

Using Stochastic Petri Nets for Reliability Evaluation of Subsea Annular BOP

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Abstract

This paper performs reliability evaluation of the subsea annular blowout preventer (BOP) based on stochastic Petri nets. The presented model has been analyzed based on its isomorphic continuous time Markov chain. According to the derived equations, transient availability, reliability and MTTF of the annular BOP are obtained. Availability will reach a stable value very quickly while reliability decreases slowly over time. The effects of failure rates on MTTF are researched. The results demonstrate that in order to effectively improve MTTF, great efforts should be made to reduce the failure rates of blue pod or yellow pod.

Keywords: *Subsea annular blowout preventer; stochastic Petri nets; reliability; availability; MTTF*

1. Introduction

The subsea blowout preventer (BOP) stack is designed to deal with the erratic pressures caused by kicks or blowouts. It plays an important role in providing safety for drilling workers, rigs and natural environment. Once the BOP fails, kicks or blowout in the process of drilling will lead to serious consequences. For example, the semisubmersible drilling platform Deepwater Horizon in the Gulf of Mexico exploded and sank on April 20, 2010. This tragedy not only caused huge property losses and casualties, but also brought irreparable disaster to the ecological environment of the Gulf of Mexico. One important reason for this accident is that the subsea BOP did not prevent the blowout from coming out [1]. Therefore, reliability research of subsea BOP is of significance and it attracts more and more attentions recently.

The research about reliability of subsea BOP has not been extensive according to the references review. Holand *et al.*, [2, 3] collect reliability data about deepwater BOP failures in the outer continental shelf of Gulf of Mexico and established fault trees for reliability analysis. Fowler *et al.*, [4] study the safety of well-control equipments using failure modes and effects analysis (FMEA) and fault tree analysis (FTA) methods. However, FTA and FMEA belong to static analysis methods, which do not take into account the changes of system status over time as well as component fault sequence on the system reliability. Hence, these traditional models are not able to describe the dynamic characteristics of system. Besides, the repair actions after system failures can't be reflected in the models [5, 6]. In order to overcome the limitations of FTA and FMEA methods, many methods have been employed to research the reliability of subsea BOP system. Cai *et al.*, [7] present a Markov model of subsea BOP stack and research the effects of stack configurations and mounting

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types. Reliability of control system of subsea BOP system is also studied using Bayesian networks [8]. The reliability model of subsea annular BOP is not presented in the review.

Petri nets are introduced by C. A. Petri, which is a kind of system tools based on graphical modeling and analysis. It is suitable for performance analysis, since it can well describe the dynamic behaviors of the system. At present, Petri nets method has been widely used in system modeling and performance analysis of transportation, equipment maintenance and complex mechanical equipment. Petri net is a kind of directed network and it can reflect the states change of the system and the process of events. So, it is good at describing the transmission relation of faults [9].

This paper presents a stochastic Petri net (SPN) model of subsea annular BOP for reliability analysis. Based on the model, the reliability and availability are obtained. The remainder of the paper is organized as follows. Section 2 establishes the SPN model. Section 3 proposes the method to analyze the model. Section 4 is the results and discussions. Section 5 summarizes the paper.

2. System Description and Modeling

2.1. Stochastic Petri Net

Petri net (PN) is introduced by German scholar, C. A. Petri in 1962. A PN is made up of places, transitions and directed arcs. In the model, places denoted by circles are used to describe possible states or conditions (conditions) of the system; transitions denoted by rectangles are employed to describe the events to change the system states; directed arcs connect places with transitions, describing the development directions of states and events. Tokens are contained in places and denoted by black dots. Move of tokens means the change of system states. It means that the system enters into this state when there is a token in a place. Petri net is a strong and effective tool to describe the cause and effect, parallelism, conflict, asynchronous and of the system, which is expert at describing the behavior of the system and analyze the performance [10].

SPN introduces time parameters and random variables into PN model so that it can analyze the dynamic behaviors of the system. In SPN, the firing time of transitions follows exponential distributions. It means that when a transition is enabled it will be fired in a period of delay time subjected to exponential distribution. SPN is a 6-tuple [11], $SPN = (P, T, F, M_0, W, \lambda)$, where

- (1) $P = (P_1, P_2, P_3, \dots, P_m)$ refers to the finite set of places;
- (2) $T = (T_1, T_2, T_3, \dots, T_q)$ refers to the finite set of transitions;
- (3) $F \subseteq (P \times T) \cup (T \times P)$ is the set of directed arcs;
- (4) $M_0 = \{M_{01}, M_{02}, \dots, M_{0k}\}$ is the initial marking;
- (5) $W \rightarrow \{0, 1, 2, \dots\}$ is a weight function;
- (6) $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_l\}$ is the set of firing rates of transitions.

2.2. System Modeling

In deepwater drilling activities, a traditional BOP stack is equipped with 2 annular BOPs and 4 ram BOPs. In order to improve the reliability, modern configuration of deepwater BOP

system will add one more casing shear ram BOP and a test ram BOP. The configuration of two annular BOPs can meet the requirements of redundant design and the upper annular BOP is the major device for well control. The schematic of subsea annular BOP is shown in Figure 1.

As shown in Figure 1, open and close of the annular BOP is controlled by underwater control pods, which are in charge of high-pressure fluid in hydraulic circuit. Control pod is a valve manifold used for underwater operation function, including all the pressure regulating valves and control valves. In order to achieve redundancy, hydraulic control system is equipped with a blue control pod and a yellow control pod. Only one pod operates while the other one is in hot standby under normal circumstances. For the sake of simplicity, yellow pod is assumed to be the standby one. During the drilling activities, if the blue pod fails, the yellow pod will automatically take the place of failed pod. So, yellow pod can perform all the control functions instead of the blue pod. The failed control pod will be pulled out of the water for repair.

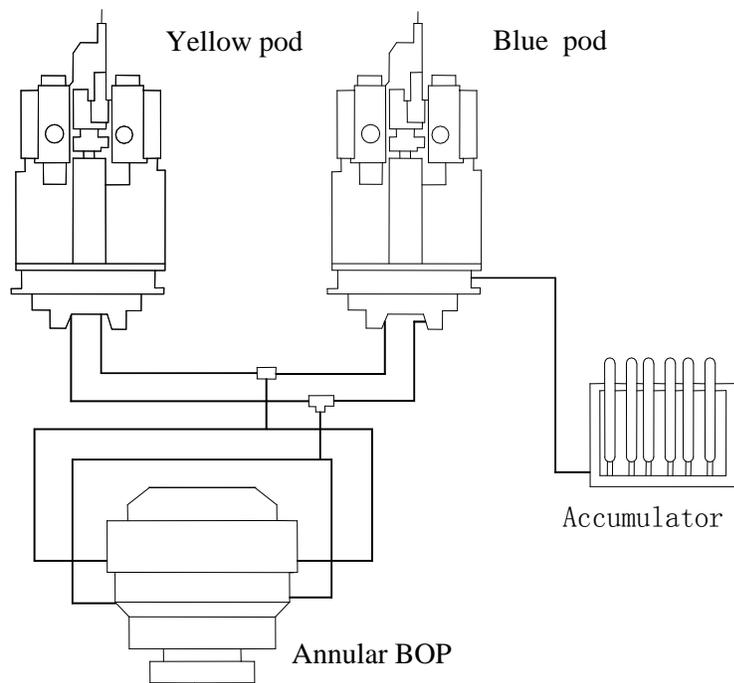


Figure 1. Control of Subsea Annular Blowout Preventer

According to the research report by Holand, there are three main failure modes for subsea annular BOP, namely "unable to close", "unable to open" and "internal hydraulic leakage" [2]. Besides, failures of control system also can make the annular BOP out of control. The BOP can't be closed or opened if there are failures of the accumulator, hydraulic circuits or valve parts. Based on the possible failure modes and its working states, the SPN model of subsea annular BOP is presented in Figure 2. In the model, place P0 denotes that the system is in normal operation. Place P1 refers to the failure of BOP system while P2 means that the blue pod fails and the BOP can still work. The meanings of transitions are illustrated in Table 1 and firing rates values are determined according to Reference [2].

Table 1. Meaning of Transitions and their Firing Rates

Components	Meanings	Firing rates values
T1	Occurrence of internal leakage for BOP	$\lambda_1 = 1.4e - 4 / h$
T2	BOP is unable to close	$\lambda_2 = 3.6e - 5 / h$
T3	BOP is unable to open	$\lambda_3 = 7.2e - 5 / h$
T4	Occurrence of leakage in line pipes	$\lambda_4 = 1e - 5 / h$
T5	Occurrence of leakage in accumulators	$\lambda_5 = 1.7e - 9 / h$
T6	Blue pod fails	$\lambda_6 = 9.91e - 4 / h$
T7	Yellow pod fails	$\lambda_7 = 9.91e - 4 / h$
T8, T9, T10	Repair the failures	$\mu = 1.02e - 2 / h$

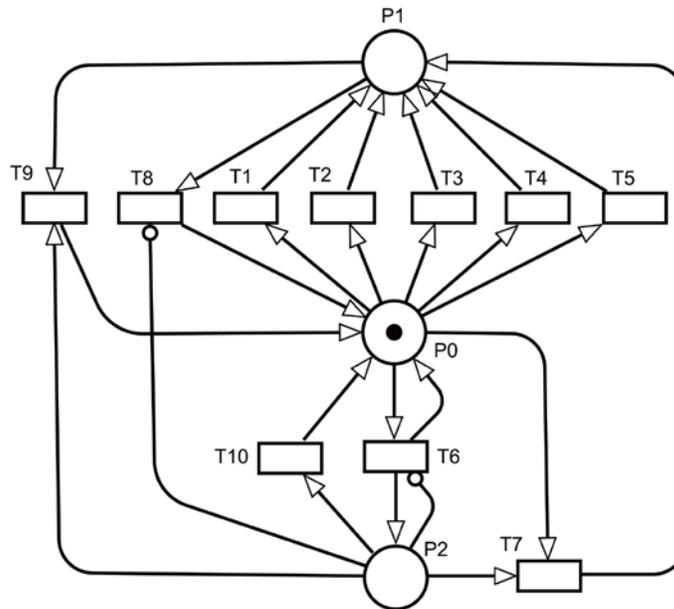


Figure 2. Stochastic Petri Nets Model of Subsea Annular Blowout Preventer

3. Model Analysis

Because a SPN model is isomorphic to a continuous time Markov chain, the system performance analysis can be performed based on it [12]. As shown in Figure 2, there is a token in place P0 at the initial time, which means the subsea annular BOP is in normal operation. The initial marking is $M_0 = (100)$, the figures denote the number of tokens in place P0, P1 and P2, respectively. All the possible markings are $M_0 = (100)$, $M_1 = (101)$, $M_2 = (011)$ and $M_3 = (010)$. Therefore, the isomorphic Markov model is presented in Figure 3.

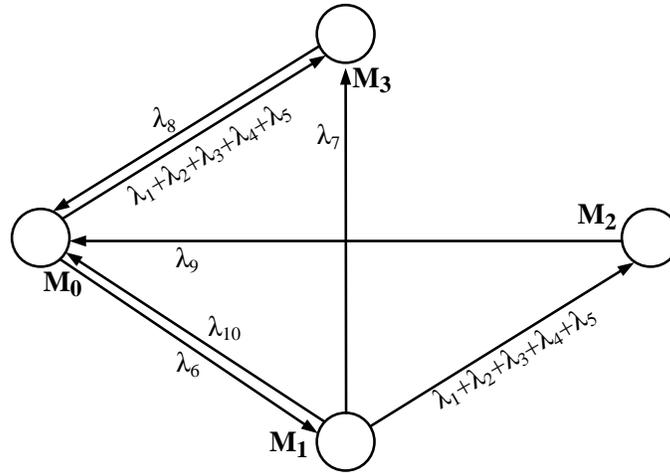


Figure 3. The Isomorphic Markov Model of the SPN Model

Transition matrix Q of the Markov model is shown in Equation (1).

$$Q = \begin{bmatrix} -(q1 + \lambda_6) & \lambda_6 & 0 & q1 \\ \lambda_{10} & -(q1 + \lambda_7 + \lambda_{10}) & q1 & \lambda_7 \\ \lambda_9 & 0 & -\lambda_9 & 0 \\ \lambda_8 & 0 & 0 & -\lambda_8 \end{bmatrix} \quad (1)$$

where , $q1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$. Let $P = (P_0, P_1, P_2, P_3)$ denote the steady-state probabilities of state $M_i (i = 0,1,2,3)$, which can be calculated by solving the equations [13].

$$\begin{cases} PQ = 0 \\ P_0 + P_1 + P_2 + P_3 = 1 \end{cases} \quad (2)$$

When it enters into state M_2 and M_3 , the system will fail. Hence, the steady-state availability can be obtained as follows:

$$A = P_0 + P_1 \quad (3)$$

Transient probabilities of the states can be derived through the Equation (4):

$$\begin{cases} P(t)Q = \frac{dP(t)}{dt} \\ \sum_{i=0}^3 P_i(t) = 1; P_0(0) = 1, P_i(0) = 0 (i \neq 0) \end{cases} \quad (4)$$

where $P(t)$ is the transient probability vector; $P_i(t)$ is the element in $P(t)$ and it refers to the transient probability of state M_i .

Transient availability of the system is :

$$A(t) = P_0(t) + P_1(t) \quad (5)$$

As availability is the probability that a system is performing its required function at any moment in time. Reliability describes the ability of a system to perform its required functions under stated conditions for a specified period of time. No failures and subsequent repairs are

allowed for an entire time interval. Therefore, to calculate the reliability of repairable systems, the repair transitions have to be omitted [14].

Mean time to failure (MTTF) is an important reliability index, which is the average time from running to failure for the system. The failure states M_2 and M_3 are defined as absorbing states. When the elements related to absorbing states are deleted in matrix Q , Q_R is obtained as follows.

$$Q_R = \begin{bmatrix} -(q1 + \lambda_6) & \lambda_6 \\ \lambda_{10} & -(q1 + \lambda_7 + \lambda_{10}) \end{bmatrix} \quad (6)$$

Then the equation is established,

$$P_R^*(s)Q_R = [sP^*(s) - P(0)]_R \quad (7)$$

where, $P^*(s)$ is the Laplace transform of $P(t)$; $P_R^*(s)$ is the remaining vector with the elements about absorbing states deleted. So, the expression to calculate MTTF is,

$$MTTF = \sum P_j^*(0) \quad (8)$$

where, $P_j^*(0)$ is the elements in $P_R^*(s)$, when s is equal to 0

4. Results and Discussion

Based on the equations derived in the former section, transient availability and reliability of the subsea annular BOP is plotted in Figure 4 and Figure 5. Figure 4 show that availability decreases from 1 to a stable value over time. It decreases very quickly in the first 10 days and reaches the stable value 0.9989 in about 25 days. As shown in Figure 5, reliability of the annular BOP decreases slowly and reaches 0 in about 6000 days.

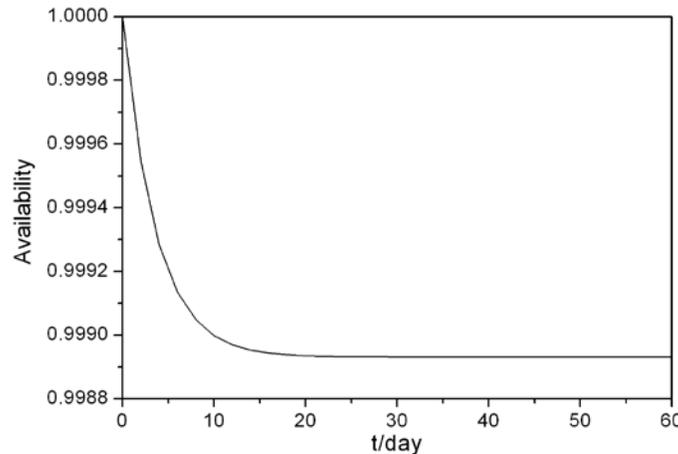


Figure 4. Transient Availability of Subsea Annular Blowout Preventer

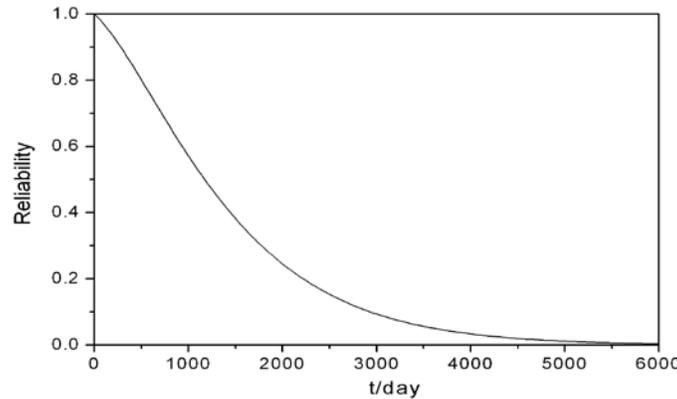


Figure 5. Transient Reliability of Subsea Annular Blowout Preventer

According to Equations (6)-(8), MTTF of the subsea annular BOP is 1436 days. MTTF of the system depends on its failure rates, but the values might not be accurate due to various factors [15-18]. Uncertainty analysis of the failure rates is performed to research the influences on MTTF. Assume that the failure rates are subject to an uncertainty of $\pm 20\%$. The upper and lower bounds of the MTTF are plotted in a histogram form as shown in Figure 6. It shows that MTTF decreases as failure rates decrease. However, failure rates have different effects on MTTF. Failure rates of blue pod and yellow pod have the greatest influence degree while the failure of accumulator has the lowest influence degree. The influence on availability is in the following order: $\lambda_6 = \lambda_7 > \lambda_1 > \lambda_3 > \lambda_2 > \lambda_4 > \lambda_5$. If the failure rate of blue pod or yellow pod is reduced by 20%, MTTF will be extended by 120 days. Therefore, in order to improve MTTF, great efforts should be made to reduce the failure rates of blue pod or yellow pod.

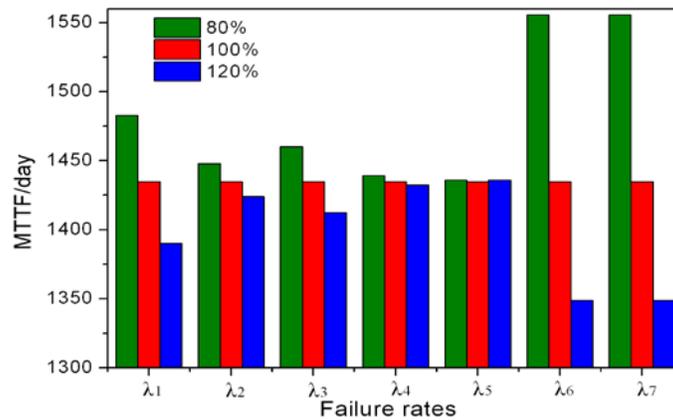


Figure 6. Effects of the Failure Rates on MTTF

5. Conclusions

(1) This paper presents a SPN model of subsea annular BOP based on its failure modes and working states. The method to perform reliability analysis of the model is introduced and the expressions to obtain the reliability index are derived.

(2) Transient availability and reliability of the BOP is obtained. Availability decreases very quickly in the first 10 days and reaches the stable value 0.9989 in about 25 days. Reliability of the annular BOP decreases slowly and reaches 0 in about 6000 days.

(3) Failure rates have different effects on MTTF of the subsea annular BOP. Failure rates of blue pod and yellow pod have the greatest influence degree while the failure of accumulator has the lowest influence degree.

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