

# A Study on Super Resolution Estimation Accuracy Method of DOA using Signal Model Errors Effects

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## **Abstract**

*To extract the direction of arrival of multiple sensors from analysis data by a sensor array is a multiple parameter estimation problem. Some important algorithms for spatial temporal processing have been developed in the past decades. A new direction of arrival estimation method using effects of model errors and sensitivity analysis is proposed. Two subspace are used to form a signal space whose phase shift between the reference signal and its effects of model error signal. Since a desired signal is obtained after interference rejection through correction effects of model error, the effect of channel interference on the estimation is significantly reduced. The proposed method in the number of signal sources detectable is not bounded by the number of antenna elements used. Through simulation, we show that the proposed method offers significantly improved estimation resolution and accuracy relative to existing method.*

**Keywords:** *Estimation, MUSIC, Resolution, Model error, Target Position*

## **1. Introduction**

The smart antenna lies mainly in the signal processing algorithms. Figure 1 is smart antenna The growth in demand for smart antennas is fueled by two major reasons [1]. First, the high speed digital signal processing is burgeoning at an alarming rate. The technology required in order to make the necessary rapid and computationally intense calculations has only emerged recently. Smart antenna is limited in their capabilities because adaptive algorithms were usually implemented in analog hardware. The global demand for all forms of wireless communication and sensing continues to grow at a rapid rate. Smart antennas are the practical realization of the subject of adaptive array signal processing and have a wide range of interesting applications [2].

Smart antennas have numerous important benefits in wireless application as well as in sensors such as radar. In the realm of mobile wireless applications, smart antennas can provide higher system capacities by directing narrow beams toward the users of interest, while nulling other users not of interesting . Smart antennas were that the deleterious effects of multipath can be mitigated. Smart antennas can be used to enhance direction of arrival techniques by more accurately finding angles of arrival [3].

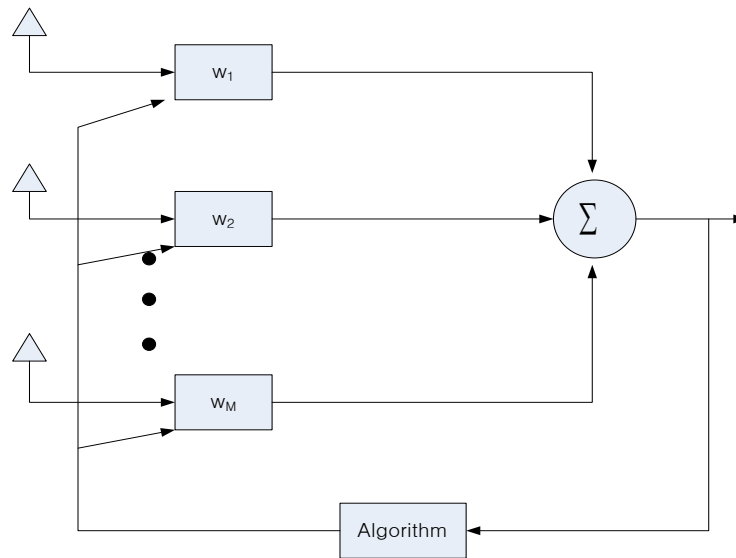
An array of spectral estimation method can be incorporated, which are able to isolate the angle of arrival with an angular precision that exceeds the resolution of the array. Wireless communication systems will use smart antennas to improve the performance and the spectrum efficiency of the system [4]. Moreover, in future systems, wideband signals will be used for requirements of higher data rate services. In this letter the direction-finding smart antennas approach is considered [5]. This approach has two phases: first, the directions of users are

estimated during reception (DOA estimation) and then the direction information is used to calculate the weights for array transmission.

The wideband DOA estimation for wideband smart antennas in future wireless communication systems is considered. Most of the results for DOA estimation have been obtained for narrowband signals [6]. These have the task of extracting the required signal parameters of multiple signal sources from the data received at the antenna array sensors. These parameters may be the number of signal sources, the signal frequencies, direction of arrival [7]. This is multi dimension estimation problem, and thus the necessary signal processing algorithms are complex. The complexity of these estimation algorithm is further magnified when array model errors such as the inequality of sensor channel gain and phase characteristics, sensor location errors, and the presence of inter sensor mutual coupling are included in the numerical model[8]. One way of reducing the complexity of the estimation algorithm is to characterize the array model more accurately by estimating the net effect of the various errors and using this real array model in the parameter estimation. This requires signal sources with known parameters covering the utilized array manifold [9].

In this paper, a new DOA estimation method that uses effects of model error (EME) is proposed. The major difference between the proposed DOA estimation method and existing method is that in the proposed DOA estimation method, the target DOA is estimated after interference rejection using effects model error. In existing method, the DOA estimation is based on computing the spatial signatures.

In the proposed DOA estimation method, the target DOA is estimated from the phase shift introduced in the target signal by effects model error, which is a function of the target DOA. Since the phase shift is estimated after effects of model error, all signals and interference other than the target one can be efficiently rejected before DOA estimation. Thus their interference on the DOA estimation is reduced. In this way, the estimation resolution and accuracy of the proposed method are better than those for existing method.

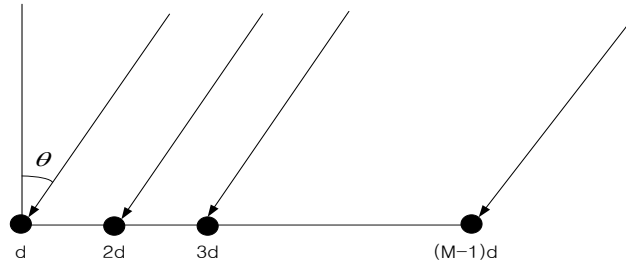


**Figure 1. Smart Antenna System**

## 2. Array Antenna Signal Model

Consider an antenna composed of  $M$  antenna sensors arbitrarily located in spatial and assume that a signal impinges on the array. Figure 2 is  $M$ -element linear array. Then the output of  $m^{\text{th}}$  the sensor can be written as follow [10-12]

$$X_m(t) = \sum_{n=1}^p \alpha_m s_n(t - \tau_{mn}) + n_m(t) \quad (1)$$



**Figure 2. M –element linear Array**

Where  $s_n(t)$  ( $n = 1, 2, \dots, p$ ) is the receive signal,  $\alpha_m$  and  $\varphi_m$  are the gain and phase delay associated with the  $m^{\text{th}}$  sensor,  $\tau_{mn}$  is the delay relative to a reference point associated with the signal propagation from the  $n^{\text{th}}$  source to the  $m^{\text{th}}$  sensor. The array output can be expressed as [13]

$$X(t) = C \Lambda A(\theta) S(t) + N(t) \quad (2)$$

where  $X(t)$  is the received signal vector,  $C$  is the mutual coupling matrix,  $\Lambda$  is complex diagonal matrix,  $S(t)$  is the signal vector,  $A(\theta)$  is array manifold matrix,  $a(\theta_m)$  is steering vector corresponding to the  $m^{\text{th}}$  source at direction  $\theta_m$ , and  $N(t)$  is additive white noise.

$$X(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T \quad (3)$$

$$S(t) = [s_1(t), s_2(t), \dots, s_M(t)]^T \quad (4)$$

$$N(t) = [n_1(t), n_2(t), \dots, n_M(t)]^T \quad (5)$$

$$A(\theta) = [a(\theta_1), a(\theta_2), \dots, a(\theta_M)]^T \quad (6)$$

$$\Lambda = \text{diag}[\alpha_1 e^{-j\omega\beta_1}, \alpha_2 e^{-j\omega\beta_2}, \dots, \alpha_M e^{-j\omega\beta_M}] \quad (7)$$

There are mutual coupling effects between the antenna sensors that make up the array. When antenna sensors are in close proximity, less than half a wavelength, the impedance and polar response of each sensor is affected by the electromagnetic coupling to its adjacent sensors. This will distort the radiation pattern of the array and hence modify the array manifold. The output of the  $m^{\text{th}}$  sensor, in the absence of other sources of error, can be written as follow

$$X_m(t) = \sum_{i=1}^M C_{m,i} S(t) + N(t) \quad (8)$$

Where  $C_{m,i}$  ( $i, m = 1, 2, \dots, M$ ) is the mutual coupling factor and models the mutual coupling effect of the  $i^{\text{th}}$  sensor on the  $m^{\text{th}}$  sensor and  $a_i(\theta)$  is the  $i^{\text{th}}$  row of the steering matrix. Thus the output of the array is given by [14-15]

$$X(t) = C A(\theta) S(t) + N(t) \quad (9)$$

Where  $C$  is referred to as the mutual coupling matrix. The matrix  $C$  has no special structure; however, if the array is uniform, then the matrix will be structured because coupling between adjacent sensors is common to sensors occupying a similar position in the array and coupling between non-adjacent sensors may be ignored. When both types of error are considered the array output can be written as follow

$$X(t) = C \Lambda A(\theta) S(t) + N(t) \quad (10)$$

### 3. DOA Estimation with Effects of Model Error

If signals receive on an antenna array comprising  $M$  sensor, then the array output covariance matrix can be written as follow [16-17]

$$R = E[XX^H] = A(\theta)R_{xx}A^H(\theta) + \sigma^2I \quad (11)$$

Where  $R_{xx}$  is signal covariance matrix,  $()^H$  is Hermitean. In subspace DOA estimation methods, the array output covariance matrix  $R$  can be partitioned into two subspace, the signal subspace and the noise subspace, using eigenvalue decomposition, such that

$$R = E_s \gamma_s E_s^H + \sigma^2 E_n E_n^H \quad (12)$$

Where  $E_s$  and  $E_n$  are signal subspace and noise subspace respectively. When we consider array response errors, the array manifold can be written as follow

$$\ddot{A}(\theta) = \Lambda C A(\theta) \quad (13)$$

The array manifold will affect the orthogonality between the two subspace. In particular, due to the effects of the array model response errors, the real array manifold  $\ddot{A}(\theta)$  is different form the ideal array manifold  $A(\theta)$ . Thus the accuracy and resolution of he DOA estimation will be degraded by the errors in the array manifold. We refer to the different between the ideal and real array parameters. The spatial spectrum of MUSIC and DOA estimation can be written as follow

$$SE(\theta, \mu) = \frac{1}{A(\theta, \mu)E_n^H(\mu)E_n(\mu)A^H(\theta, \mu)} \quad (14)$$

Where  $\mu$  is real array parameters,  $\mu_0$  is ideal array parameters. We can get the exact estimate of the true DOAs. When  $\mu_0 \neq \mu$ , the peaks of  $SE(\theta, \mu)$  will no longer be the true DOAs and the accuracy of DOA estimations can be degraded by even small model errors. Therefore, we will consider the effect of model errors on both accuracy and resolution of DOA estimations. Let  $\mu - \mu_0$  is the model error.

$$\mu - \mu_0 = \sigma_u \vartheta \quad (15)$$

Where  $\vartheta$  is random vector with zero mean and unit variance and  $\sigma_u$  is a positive scalar.

$$\sigma_{u(DOA)} = \sqrt{\frac{1}{N} \sum_{i=1}^N \sigma_i^2} \quad (16)$$

Where  $\sigma_{u(DOA)}$  represents the mean square value of DOA errors caused by modeling errors and  $N$  is the number of signals

#### 4. Simulation

In this section, through simulation, we showed that we compare general method with proposed method. General methods used ML and MUSIC method. Consider a uniform linear array with 9 sensors, each sensor separated by half wavelength. Figure 3 shows the Root Mean Square Error(RMSE) of the DOA estimation of a signal arriving using the ML, MUSIC and proposed method for different SNR. The SNR is varied from 0dB to 25dB. In Figure 3, ML method showed  $10^{-1}$  in the case of SNR=10dB, MUSIC method shows  $10^{-1.6}$  in the case of SNR=10dB, and proposal method shows  $10^{-1.8}$  in the case of SNR=10dB. Proposal method improved 8dB than ML method in this paper. Proposal method improved 2dB than MUSIC method. ML method shows SNR 10dB in the case of MMSE  $10^{-1}$ . MUSIC method shows SNR 6dB in the case of MMSE  $10^{-1}$ . Proposal method shows SNR 4dB in the case of MMSE  $10^{-1}$ .

Figure 4 shows DOA estimation using ML method in angle  $[-40^\circ, 0^\circ, 5^\circ]$ . The ML method had not estimation a desired signal in angle  $[5^\circ]$ . Figure 4 estimated only two target, and decrease direction of arrival resolution. Figure 5 shows DOA estimation using MUSIC method in angle  $[-40^\circ, 0^\circ, 5^\circ]$ . The MUSIC method had not estimation a desired signal in angle  $[5^\circ]$ , and decrease direction of arrival resolution. Figure 6 shows DOA estimation using proposed method in angle  $[-40^\circ, 0^\circ, 5^\circ]$ . When we compare Figure 6 with Figure 4 and Figure 5, the proposed method all estimated a desired signal. Table 1 appeared DOA estimation according to class method. Where, X is not doing DOA estimation.

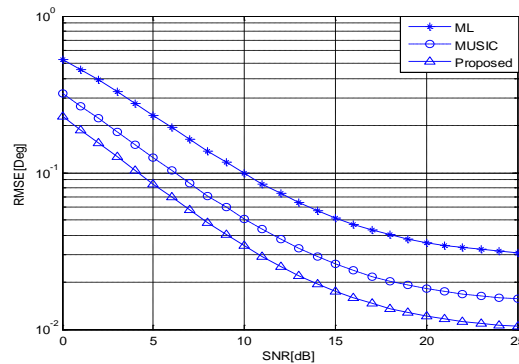
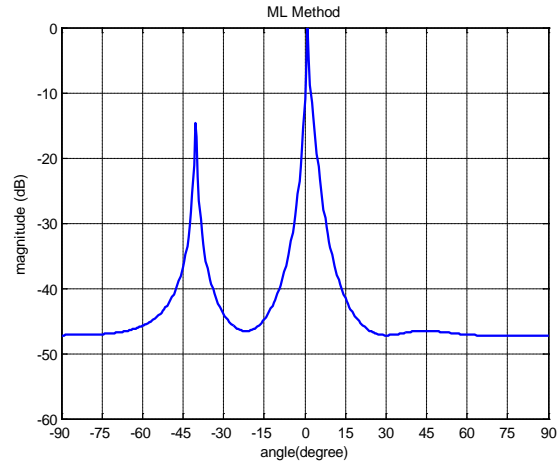


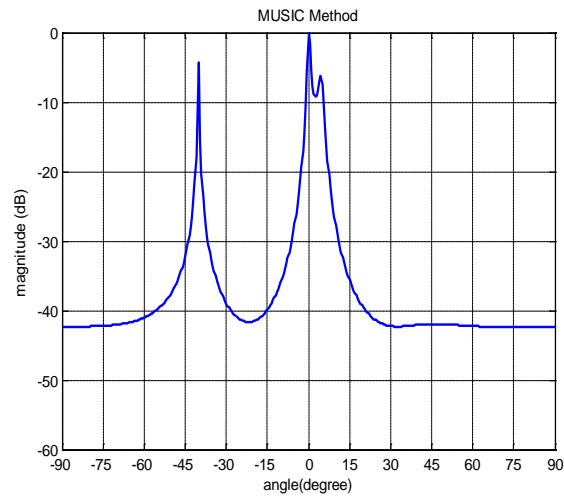
Figure 3. DOA of MUSIC at  $-10^\circ$  and  $10^\circ$

Table 1. DOA Estimation in Simulation

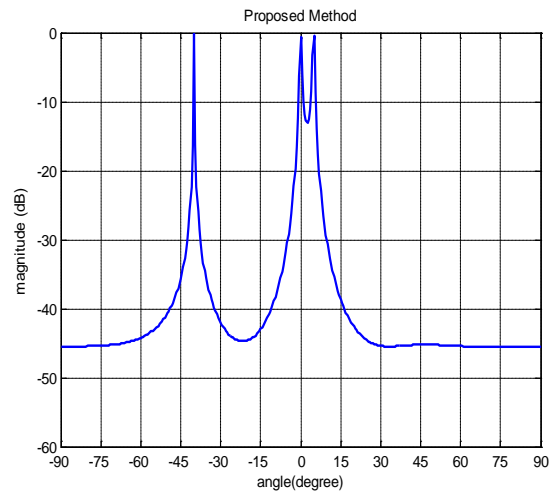
Method	DOA Estimation $[-40^\circ \ 0^\circ \ 5^\circ]$
ML	$[-40^\circ \ 0^\circ \ X]$
MUSIC	$[-40^\circ \ 0^\circ \ X]$
Proposed	$[-40^\circ \ 0^\circ \ 5^\circ]$



**Figure 4. DOA of ML Method**



**Figure 5. DOA of MUSIC Method**



**Figure 6. DOA of Proposed Method**

## 5. Conclusion

A new DOA estimation method based upon effects of model errors has been proposed. In the new method, two subspace are used to obtain an optimum estimation of the desired signal whose phase relative to the model error signal is a function of the target DOA. The target DOA is estimated from the effects of model error between the estimated signal in array antenna and its correction effects of model error signal. In this way, the effect of interference on DOA estimation is reduced. Through simulation, the performance showed that the proposed method leads to increased resolution and improved accuracy of DOA estimation relative to those achieved with existing method.

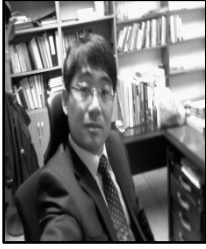
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