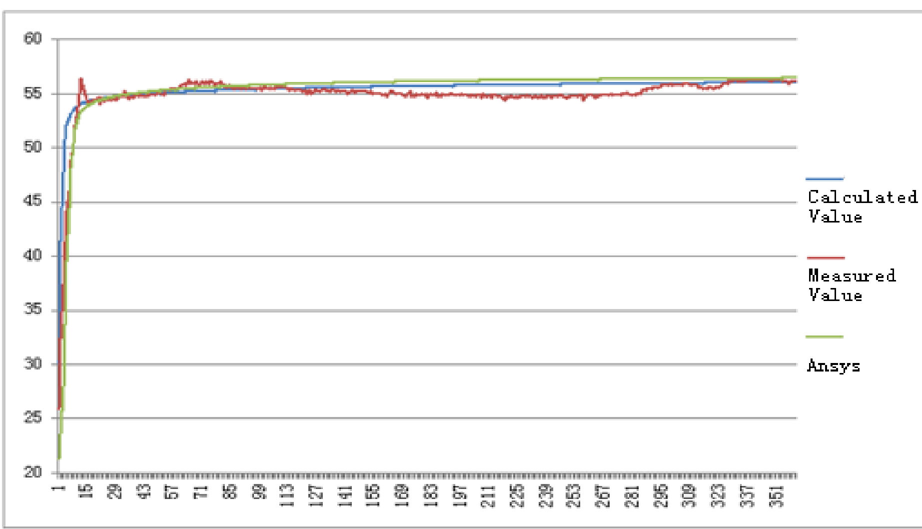

Figure 1. Contrast of Calculation Results without Seepage

Figure 1 gives a data comparison about the outlet temperature of the heat pipes within the pile, demonstrating the contrast of the measured data, the ANSYS and our programmed calculation result. The degree of data matching reveals that the closer our numerical calculation result is to the measured value, the lower the error rate is. It has verified the rationality of our calculation format.

4.2 Example 2

In consideration of the plane physical model for the convective heat transfer in porous media (as indicated in Figure 2), assume that the underground water flows in a 6m*1m soil body, the mean velocity (Darcy velocity) on the section is 1.5e-5m/s and the porosity is 0.3 with the thermophysical parameters as shown in Table 4.2. The initial temperature is 20℃, then the temperature of the admission section is 35℃ when t=0. Except the admission section, all of the other three sides of the soil body are heat-insulation. Values of the main parameters are given as below for the solving process.

Table 2. Relevant Parameters of a Single Drilled Ground Heat Transfer Model

Thermal conductivity of the soil body（W/m•K）	Specific heat capacity of the soil body（J/kg•K）	Density of the soil body（kg/m^3）
2	800	1800
Thermal conductivity of the water（W/m•K）	Specific heat capacity of the water（J/kg•K）	Water density（kg/m^3）
0.6	4200	1000

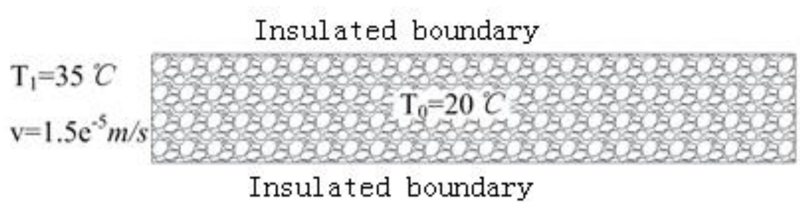

Figure 2. Sketch of Physical Model for Heat Transfer in Porous Media

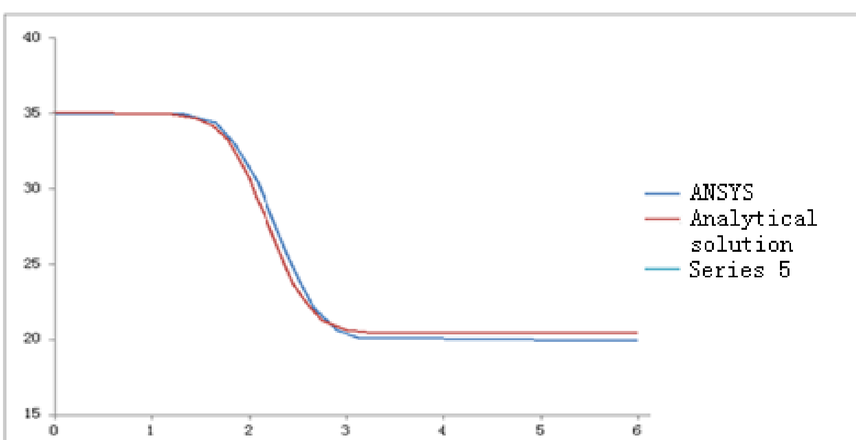

Figure 3. Contrast of Calculation Results with Seepage

Figure 3 shows the contrast of the analytical solution, the ANSYS and our calculation result, revealing clearly that the calculation result through our format is relatively closer to the analytical result.

4.3 Example 3

When considering the physical model, make sure to choose those representative bored piles in the experiment. Connect the buried double U-bent tubes (or the single W-type tubes) in series within the pile and strap the heat exchange tubes along the inside of the reinforcement cage. Plan to choose such test post with a diameter of 700mm and a length of 43m-44m. The outside diameter of the heat pipe is 36mm, while the inside diameter is 26mm. Also assume that the initial temperature of the soil layer is 16.3℃ and the underground water flows in this layer. The underground water is distributed in the region that is between 10.83m and 18.28m under the ground with the mean velocity (Darcy velocity) on the section up to 1.39e-6m/s and a porosity of 0.3. Soil body has a mean density of 1940 kg/m3 with a specific heat capacity of 1400 J/kg·K and a thermal conductivity of 1.53 W/m·K. Then calculate the concrete based on C40 concrete, whose density is 2400kg/m3, the specific heat capacity is 960J/kg·K and the thermal conductivity is 2.3W/m·K. Also assume that the flow rate within the heat pipe is 0.8m3/h, then adopt the method of constant power in the experiment for the thermal response of buried pipes in the piles under the condition that the temperature difference is set as 2.5℃.

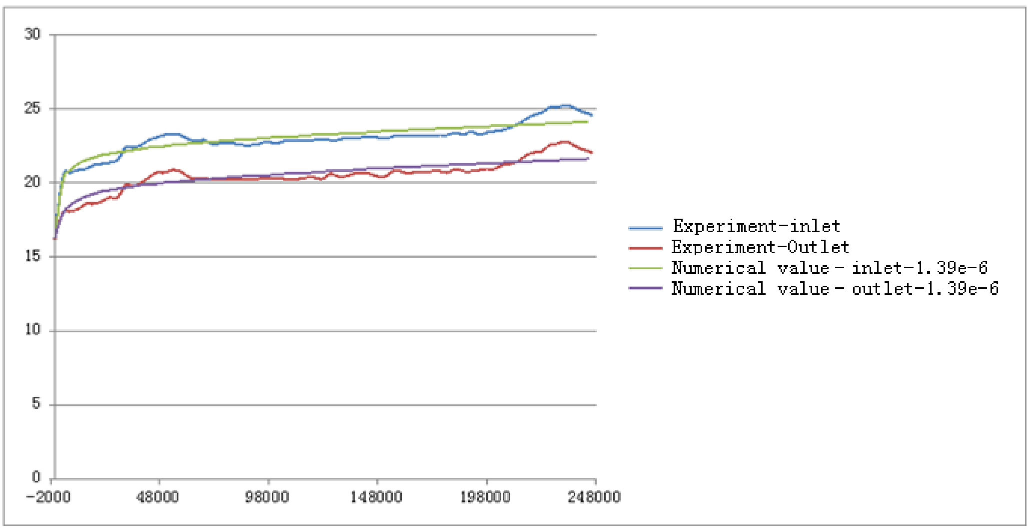

Figure 4. Contrast of the Experimental Data and the Numerical Results

Figure 4 gives the contrast result of the experimental data and the numerical results, comparing separately the data about the inlet and outlet of the heat pipe. On the whole, it shows that the numerical calculation result has coincided exactly with the experimental result, proving the rationality of our calculation format.

Velocity	Outlet temperature
0	21.80981
5.00E-08	21.64365
1.00E-07	21.64409
3.00E-07	21.64583
3.64E-07	21.64618
5.00E-07	21.64662
1.39E-06	21.64081
2.00E-06	21.62838
3.60E-06	21.56536
5.00E-06	21.48284
8.20E-06	21.27748
5.00E-05	20.73053

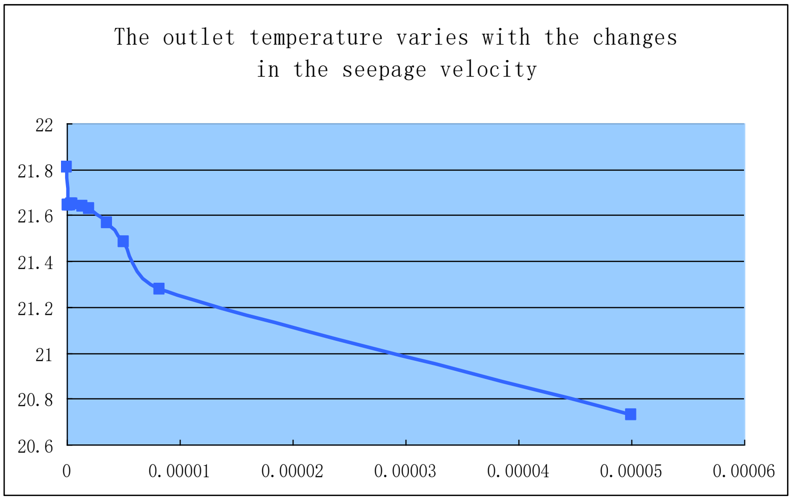

Figure 5. Relationship between the Outlet Temperature of the Heat Pipe and the Seepage Velocity

We have calculated various seepage velocities one by one with the data as shown in Figure 5, which reveals that when the flow velocity increases, the outlet temperature of the heat pipe would decrease constantly, reflecting that more heat would be taken away by the underground water when it flows more quickly, indicating also that the effect of the heat transfer between the fluid inside the heat pipe and the pile is getting better.

5. Conclusion

This paper provides a new finite element-based solution format for piles and U-bent tubes,

makes a calculation on the term of heat transfer between the pile and the U-bent tube through the point integration method,and defines the value of the convective heat transfer coefficient h through the definition of the Nussellt number. Its rationality has been proved through the illustrative examples. In this way, the model for the convective heat transfer in porous media will be improved to increase its calculation accuracy. In our future researches, we shall try to further improve this model through the introduction of more practical examples ,and make it play its role in the real application.

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