

Prediction of Solar Global Radiation in Bogotá Colombia Based on Mathematical Models

Nicolas Marrugo¹ and Dario Amaya²

¹Research assistant, ²Main researcher of GAV research group of Nueva Granada Military University
{¹tmp.nicolas.marrugo, ²dario.amaya}@unimilitar.edu.co

Abstract

Nowadays, sub developed countries are presenting a growing tendency of technological implementation based on renewable energy. For which is very common to make meteorological factors predictions, to guarantee a better performance in accord to the climate variability. With this work the validation of the global solar radiation predicting models involving geographic and / or meteorological factors for the city of Bogota DC is made. This validation is based on the comparison of the results from the models with the experimental data and NASA, IDEAM, SWERA, METEONORM, CAR data bases. The results obtained from this work identify the error of the used models, defining the most appropriate one to predict the global solar radiation including the location and weather of the northern area of Bogota.

Keywords: Global solar radiation, Prediction algorithm, Mathematical models

1. Introduction

Nowadays, all countries worldwide have as main environmental objective, reduce the greenhouse gases emissions. One of the most used alternatives to achieve this goal, is the implementation of renewable energy. Proof of this, is that in 2014 this type of energies supplied 657 Gigawatts that represents the 19.1% of the global energetic consumption [1]. Among the countries that contribute more in renewable energy generation are China, United States, Germany, Italy and Spain [2, 3].

The type of renewable energy that has been implemented more frequently in the last few years is solar energy, with an increase in its usage of 40% in the past 10 years [4]. Being Germany, Italy, China and Japan the more developed countries in solar energy production. For that reason, global solar radiation is considered as one of the most important parameters in meteorology for being a useful factor in the implementation of photovoltaic systems [5].

The importance of solar radiation lies in that by knowing its value for a specific area, allows the appropriate design of energy systems for buildings and photovoltaic systems. The environmental thermic evaluation even helps to create green architectural and interior designs [6], [7].

Due to its importance, many tools were developed to predict the solar radiation for a specific location, taking into account the environmental characteristics of the area. Some examples are the applications such as SWERA or METEONORM software, based on mathematical models to make the prediction of meteorological variables [8], [9]. These models have a degree of error, which increases or decreases depending of the geographical location.

Unlike developed countries, in Latin America, atlas of the meteorological variables are used as a prediction tool for solar radiation. These atlas use mathematical models based on meteorological data collected from stations and measurements of each country. An example of these tools, are the “atlas de radiación solar de Colombia” of the year 2005,

“atlas de fuente solar brasileño” of the year 2002 or “mapas de radiación solar global en Argentina” of the year 2005[10], [11], [12].

In order to make predictions of solar radiation in Colombia, the angstrom model was implemented by the IDEAM in the year 2005. The collected data was compressed in the “atlas de radiación solar de Colombia”, but due to the global warming these predictions of meteorological variables have changed for the year 2015.

This paper proposes the validation of 13 mathematical models, some of which take into account meteorological factors. To do the validation, the models are compare with databases from NASA, METEONORM, SWERA, IDEAM and CAR. With the purpose of identifying the most relevant models in order to make a global solar prediction in Bogota, Colombia.

2. Methodology

In order to make the prediction of global solar radiation in Bogotá, the 13 different models were classified in those that involve geographical location, those that use location with temperature or relative humidity and finally those that implicate a combination of all of the previous variable. The groups, according to the factors used in the global solar radiation calculation, can be seen in Figure 1.

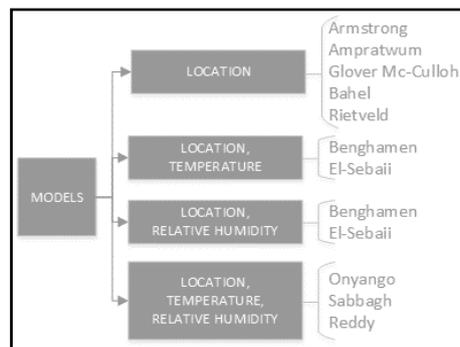


Figure 1. Mathematical Models Classification

Before using the models to calculate de global solar radiation, first the angular position (α), solar declination (δ), distance from the sun to the earth $\left(\frac{R}{R_0}\right)^2$ and latitude (φ) must be define. With these parameters the hour angle from the solar declination and the latitude using equation 1 is calculated, where the latitude is 4.58 grades corresponding to Bogota city.

$$\omega = \cos^{-1}(-\tan^{-1} \varphi \tan \delta) \quad (1)$$

All previous parameters are used by the selected models to predict the global solar radiation in Bogota.

2.1. Models without Meteorological Factors

The models without meteorological factors take into account astronomic and geographic variables. The astronomic parameters used are the astronomical duration of the day (N)and the extraterrestrial solar radiation(H_0). The geographic factors are average global radiation (H)and the number of hours of solar brilliance(n).

The first model was develop by Angstrom in 1990, it is the base to calculate solar radiation and has been used to develop future models. It involves the average monthly values of irradiation and solar brilliance as shown in equation 2[11].

$$\frac{H}{H_0} = a + b \left(\frac{S}{S_0} \right) \quad (2)$$

The coefficients (a and b) are constants obtained from the irradiation data with average values corresponding to 0.18 and 0.62 respectively[12].

The term (S) refers to the number of hours of solar brilliance measured on the surface; (S₀) is the duration of the day calculated from the astronomical considerations which is given by equation 3 [13].

$$S_0 = \frac{2}{15} (\cos^{-1} \tan \varphi \tan \delta) \quad (3)$$

The term (H) is the daily global solar radiation on the surface; (H₀) is the extraterrestrial daily global solar radiation for a determined day on the desired location and is calculated by equation 4.

$$H_0 = \frac{24}{\pi} I_{sc} \left(\frac{R}{R_0} \right)^2 \left(\cos \varphi \cos \delta \sin \omega + \frac{2\pi\omega}{360} \sin \varphi \sin \delta \right) \quad (4)$$

Where $I_{sc} = 1353(W/m^2)$ is the solar constant according to NASA.

The second model, shown in equation 5, derives from the angstrom model. It was developed by Ampratwum[14].

$$\frac{H}{H_0} = 0.6376 + 0.2490 \log \left(\frac{S}{S_0} \right) \quad (5)$$

The third model was developed by Glover and Mc-Culloch where they added the latitude to the angstrom model, represented by equation 6 [15].

$$\frac{H}{H_0} = 0.29 \cos \varphi + 0.52 \left(\frac{S}{S_0} \right) \quad (6)$$

The fourth model, presented in equation 7, was estimated and developed by Bahel. The estimation was made through the recompilation data of hours of sun light and global solar radiation from 48 different locations around the world.

$$\frac{H}{H_0} = 0.16 + 0.87 \left(\frac{S}{S_0} \right) + 0.61 \left(\frac{S}{S_0} \right)^2 + 0.349 \left(\frac{S}{S_0} \right)^3 \quad (7)$$

The fifth model was develop by Rieltveld, which determined the average values of coefficients (a and b) affected by the local climate conditions. Taking the previous into account, is possible to express (a) as a lineal factor and (b) as an hyperbolic factor forming the equation 10 [16].

$$a = 0.10 + 0.20 \left(\frac{S}{S_0} \right) \quad (8)$$

$$b = 0.38 + 0.08 \left(\frac{S}{S_0} \right)^{-1} \quad (9)$$

$$\frac{H}{H_0} = a + b \left(\frac{S}{S_0} \right) \quad (10)$$

2.2. Models with Meteorological Factors

The models with meteorological factors include astronomic, geographic and meteorological variables. The astronomic parameters used are the astronomical duration of the day (N) and the extraterrestrial solar radiation (H_0). The geographic factors are the average global radiation (H) and the number of hours of solar brilliance (n). Finally, the meteorological factors are the daily average temperature (T), daily maximum temperature (T_{max}), daily minimum temperature (T_{min}) and daily average relative humidity (R).

The sixth, seventh and eighth models were developed by El-Sebaei. These were based in the angstrom model structure but contemplating the daily average temperature, the daily average relative humidity and the variation of temperature during the day. These models are represented by equations 11, 12, 13 respectably [17].

$$\frac{H}{H_0} = -1.92 + 2.60 \left(\frac{S}{S_0} \right) + 0.006T \quad (11)$$

$$\frac{H}{H_0} = -1.62 + 2.24 \left(\frac{S}{S_0} \right) + 0.332R \quad (12)$$

$$\frac{H}{H_0} = -0.08 + 0.21(T_{max} - T_{min})^{0.5} - 0.012 \left(\frac{S}{S_0} \right) \quad (13)$$

The ninth and tenth models were developed by Benghanem, also based on the angstrom model structure, but included the daily average temperature, maximum temperature and maximum relative humidity of the day. These models are represented by equation 14 and 15 respectably [18].

$$\frac{H}{H_0} = 0.6369 + 0.037 \left(\frac{T}{T_{max}} \right) \quad (14)$$

$$\frac{H}{H_0} = 0.7556 - 0.1353 \left(\frac{R}{R_{max}} \right) \quad (15)$$

The eleventh model was developed by Onyango, which includes the daily average relative humidity and maximum temperature to estimate the global solar radiation through the equation 16. [19].

$$H = H_0 \exp \left[\varphi \left(\frac{S}{S_0} - \frac{R}{15} - \frac{1}{T_{max}} \right) \right] \quad (16)$$

The extraterrestrial radiation was defined by equation 17.

$$H_0 = (1.7 - 0.458\varphi) \left(\frac{20S_0}{1 + 0.1\varphi} + w_{i,j} \cos \varphi \right) 41.858 \quad (17)$$

The term $w_{i,j}$ is a constant calculated for a given month.

The twelfth model was developed by Sabbagh, which presents the irradiation as a function of climatological and geographical data (Maximum temperature of the day, daily average relative humidity, latitude and longitude). It is represented by equation 18.

$$H = 1.53K \exp \varphi_{radianes} \left(\frac{S}{S_0} - \frac{R^{1/3}}{100} - \frac{L_{radianes}}{T_{max}} \right) \quad (18)$$

The value of K is define by equation 19.

$$K = [\lambda S_0 + \psi_{i,j} \cos \varphi] 100 \quad (19)$$

The symbol λ is a constant that represents the latitude of the location and $\psi_{i,j}$ is the geographic factor of the season, where index i=1 refers to general places and i=2 to places near the sea. The j index corresponds to the number of month, as table 1 indicates [20].

Table 1. Geographical Factor of Season

Month	ψ_1	ψ_2
January	1.28	1.48
Febrerary	1.38	1.77
March	1.54	2.05
April	1.77	2.15
May	2.05	2.05
June	2.3	2.05
July	2.48	2.1
August	2.41	2.17
September	2.36	2.14
October	1.73	1.96
November	1.18	1.6
December	1.17	1.43

The thirteenth model was developed by Reddy. He proposed a complex equation to calculate the daily global radiation in ($Cal/cm^2/day$) contemplating the average daily temperature and average daily relative humidity, as shown in equation 20 [21].

$$H = K \left(0.6 + 0.02T \frac{S}{S_0} - 0.04\sqrt{R} \right) - R(4.3 - \sqrt{T}) \quad (20)$$

2.3. Experimental Measurements

In order to make the proposed comparison using theoretical values of the radiation, temperature and humidity, the corresponding measurements were taken for the models using the following devices. To collect the data the radiometer hd2102 delta ohm was used, which measures the global solar radiation between 0nm and 200nm of spectral range of the irradiance wavelength. The hygrometer hd2101r delta ohm was used to measure the relative humidity and temperature. These 3 values were captured for 30 days, between November 11 of 2014 and December 10 of 2014.

2.4. Databases

Finally, 5 databases were chosen to complete the comparison of the theoretical value for the selected models. At an international level NASA, SWERA, and METEONORM are the most commonly used, and at a national level IDEAM and CAR, generating the values shown in table 2.

Table 2. Meteorological Factors for Bogota Taking from Databases

November de 2014			
Databases	Daily global radiation (Kwh/m^2)	Average relative humidity (%)	Average temperature ($^{\circ}C$)
NASA	8.4	8.1	17-22
SWERA	3.5-4.0	75-80	15-20
METEONORM	4.5	NA	13-18
IEDEAM	3.5-4.0	88-91	18.5-19.5
CAR	3.279	70-80	11-13

3. Results

To analyze the resulting error from the models, in this case in particular, the values for the average daily per month from the radiometer and hygrometer were collected. The monthly average daily obtained was $4.46(Kwh/m^2)$ for the global radiation, 66.32% for relative humidity and 66.32% for temperature during the month of November of 2014 in Bogota.

Based on the above, the software MATLAB® was used to determine the behavior of the daily error of global solar radiation through the selected models and experimental data.

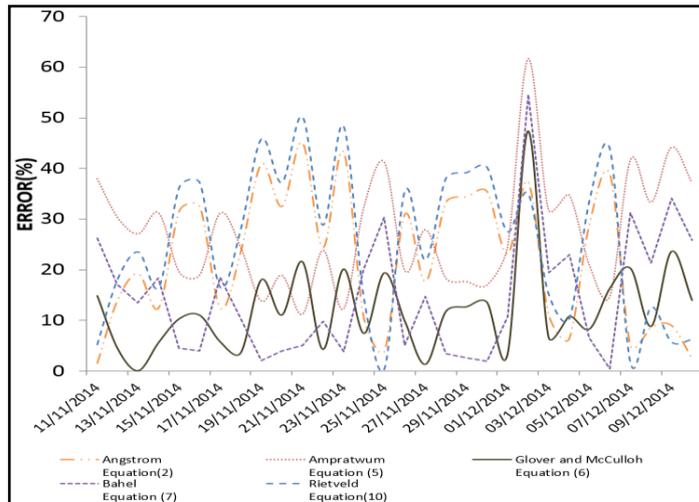


Figure 2. Behavior of Daily Error of the Models without Meteorological Factors

In Figure 2 is possible to highlight the Glover and Mc-Culloch model related with equation (6), due to having the lowest average error with a value of 12.22%.

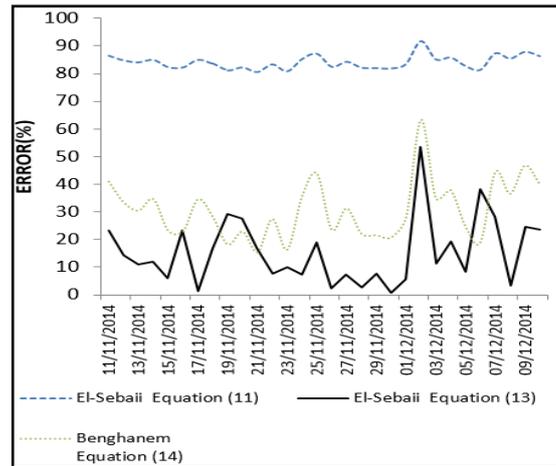


Figure 3. Behavior of Daily Error of the Models Involving Temperature

In Figure 3, The El-Sebaïi model related with equation (13), indicates its importance due to having an average error of 15.45%

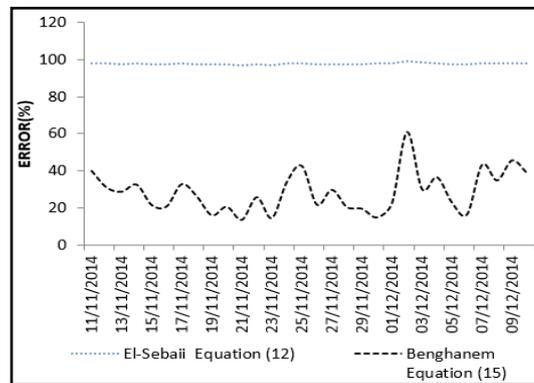


Figure 4. Behavior of Daily Error of the Models Involving Relative Humidity

In Figure 4, the Benghanem model related with equation (15) calls the attention for having an average error of 28.7%

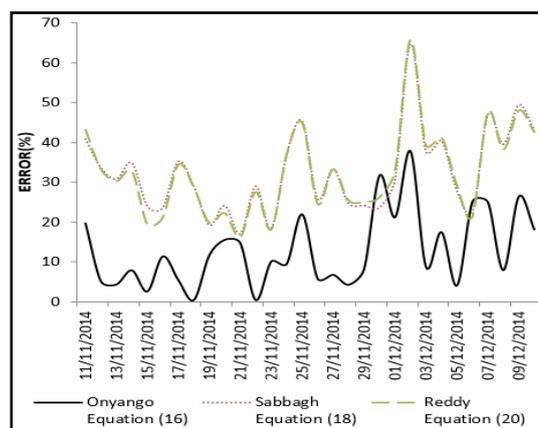


Figure 5. Behavior of Daily Error of the Models Involving Relative Humidity and Temperature

The Onyango model related with equation (16), has a low average error of 12.95%, as can be seen in Figure 5.

After analyzing the daily behavior, the next step was to make the comparison between the daily average global solar radiation of month obtained experimentally and the data obtained theoretically through the selected models. This comparison generates the error shown in Figure 6.

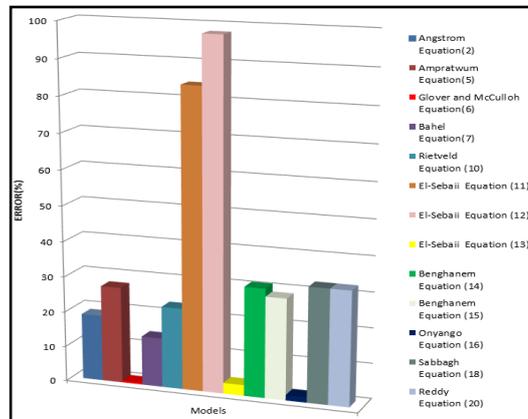


Figure 6. Average Daily Error of Given Month of Selected Models

From the previous Figure, 3 models with an average daily error per month of less than 5% can be noticed. From this analysis it can be highlighted that the Glover and McCulloch (equation 6) model has the least percentage of error with a 0.41%, followed by El-Sebaïi (equation 13) with an error of 3.05% and Onyango (equation 16) with an error of 1.53%. After identifying the most relevant models, a comparison with the national and international databases was performed. The results are presented in Figure 7.

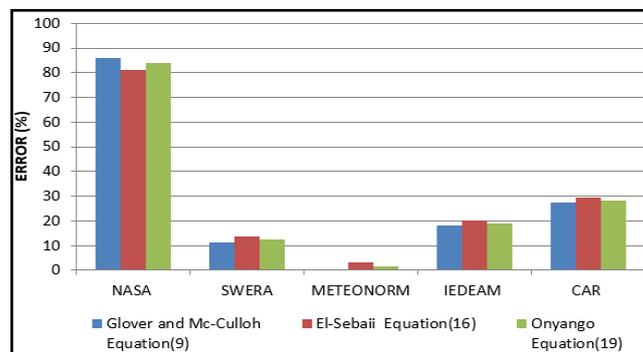


Figure 1. Error of the Most Relevant Models Compared with the Databases

The previous figure shows that the most relevant models have an average error of 1.6% regarding to international software METEONORM and an error of 19.10% regarding to national databases. This errors are shown in Table 3.

Table 3. Average Error of Most Relevant Models Regarding to the Databases

DATABASES	ERROR
NASA	83.64%
SWERA	12.54%
METEONORM	1.61%
IDEAM	19.10%
CAR	28.31%

4. Conclusions

After analyzing the 13 selected models to calculate the global solar radiation in Bogota, Colombia, 3 models were highlighted given the final results. These models were compared with experimental data, and with national and international databases. The previous with the purpose of getting the most accurate prediction of global solar radiation for the year 2014 and onwards.

From the study, the model developed by Glover and Mc-Culloch caught the attention, because it had the minor error of all models. This model does not involve meteorological factors, reason why it can only be used to predict the global solar radiation in general, without taking into account the environmental conditions of Bogota.

The Onyango model is the most important of all models, because it involves meteorological factors as relative humidity and temperature. With these features the model can be used to predict the global solar radiation for specific zones, which is perfect in the case of Bogota due to having different micro climates caused by its urban morphology.

On the other hand, the El-Sebaili model which only involves the variation of temperature to calculate the global solar radiation in Bogota, can only be used to make predictions in a specific period of time. The reason lies in it using the minimum temperature along with the maximum temperature of the day, which makes the prediction to get amplified.

The Onyango is the most relevant model to do the prediction of global solar radiation due to having a margin of error of 1.61% when compared with the software METEONORM, a tool that contains more accurate and updated information than national databases. This model could be used for developing prediction programs of meteorological factors in Bogota. Programs that will serve as an updated information source for support of urban projects based on the environmental conditions of Bogota, Colombia.

Acknowledgments

The authors would like to offer their special gratitude to the Research Vice-chancellorship of Nueva Granada Military University for financing the research project IMP-ING 1576 titled: "Abastecimiento energético autónomo mediante SFRC para áreas de laboratorio en el campus de Cajicá", 2015.

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Authors



Nicolas Marrugo, he received his B.Sc. degree in mechatronics engineering from Piloto University, Colombia, in 2014. Currently, he is a research assistant in GAV research group at Nueva Granada Military University. His research interests include renewal energies, automation system.



Dario Amaya Hurtado, he was educated at UAN, Bogotá, Colombia receiving the B Sc. degree in Electronics Engineering in 1995 and the M.Sc. degree in Teleinformatic in 2007 by the Faculty of Engineering at the Francisco José de Caldas District University, UFJC in Bogotá, Colombia. He was awarded the Ph.D. degree in 2011 in Mechanical Engineering at Campinas State University, São Paulo, Brazil, working on hybrid control – He has worked as a professor and researcher at the Military University, Colombia since 2007.