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Research on Traffic Congestion Mathematical Model in Traffic Signal Control System

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Abstract

Based on the thought of Logistic cumulative probability, this paper establishes the traffic state probability model under snow-ice condition on artery, and puts forward the corresponding parameters calibration method and model evaluation method; finally, using traffic flow survey data under snow-ice condition on Huanghe road and Hongqi street in a city to analysis and forecast traffic state, put forward traffic state prewarning index, and combining the analysis results, present the corresponding traffic management strategy and guidance suggestion.

***Keywords**: urban artery, state probability model, snow-ice conditions, traffic flow characteristics*

1. Introduction

The randomness of traffic flow leads to uncertainties of traffic state [1-5]. Here we introduce probability analysis method to construct from the angle of traffic prediction the probabilistic model for identifying the traffic flow running state on urban trunk roads under ice and snow conditions, describing quantitatively the change rule of such state [6-7].

Traffic state is the self-organized presentation of traffic flow phenomena. The features of traffic flow are important indicator to reflect such presentation [8-10]. So studies on the change law of traffic state under ice and snow conditions must be done by using feature parameters of traffic stream as breakthrough point. Mutual mapping relationship exists essentially between those parameters and traffic flow state. The variation of traffic flow states is macroscopic manifestation of the change of traffic flow features; while the change of those features will definitely result in the cumulated quantitative change of traffic state; finally the qualitative change causes the change of traffic flow state [11].

Based on the above discussion, we proposed a traffic state probability model, which uses urban road traffic flow testing data to predict the traffic flow running state in certain time range. The model can describe the likeliness to have changing traffic state and its change law, which will work better on the transportation system.

2. Traffic State Probability Model Research Ideas

2.1Probability Analysis Method for State Variable

When dependent variable is classified variable e.g. traffic state, cumulative logistic regression analysis model is a significant method used to find out the occurring possibility of dependent variable which has order relation subject to the combined effect of several independent variables [12-13]. The regression model between categorical variable and influential factors is built as:

$$y = a + \beta x + \varepsilon \qquad (1)$$

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$$y^* = a + \beta x \quad (2)$$

Generally, after the distribution function for dependent variable and independent variable is determined, it's possible to get the cumulative probability of ordering variable. Suppose distribution function $F(\bullet)$; then we can get the cumulative probability after the combination of independent variables like:

$$\begin{aligned} P(y \le j \mid x) &= P(y^* \le \mu_j) = P[(\alpha + \beta x + \varepsilon) \le \mu_j] \\ &= P[\varepsilon \le \mu_j - (\alpha + \beta x)] \\ &= F[\mu_j - (\alpha + \beta x)] \end{aligned} \quad (3)$$

When the dependent variable has many possible states, the probability of occurrence of the j state is:

$$P(y = j \mid x) = P(y \le j \mid x) - P(y \le j-1 \mid x) \quad (4)$$

2.2 Analysis by the Probability Model for Predicting Traffic State

During the study about how dependent variable is affected by independent one, when dependent variable is multi-class ordering variable, the research is transformed to focus on the probability of each status, which is a new effective research thinking. Traffic flow state is the actual morphology under the composite effect of traffic flow feature parameters and road environment. Road traffic state is a multi-class independent variable. Its influential factors are acting very complicatedly. So we don't think it's appropriate to use the least square method to regress a model, but preferably to analyze classified variables with non-linear function. The change of traffic flow running state leads to orderly transition of different traffic flow state; that is, definite consecutive order relation exists between smooth state, moderately congested state, congested state and heavily congested state. The cumulative probability model should be the commonest and most proper approach to analyze the ordering dependent variable.

Cumulative probabilistic model transforms classified variable analysis problem to the analysis of any possible occurrence of events. In the paper we draw lessons from the basic idea of logistic model analysis method to discuss the probability of traffic flow state changing in icy and snowy circumstances. To put it simple, we talk about the probability of traffic flow being smooth, moderately congested, congested and seriously congested, under the combined effect of traffic flow parameters.

From probability theory, we know that state probability should be continuous curve in (0,1). Traffic states are of continuously changing order classifying relation. We use y=1,2,3,4 to stand respectively for smooth, moderately congested, congested and heavily congested state; $P(y \le j)$ means the summation of probability of the first j^{th} occurring traffic state, i.e. $P(y \le j)$ is the cumulative probability of traffic states.

$$P(y \le j) = P(y = 1) + P(y = 2) + \ldots + P(y = j) \quad (5)$$

Based on the analysis of traffic state parameters in the model, we find linear relationship between traffic state parameters and traffic states. Assume the relationship is described in the following expression:

$$y = f(x) = a + \beta X \quad (6)$$

Considering the above equation is function relation between y and x, to find out the percentage of traffic state changing because of the combined effect of traffic flow parameters, we convert the above equation into:

$$P(y^*) = \frac{e^{f(x)}}{1 + e^{f(x)}} \in (0,1) \quad (7)$$

According to statistical sciences, $P(y^*)$ continues along with $e^{f(x)}$ and increases monotonically, whose value domain is (0,1); then we get $P(y^*)$ is the distribution function of $e^{f(x)}$, marked as $F(\bullet)$. Under the combined effect of given traffic flow parameters, the cumulative probability of traffic states is:

$$P(y \le j|x) = P(y^* \le \mu_j) = F[\mu_j - f(x)] = \frac{e^{[\mu_j - f(x)]}}{1 + e^{[\mu_j - f(x)]}} \quad (8)$$

$$1 - P(y \le j|x) = \frac{1}{1 + e^{[\mu_j - f(x)]}} \quad (9)$$

Further, to obtain traffic state probability:

$$\ln\left(\frac{P(y \le j|x)}{1 - P(y \le j|x)}\right) = \mu_j - f(x) = \mu_j - (\alpha + \beta X) \quad (10)$$

In the traffic state probability model, the solution of the model parameters is generally nonlinear. Use the actual survey data based on a certain parameter estimation method to calibrate the model parameters.

2.3. Estimation of Parameters of Traffic State Probability Model

After model structure and dependent variable are decided, it's required to determine the model's regression parameters. We use maximum likelihood method calibrate those parameters.

Assume there are n random samples $(y_1, y_2, ..., y_n)$. Make $P(y_j = 1 | x_j)$ the conditional probability of the state y=1 after the condition of x_j is given. Then we can get the possibility of traffic state having change:

$$P(nj) = p_j^Y (1 - p_j)^{n-Y} \quad (11)$$

With the calibration result of parameters of the probability model for predicting traffic state, we can have the probability model such alike under ice and snow condition.

3. Precision Evaluation of the Traffic State Probability Model

Once model's structure dependent variables are chosen and parameters are calibrated, the model is basically established. But it's necessary to evaluate whether the model can work precisely and effectively in practical use, i.e. precision evaluation. The assessment of model precision is an important step to guarantee the model's rationality, accuracy and practicability.

3.1. Evaluation of the Model's Fitting Degree

In statistics, Pearson χ^2 statistical magnitude is one common way to check if regression analysis results are reasonable. In the paper we utilize Pearson χ^2 statistical method to evaluate the fitting of the cumulative probability model, as done in the following way:

$$\chi^2(J-1) = \sum_j^J \frac{(Y_j - NP_j)^2}{NP_j} \quad (12)$$

3.2. Evaluation of the Model’s Prediction Precision

The evaluation here of the model’s prediction precision relies on how much the prediction results of traffic state approximate to the actual traffic state. That is an intuitive and effective way used often to appraise the model’s accuracy. We use odds ratio to examine the prediction precision of the model.

By the probability model for traffic state, we can know the possibility of traffic state i happening is:

$$P(i) = P(y = i \mid x) = \frac{e^{f_i(x)}}{1 + \sum_{r=1}^{J} e^{f_r(x)}} \tag{13}$$

The probability of occurrence of traffic state i is :

$$1 - P(i) = P(y \neq i \mid x) = \frac{1}{1 + \sum_{r=1}^{J} e^{f_r(x)}} \tag{14}$$

4. Experiment Design and Discussion

We use some sections on Huanghe Road and Hongqi Street in one city as study target to validate the probability model’s accuracy and availability.

4.1Data Survey

For the purpose, we chose the area between upstream and downstream stop line of urban trunk road segment as the field of study. We need to collect data in that field under ice and snow condition such as road traffic friction coefficient, time sequence of traffic flow velocity and density on average in each lane, vehicle queuing length on average at the final end of each lane, and record time interval in each statistics.

We chose the section between Huanghe Road section (Huangshan Road to Songshan Road) and Hongqi Street section (Liaohe Road to Huaihe Road) as study field to examine traffic state of one city in icy and snowy cases with the proposed model. Of that, Huanghe Road is two-way six-lane, isolated with central barriers; Hongqi Street is also two-way six-lane, at each side with auxiliary roads which are separated by green belts, but without separators. The location of the examined field is described hereunder. It is shown in fig1-2.

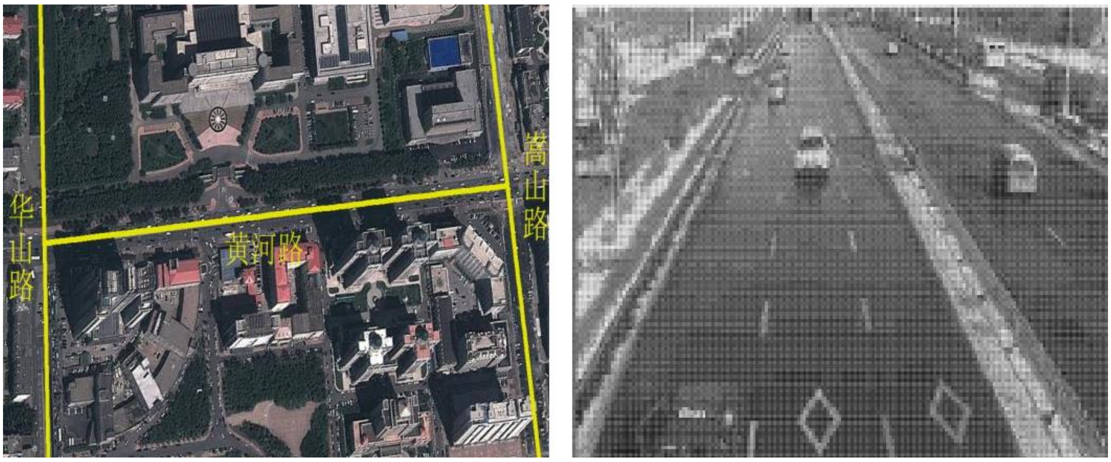

Figure 1. Location and Investigation of the Huanghe Road Section in One City

4.2. Data Processing

4.2.1 Traffic Flow Speed. We observe the time when vehicles go to and out of the field by checking car plate or type and color from two cameras; then with reference to the length of the research field, we can get vehicle speed. Speed data fetched directly from videos cannot be utilized directly. The time difference of one vehicle at two cross sections is important information which can be used to calculate the traffic flow speed. If the time difference is big and largely different from other vehicles', it may not be normal driving speed. In this case, there're two causations:

(1) one car paused in the field maybe because of getting things done, incident etc.; for the moment, the car's running state can't be indicative of real traffic flow state, which should be discarded;

(2) some videos were missed from detection or car came back to the place after driving away; the time difference fetched for that moment should be outcome of it going back and forth for several times; however it's not possible to know the car's driving speed and thus it's useless data. To ensure the quality of speed data, in light of great variations in the driving speed of vehicles in different traffic states on frozen days, we take method as in (15) to remove useless data with huge discrete degree in the same category. In the same class, speed data sufficing the above equation are retained, without invalid ones.

$$|(v_i - \bar{v} / \bar{v}| \le 3\sigma \quad (15)$$

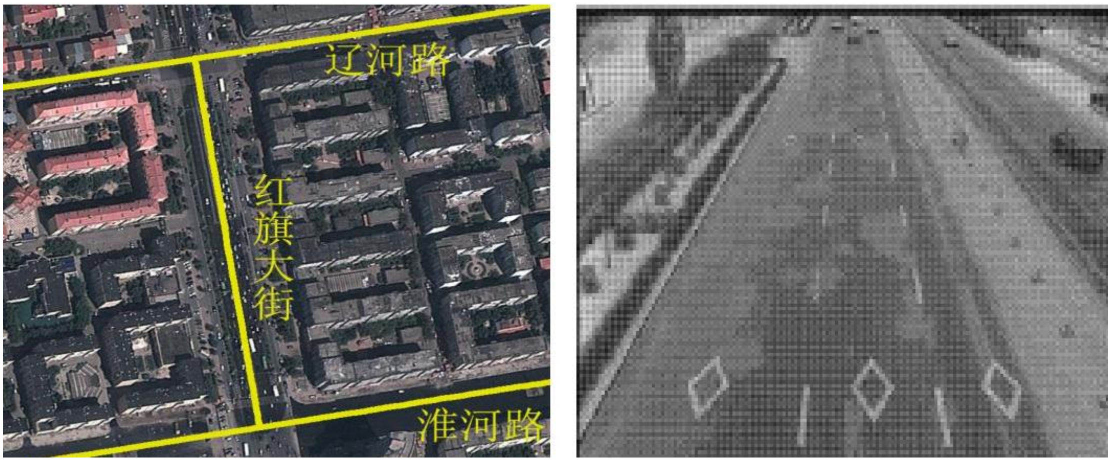

Figure 2. Location and Investigation of the Hongqi Street Section in One City

After selecting the research section, according to the survey method, the paper chooses the position of the two ends of the section to choose the good attachment to the camera. The traffic flow of the corresponding section is monitored, and the friction coefficient is measured, and the length of the section is measured.

4.2.2 Density of Traffic Flow. With traffic volume in the research area, we can compute the density of traffic stream.

4.2.3 Queuing Length. Demarcate in videos with border trees to tag the queuing position of vehicles. Measure at site the distance among border trees and the queuing length of vehicles can be obtained.

In actual investigation, the time interval for Huanghe Road is 3mins and 5mins for Hongqi Street. After data investigation and processing, we get statistical results as follow in talbe1-2.

Based on studies about traffic flow state parameters under ice and snow conditions, we process investigation data here and transform them as to get like:

Table 1. Statistical Results of Traffic Flow Parameters (Huaihe Road)

number	Traffic state	φ	v_t (km/h)	p_t (pcu/km)	l_t (m)	t (min)
1	--	0.62	60.83	30.47	113.5	--
2	--	0.62	62.56	36.01	87.2	6
3	2	0.62	58.66	41.55	129.6	3
4	2	0.62	61.22	44.32	152.4	3
5	2	0.62	60.45	47.09	164.7	6
6	2	0.38	39.63	53.18	140.8	3
7	3	0.38	23.54	55.4	176.2	9
8	3	0.38	21.37	57.62	188.2	6
9	3	0.38	29.86	48.75	192.0	3
10	2	0.38	32.63	53.18	203.2	6
11	2	0.38	23.54	55.4	246.0	6
12	2	0.13	24.79	64.26	264.3	12
13	3	0.13	19.21	57.62	268.5	15
14	3	0.13	20.34	59.83	230.0	12

Table 2. Statistical Results of Traffic Flow Parameters (Hongqi Street)

number	Traffic state	φ	v_t (km/h)	p_t (pcu/km)	l_t (m)	t (min)
1	--	0.38	43.95	31.02	116.2	--
2	--	0.38	42.42	33.24	134.5	5
3	2	0.38	42.07	35.46	99.7	15
4	2	0.38	37.87	37.67	128.0	5
5	3	0.38	23.54	55.4	133.6	10
6	3	0.40	21.37	57.62	109.5	10
7	3	0.40	21.08	59.83	115.2	15
8	2	0.40	35.92	39.89	88.9	5
9	2	0.38	39.63	53.18	84.0	20
10	3	0.38	23.54	55.4	129.0	10
11	3	0.38	21.37	57.62	157.3	15
12	3	0.38	29.66	59.83	148.2	20
13	2	0.38	38.61	44.32	113.3	15
14	2	0.38	34.59	46.54	104.5	10

With the results of calibrated parameters of traffic state probability prediction model, the probability prediction model of traffic state can be acquired like:

$$\ln\frac{P_1}{P_2+P_3+P_4}=0.098-1.708\varphi+1.982\frac{v_t}{v_0}+1.377\frac{v_{t-1}}{v_0}+0.826\frac{v_{t-2}}{v_0}-3.247\frac{\rho_t}{\rho_0}$$
$$-3.025\frac{\rho_{t-1}}{\rho_0}-1.228\frac{\rho_{t-2}}{\rho_0}-6.248\frac{l_t}{L}-0.076\frac{t}{\Delta t} \quad (16)$$

4.3. Analysis of Results

4.3.1Analysis of the Model's Fitting Degree. Fitting degree is magnitude used to verify if results got by the model match with or approximate historical data. A better degree of fitting is good criterion for assured precision of the model. After the fitting of traffic flow data on roads mentioned above under icy conditions, we get results as follows in table3:

Table 3. Results of Fitting Degree of Traffic State Probability Model

number	$\chi^2_{0.05}(3)$	Huaihe Road		Hongqi Street	
		Fitting degree $\chi^2(J-1)$	Fitting results	Fitting degree $\chi^2(J-1)$	Fitting results
1	7.815	3.181	Satisfying (excellent)	3.126	Satisfying (excellent)
2		7.605	Satisfy (better)	4.861	Satisfying (excellent)
3		7.605	Satisfy (better)	6.660	Satisfy (better)
4		7.610	Satisfy (better)	6.836	Satisfy (better)
5		7.433	Satisfy (better)	7.802	Satisfy (better)
6		7.127	Satisfy (better)	7.686	Satisfy (better)
7		6.812	Satisfying (excellent)	5.559	Satisfying (excellent)
8		6.615	Satisfying (excellent)	7.251	Satisfy (better)
9		7.129	Satisfy (better)	6.385	Satisfy (better)
10		4.385	Satisfy (better)	4.962	Satisfying (excellent)
11		5.332	Satisfying (excellent)	3.268	Satisfying (excellent)
12		5.687	Satisfying (excellent)	5.194	Satisfying (excellent)
13		7.615	Satisfy (better)	5.029	Satisfying (excellent)
14		6.302	Satisfy (better)	6.271	Satisfy (better)
15		5.232	Satisfying (excellent)	--	

From them, we see the prediction model has better fitting performance, possibly in good match with the actual traffic flow state in snowy circumstances.

4.3.2Analysis of the Model's Prediction Precision and Warning of Traffic State

The model's fitting degree helps it realize good match with historical data. Its prediction precision is a sign of representation model to indicate the change tendency of traffic flow state and evaluation criterion to ensure the model's practicability and correctness. We create the possibility prediction model of traffic state based on historical survey data got from mentioned roads under icy and snowy conditions. For the time interval $t/\Delta t=1$、$t/\Delta t=2$、$t/\Delta t=3$、$t/\Delta t=5$、$t/\Delta t=10$、$t/\Delta t=15$, we select four groups of data to predict traffic state and examine the model's precision as per changing results of actual traffic state, which are found hereunder. It is shown in table4.

The model's results match better with the actual traffic state, and the same with predictive time in one statistic cycle (3mins for Huanghe Road; 5mins for Hongqi Street). With increasing predictive time, the model's prediction results have certain errors. According to change rule of traffic state odds ratio p, when the changing curve goes up, the model's prediction results conform better to the factual case; at the bottom of the curve, the results have some errors; when P becomes descending, it suggests that traffic flow changes from one state to another, i.e. the change rule of odds ratio is warning signal for the change of traffic state.

Table 4. Accuracy Test Eesults of Traffic State Probability Model

Number	Data source	Time interval t/Δt	Model prediction results				Predictive state	Actual traffic condition	$\frac{P(i)}{1-P(i)}$
			P_1	P_2	P_3	P_4			
1	Huaihe Road Tenth data	1	0.20	0.55	0.24	0.01	2	2	1.22
		2	0.22	0.53	0.24	0.01	2	2	1.13
		3	0.21	0.52	0.26	0.01	2	2	1.08
		5	0.18	0.49	0.32	0.01	2	2	0.96
		10	0.13	0.45	0.39	0.03	2	3	0.82
		15	0.09	0.34	0.54.	0.03	3	3	1.17
2	Huaihe Road Fifteenth data	1	0.01	0.25	0.42	0.32	3	3	0.72
		2	0.01	0.25	0.40	0.34	3	3	0.67
		3	0.01	0.22	0.38	0.39	4	4	0.64
		5	0.01	0.20	0.36	0.43	4	4	0.75
		10	0.01	0.14	0.39	0.56	4	4	1.27
		15	0.00	0.10	0.23	0.67	4	4	2.03
3	Hongqi Street Tenth data	1	0.02	0.24	0.45	0.29	3	3	0.82
		2	0.01	0.27	0.44	0.28	3	3	0.79
		3	0.01	0.31	0.42	0.26	3	3	0.72
		5	0.01	0.37	0.39	0.23	3	3	0.64
		10	0.01	0.51	0.32	0.16	2	2	10.4
		15	0.03	0.60	0.26	0.11	2	2	1.50
4	Hongqi Street Fourteenth data	1	0.18	0.52	0.26	0.04	2	2	1.08
		2	0.20	0.51	0.25	0.04	2	2	1.04
		3	0.24	0.49	0.24	0.03	2	2	0.96
		5	0.30	0.45	0.22	0.03	2	2	0.82
		10	0.46	0.35	0.17	0.02	1	2	0.85
		15	0.59	0.27	0.13	0.01	1	1	1.44

5. Conclusion

We discussed traffic flow features and illustrated the traffic flow state probability model with practical case about the traffic in ice and snow weathers. Traffic survey data were collected on Huanghe Road and Hongqi Street and used for the experiment here. Based on actual observed data, we built the real-time probability model of traffic state under such condition and evaluated its fitting degree and prediction accuracy. The warning feature of the model against evolving of traffic flow state provides foundations to traffic management and dispersion. Lastly, with reference to research results, we proposed measures regarding traffic flow guidance and control and management of urban trunk roads on snowy days, to enable secure, smooth and effective operation of urban transportation under such conditions.

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