Statistical Description and Analysis of the Concurrent Data Transmission from Massive MTC Devices

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Abstract

The concurrent data transmission from massive machine type communications (MTC) devices within a short time period makes the traffic flow of MTC more bursty. The classic Markovian traffic models under the assumption of Poisson arrival, whose inter-arrival time (IAT) is a negative exponentially distributed random variable with infinite support range, are no longer suitable for this situation. Beta distribution, a distribution with finite support range, is then proposed by 3GPP to account for this and used to serve as the IAT of MTC. The researches on the traffic modeling based on Beta distribution is still in its very initial step and is rarely reported in state of the art literatures. This article takes the initiatives to discuss some prime issues for MTC traffic modeling under the assumption of Beta arrival. By the usage of renewal theory and Volterra integral equation of the second kind with difference kernel, we present the methodology to deduce the access intensity of MTC arrival process, which is defined as the mean number of renewals by time t. Numerical results show that in the context of MTC, there is a sharp increase in the number of access requests from MTC devices which directly causes the well-known congestion problem of MTC. And the ratio of β and α of Beta(α, β) distribution to some extent can be used as a metric to represent the burstiness of Beta distribution or that of MTC. These works together provide researchers and engineers a basis to appropriately choose traffic models for different MTC applications.

Keywords: Machine type communications; Beta distribution; Access Intensity; Volterra integral equation of second kind with difference kernel

1. Introduction

Besides the call for higher data rates and more capacity to cope with the traffic prediction in the next few years, operators and manufactures in the 3rd Generation Partnership Project (3GPP) also unanimously agreed on the continuous enhancement for machine type communications (MTC) and support for diverse traffic types [1]. MTC is defined as machine to machine communications over cellular mobile networks. It has a tremendous market potential and is an important revenue stream for operators [2, 3]. Figure 1 depicts the typical network architecture and applications of MTC. The network consists of three components [3, 4], namely the MTC server(s), the communication network(s) and the huge amount of autonomous industrial MTC devices from smart metering like smart grid, smart home, Internet of vehicles (IoV), Internet of things (IoT), wireless sensor network (WSN) and *etc*. The possibility of these massive MTC devices generating their access attempts within a short time period makes the traffic characteristics of MTC quite different from the conventional network services [5]. Therefore, conventional cellular mobile networks that have been optimally designed for

Human to Human (H2H) communications should be developed to accommodate these new challenges and guarantee the quality of service (QoS) of both MTC and H2H communications.
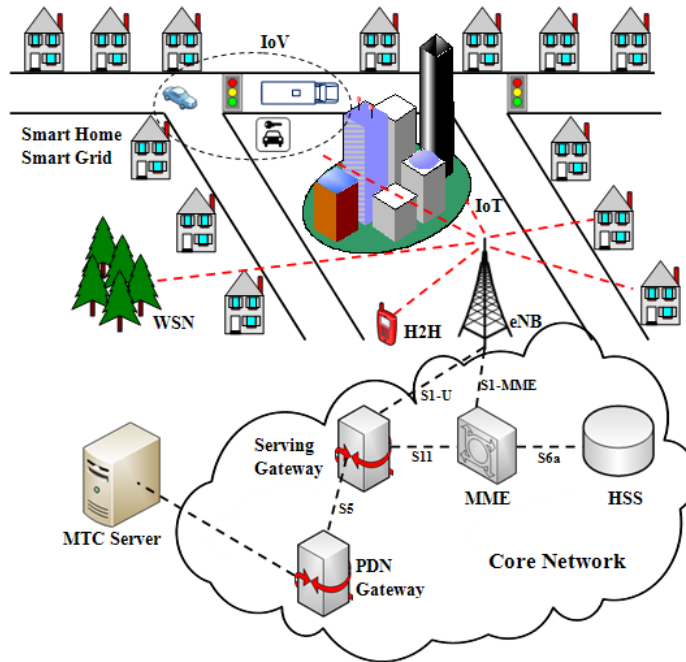

Figure 1. Typical Network Architecture and Application Scenarios of MTC

Prior to any technology enhancements for MTC, an appropriate traffic model enabling preliminary performance evaluation is of prime important. Classic Markovian traffic models under the assumption of Poisson arrival have been long-term used for stochastic traffic modeling in telecommunications [6]. These models include the simplest homogeneous Poisson process and the more complex compound Poisson process, such as the Markov arrival process and the Markov modulated Poisson process [6]. Compound Poisson process is a nature extension of homogeneous Poisson process. It assumes the arrival rate of Poisson process to be a random variable or random process rather than a constant. In recent publications, these models have been directly applied to MTC traffic modeling [7-10]. However, the concurrent data transmission from massive MTC devices within a short time period makes the arrival pattern more bursty and invalidates the Poisson assumption of these models. To capture this burstiness and evaluate the network performance under different access intensities, an appropriate statistical description of this burstiness should be concerned. As a major involver, 3GPP proposes two reference traffic models, in which the inter-arrival time (IAT) of MTC is assumed to follow uniform distribution or Beta distribution, respectively [11]. These two distributions correspond to two realistic scenarios when MTC devices access to the network in non-synchronized or synchronized manner. Uniform distribution is a special case of Beta distribution. Beta distribution is a distribution with two shape parameters and limited support range which has the flexibility to model a lot of MTC devices starting their access within a short time period. However, unlike the full understood Markovian traffic models, the researches on the traffic modeling based on Beta distribution is still in its very initial step and is rarely reported in state of the art literatures.

In this paper, we take the initiatives to discuss some prime issues for MTC traffic modeling under the assumption of Beta arrival. In Chapter 2, we briefly introduce the typical traffic characteristics of MTC and the reason why Beta distribution is more suitable for MTC. Then by the usage of renewal theory and Volterra integral equation of the second kind with difference kernel, in Chapter 3, we provide the methodology to deduce the access intensity of MTC arrival process which is defined as the number of renewals by time t. Numerical results in Chapter 4 show that in the context of MTC, there is a sharp increase in the number of access requests from MTC devices which directly causes the well-known congestion problem of MTC that has been reported in [3, 5, 11]. These works provide researchers and engineers a basis to appropriately choose traffic models for different MTC applications.

2. Typical Traffic Characteristics of MTC and Beta Distribution

In the context of MTC, the service data to be communicated have been pre-programmed inside the MTC device and MTC server before the service is deployed. So compared with H2H communications, MTC services have less randomness. The sessions are always triggered by predefined time or events [10], as shown in Figure 2, in which T is the predefined communication period that depends on specific applications and $T_1, T_2, ...$ are the inter-arrival time of random events triggering the MTC access requests or data transmission.
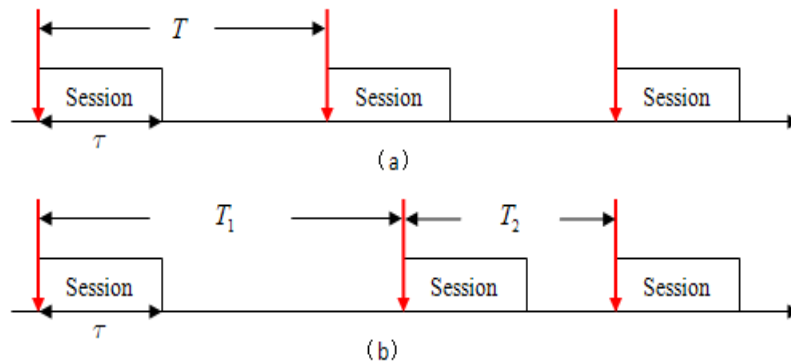

Figure 2. (a) Periodically Initiated Sessions; (b) Randomly Initiated Sessions

Usually, the average session length t is quite smaller than T or $T_1, T_2, ...$, that is, $t = T$ or $T_1, T_2, ...$. When massive MTC devices generate their access attempts within such a short time period, it would make the arrival pattern more bursty. As the inter-arrival time of Poisson process is a negatively exponential distributed random variable which has infinite support range, it is no longer suitable for this situation. Thus 3GPP suggests a distribution with finite support range, namely Beta distribution, to capture this traffic characteristic of MTC. The probability density function (PDF) of Beta distribution, for $0 \leq t \leq \tau$ and shape parameters $\alpha > 0$ and $\beta > 0$, is a power function of the variable t and of its reflection $(1 - t)$, that is [11]

$$f(t) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{t^{\alpha-1}(\tau - t)^{\beta-1}}{\tau^{\alpha+\beta-1}} \tag{1}$$

where $\Gamma(\cdot)$ is the Gama function. As any spreading time τ can be normalized to unit one with the scale operation $y = x / \tau$, thus in the rest of this paper, the spreading time τ of Beta distribution is assumed to be unit one. When $\tau = 1$, the mean and variance are [12]

$$E[t] = \frac{\alpha}{\alpha + \beta}, \quad \mathrm{var}[t] = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} \tag{2}$$

The mean of Beta distribution is a monotonous rising function of α and a monotonous decreasing function of β. The variance of Beta distribution is a monotonous decreasing function of α and β.

The PDFs of the negative exponential distribution and the Beta distribution are illustrated in Figure 3. The main differences are:

1) Beta distribution is a distribution with limited support range which is able to model a lot of MTC devices starting their access within a short time period, while exponential distribution extends itself to infinite range which is incapable of this.

2) Beta distribution is a concave function with a maximum within the support range (for shape parameters larger than one). This implies most of MTC devices would gather around at the mode (maximum value) of the PDF to transmit and behaves in a synchronized manner. While exponential distribution has no meaningful mode, it tends to distribute the access devices more evenly in the infinite time range. Therefore, to some extent, Beta distribution captures the burstiness of MTC and can be used to model the inter-arrival time of MTC within one session.

So far, the missing link is the validation of this work by measurements. When the field test results are available, the method proposed by Karl Pearson can be used to estimate the shape parameters of Beta distribution with sample mean and sample variance or higher moments [13].
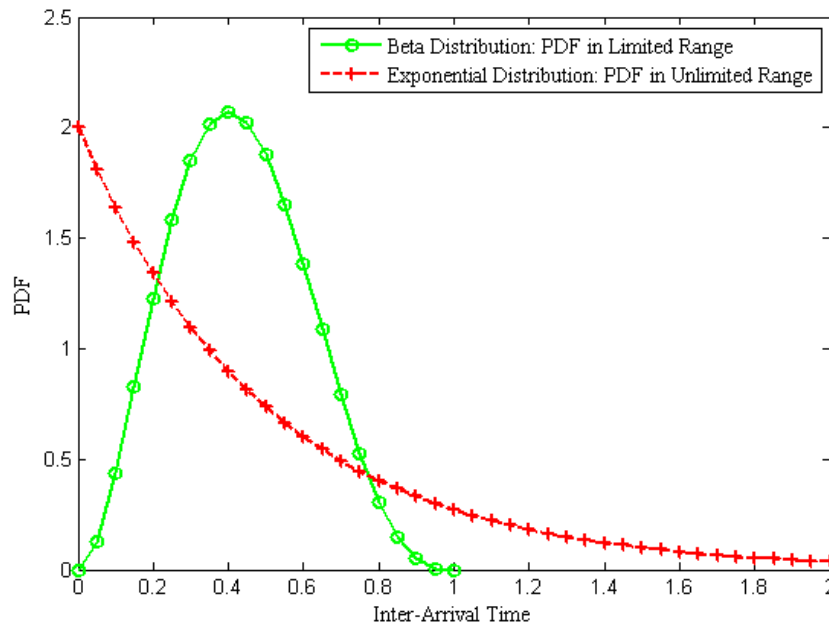

Figure 3. PDFs of Exponential Distribution and Beta Distribution

3. Access Intensity of the Arrival Process

When the IAT of MTC follows Beta distribution, it is necessary to present the main features of the arrival process under this assumption. In this chapter, by the usage of renewal theory and Volterra integral equation of the second kind with difference kernel, we evaluate the access intensity of the arrival process, which is defined as the mean number of renewals by time t.

3.1. Theoretical Analysis

If the sequence of nonnegative random variables $X_1, X_2, \ldots$ is independent and identically distributed with a PDF $f(t)$, then the counting process $N(t)$, $t \geq 0$ is said to be a renewal process. Poisson process is a special case of renewal process where $f(t)$ is exponentially distributed. In the context of MTC, $f(t)$ equals Beta distribution. In renewal theory, $N(t)$ represents the number of renewals by time t. When used in telecommunications, it represents the number of access requests of customers by time t, which can be defined as the access intensity of the arrival process.

Theoretically, the distribution of $N(t)$ can be obtained. But as it involves n-fold convolution of $f(t)$ with itself, it is always impossible to calculate analytically expect for some special cases. Therefore, we compromise to evaluate the mean value of $N(t)$. Let $m(t) = E[N(t)]$, it can be shown that there is a one to one correspondence between the IAT distribution $f(t)$ and $m(t)$, that is [14]

$$m(t) - \int_0^t f(t-x)m(x)dx = F(t) \tag{3}$$

This is the well-known renewal equation [14]. It is the Volterra integral equation of the second kind with difference kernel [15]. The general solution process for Volterra integral equation of the second kind with difference kernel can be realized in the transform domain [15]. Firstly, applying the Laplace transform to Equation (3) and taking into account the fact that by the convolution theorem the integral with kernel depending on the difference of the arguments is transformed into the product $f(s)m(s)$, we arrive at the following equation

$$m(s) = F(s) + f(s)m(s) \tag{4}$$

in which $m(s)$, $F(s)$ and $f(s)$ are the Laplace transform of $m(t)$, $F(t)$ and $f(t)$, respectively. Then, $m(s)$ is given by

$$m(s) = \frac{F(s)}{1 - f(s)} \tag{5}$$

As the right-hand function $F(t)$ is the integral of the kernel $f(t)$, that is,

$$F(t) = \int_0^t f(x)dx \tag{6}$$

According to the property of Laplace transform, $F(s) = f(s)/s$. The solution of Equation (5) can further be simplified as

$$m(s) = \frac{f(s)}{s(1 - f(s))} \tag{7}$$

Applying the Laplace inversion transform $m(t)$ can be obtained.

3.2. Moment Generation Function of Beta Distribution

To solve Equation (7), the first and most important task is to give out the Laplace transform of Beta distribution. As we know, the moment generation function (MGF) of PDF is defined as [14]

$$M(u) @ \int_{-\infty}^{\infty} e^{ux} f(x) dx \tag{8}$$

Thus, the Laplace transform of PDF $f(s) = M(-s)$. So we only have to calculate the MGF of Beta distribution.

The MGF of Beta distribution with shape parameters α and β is the confluent hyper-geometric function of first kind with parameters α and $\alpha + \beta$, that is, $_1F_1(\alpha, \alpha + \beta, u)$. [16] gives out its analytical expression. The work of deduction involves the following two propositions.

Proposition 1 Supposing n is a positive integer, the confluent hyper-geometric function of first kind with parameters $a = 1$ and $b = 1 + n$ can be represented as [16]

$$_1F_1(1, n+1, u) = \frac{n!}{u^n}[e^u - \sum_{=0}^{n-1} \frac{u^k}{k!}] \tag{9}$$

Proposition 2 (Derivative of $_1F_1(a, b, u)$) [17]

$$\frac{d}{du} {}_1F_1(a, b, u) = \frac{a}{b} {}_1F_1(a+1, b+1, u) \tag{10}$$

With Equation (9) and (10), the MGF of Beta distribution with any integral parameters can be solved as follows [16]:

1) Let $n = \beta$ and substitute it into Equation (9), we obtain $_1F_1(1, \beta + 1, u)$;

2) Find the $(\alpha - 1)$th order derivative of $_1F_1(1, \beta + 1, u)$ and scale it with a constant resulting from Equation (10), we obtain $_1F_1(\alpha, \alpha + \beta, u)$ that is the MGF of Beta distribution with shape parameters α and β.

In Table 1, we give out the MGFs of Beta distributions with low orders which will be analyzed in the rest of this paper. Besides 3GPP proposals, to choose shape parameters of Beta distribution for specific MTC applications, Karl Pearson's estimation method based on field tests will be helpful. Substituting these MGFs of Beta distributions into Equation (7), the access intensity of MTC in S-domain $m(s)$ can be obtained, then the $m(t)$.

4. Numerical Analysis

Though the analytical process to solve $m(t)$ has been given, the inverse Laplace transform of Equation (7) is always impossible. We have to resort to numerical Laurent series expansion for approximation. To do this, we notice that

$$f(0) = M(0) = 1 \qquad (11)$$

which implies $s = 0$ is a second order pole of Equation (7). This provides us the feasibility to solve Equation (7) by means of numerical Laurent series expansion. Though this is not an analytical solution, it still gives us a general view on the trend of the access intensity of MTC under different inter-arrival time distributions.

Table 1. MGF of Low Order Beta Distributions

Distribution	MGFs	Remarks
Beta(1, 1)	$\dfrac{e^{u} - 1}{u}$	3GPP proposals
Beta(3, 4)	$\dfrac{6!}{2} \dfrac{e^{u}(u^2 - 8u + 20) - 20 - 12u - 3u^2 - \frac{1}{3}u^3}{u^6}$	
Beta(2, 2)	$\dfrac{3!}{1} \dfrac{e^{u}(u - 2) + u + 2}{u^3}$	Other Beta distributions with low order parameters
Beta(3, 3)	$\dfrac{5!}{2} \dfrac{e^{u}(u^2 - 6u + 12) - u^2 - 6u - 12}{u^5}$	
Beta(2, 3)	$\dfrac{4!}{1} \dfrac{e^{u}(u - 3) + u^2/2 + 2u + 3}{u^4}$	
Beta(3, 2)	$\dfrac{4!}{2} \dfrac{e^{u}(u^2 - 4u + 6) - 2u - 6}{u^4}$	

Figure 4 depicts the access intensity $m(t)$ of MTC with the Beta distributions of 3GPP proposals, that is, Beta(1,1) and Beta(3,4). It shows that:

1) If the arrival pattern is a Poisson arrival process, $m(t)$ is strictly linear with λ as $m(t) = \lambda t$. In Figure 4 and Figure 5, we normalized l to unit one.

2) If the arrival pattern follows uniform distribution, $m(t)$ increases faster than that of Poisson arrival. This implies the mean number of customers requesting service within a time interval of uniform distribution will be greater than that of Poisson arrival.

3) If the arrival pattern follows Beta(3,4), $m(t)$ increases faster than that of Poisson arrival and uniform distribution. But at some time point, when the time interval is less than that point, $m(t)$ is less than that of Poisson arrival and uniform distribution. To some extent, this is because the cumulative density function of Beta distribution is less than that of exponential distribution within this time range.

Figure 5 depicts the access intensity $m(t)$ of MTC with lower order Beta distributions in Table 1. Without loss of generality, it shows the general trend of $m(t)$ under different Beta distributions with acceptable calculation complexity. It shows that:

1) The curves of $m(t)$ of all Beta distributions, to some extent, follow the exponential increase trend and increase faster than that of Poisson arrival. But for the same reason mentioned above, at some point, they are less than that of Poisson arrival.

2) The curves of $m(t)$ of Beta distributions with lower mean values are greater and increase faster than that of Beta distributions with larger mean values. To be more specific, the mean of Beta distribution with shape parameters α and β is $\alpha/(\alpha + \beta) = 1/(1 + \beta/\alpha)$, β/α is greater, the mean is lower and then $m(t)$ is greater. In Figure 5, the means of Beta(2,3), Beta(2,2), Beta(3,3) and Beta(3,2) are 0.4, 0.5, 0.5 and 0.6, respectively. It is easy to observe that their curves of $m(t)$ are in a descending way which verify our inference. Thus, to some extent, the ratio of β and α that is β/α can be used to represent the burstiness of Beta distribution or that of MTC.

3) When Beta distributions have equal means, like Beta(2,2) and Beta(3,3), larger value of α and β makes $m(t)$ smaller. This is caused by the reduction of the variance.

All these results unanimously show us that in the context of MTC, the number of access requests from MTC devices by t will be far greater than that of traditional H2H communications which is usually modeled by Poisson arrival. In practice, this implies increased access collisions in random access procedure and more congestion at the core network, which may seriously degrade the network performance. These conclusions lead to the well-known congestion problem caused by MTC and coincide with the reports in [3, 5].
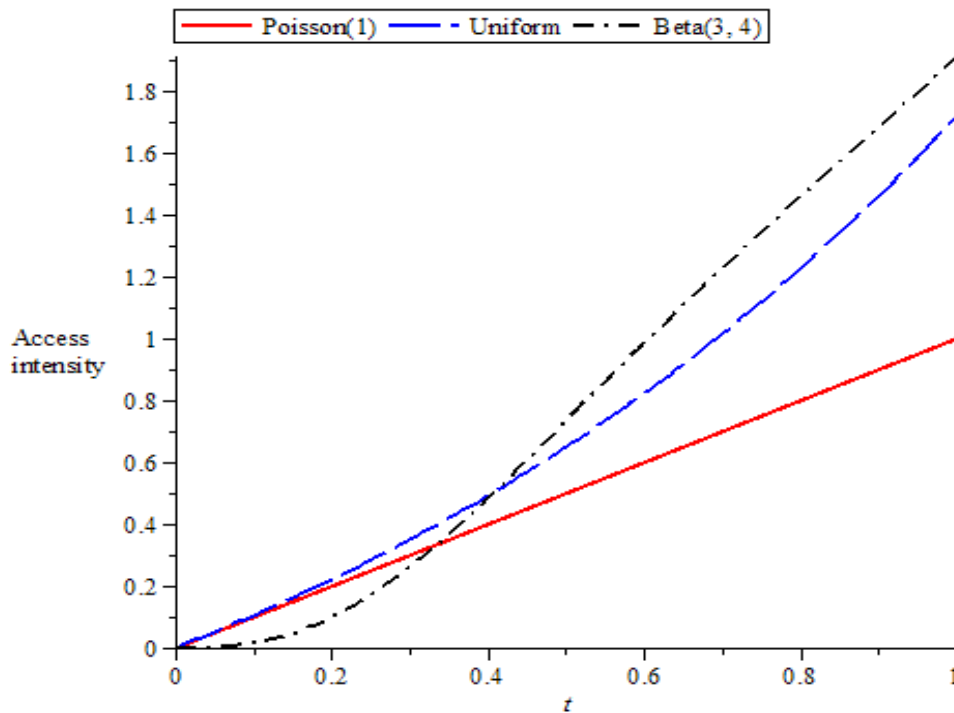

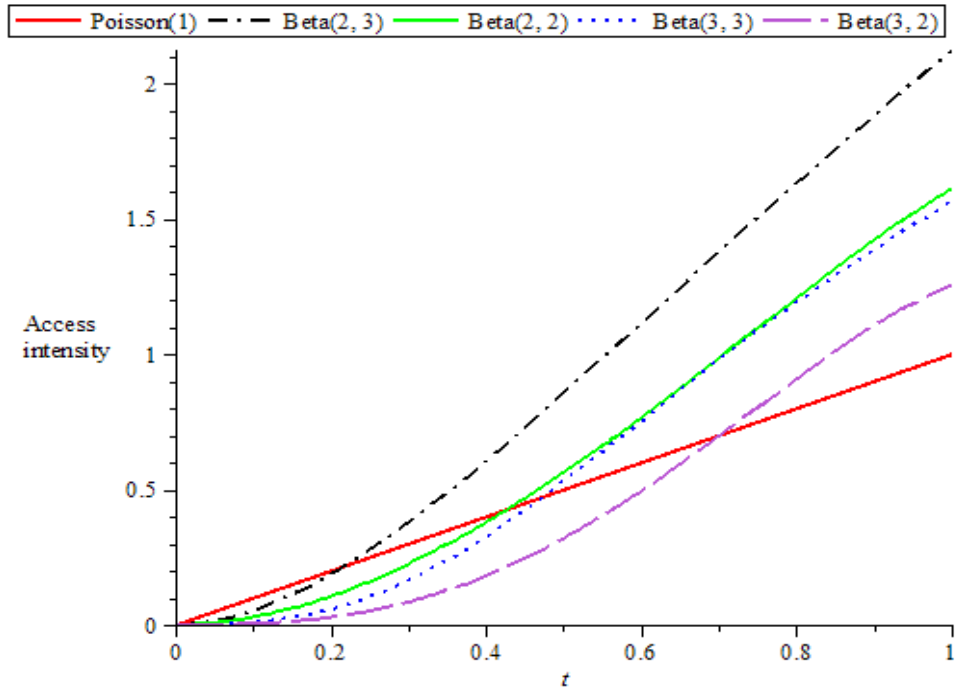**Figure 4. Access Intensity of MTC with Beta Distrbutions of 3GPP Proposals**

Figure 5. Access Intensity of MTC with Low Order Beta Distributions

5. Conclusions

By the usage of renewal theory and Volterra integral equation of the second kind with difference kernel, we present the methodology to calculate the access intensity of MTC arrival process. Numerical results show that MTC applications will undoubtedly increase the access load of network. And the ratio of β and α of Beta(α, β) distribution to some extent can be used to serve as the burstiness metric of Beta distribution or that of MTC applications.

Acknowledgements

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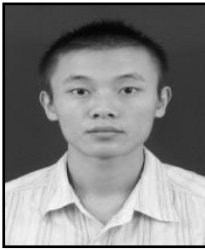
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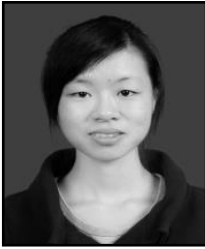
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