A Modified Denoise Approach for UCA DOA Estimation in Low SNR Case

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Abstract

When the uniform circular array (UCA) is used to estimate the direction-of-arrival (DOA) of the coherent sources, it is necessary to transform the UCA data to the interpolated uniform linear array (ULA) data. Thus, the transformed array data can be applied to the spatial processing algorithm for the coherent sources such as the forward-backward smoothing algorithm. To select a more robust transformation matrix, a modified denoise approach for UCA estimation in low SNR case is proposed in this paper. First, a denoise method is investigated to maximize the SNR in the process of the interpolated transformation. The pseudo signal to noise ratio (PSNR) obtained from the eigenvalues of the forward-backward smoothing virtual covariance matrix estimate is used as the parameter of this maximization problem. Second, a modified maximization problem and its solution are presented to obtain more accurate estimates. The simulation results demonstrate the effectiveness of the proposed method, which improve the resolution ability and the estimation accuracy of the coherent sources in the low SNR case at the same time.

Keywords: *UCA; DOA Estimation; Interpolated Array; Modified Denoise Approach*

1. Introduction

The circular array antennas have gained immense popularity in the radar system. It has proved to be a better alternative over other types of antenna array configuration due to its all-azimuth scan capability and a beam pattern which could be kept invariant. In [1], Fabio and Visa focus on the uniform circular array (UCA) Unitary root-MUSIC algorithm and propose a novel technique for reducing the bias which leads to practically bias-free DOA estimates. In [2], the authors consider the scenarios which are the time-varying terrestrial multipath propagation with the medium-to-low SNR and very short capture records (single-packet processing). The DOA estimation performance of the procedure is close to the Cramer-Rao Lower Bound. A generalized algorithm for two-dimensional angle estimation of a single source with uniform circular arrays is reported in [3]. An antenna selection method in beamspace is developed for MIMO systems using compact uniform circular arrays at the receiver in [4]. The ergodic capacity for a system employing multi-element antenna array (such as the uniform circular array) at the transmitter and receiver side is analyzed in [5]. In [6], the authors consider a blind source separation problem in which the number of sources is unknown and all or some of the sources may be networks with the frequency hopping spread spectrum capability. In this proposed method, there is no need to have several antennas for the DOA estimation and it significantly reduces implementation cost and complexity of the algorithm. In [7], Tewfik and Hong have shown that it is possible to extend the Root-MUSIC to UCA using the phase mode excitation concept. In [8], the authors analyze the spatial smoothing technique with UCA. In [9], the authors propose the real beam space MUSIC to UCA that yields the reduced computation and the better resolution. Generally, when the uniform circular array (UCA) is used to estimate the direction-of-arrival (DOA) of the coherent sources, it is necessary to transform the UCA data to the interpolated uniform linear array (ULA) data. However, how to select a more robust transformation matrix in low SNR case has not been investigated sufficiently yet. In this paper, a modified denoise approach for UCA DOA estimation in low SNR case is investigated.

2. The Interpolated Array Technique of UCA

Consider a UCA with N identical and the omni directional sensors. Let r be the radius of the array and d be the circumferential spacing between the adjacent elements. Let θ denote the angle (azimuth angle) of the interfering signal source measured in the plane containing the elements. We assume for simplicity that the interfering signal sources are in the same plane as the UCA. The scenario is shown in Figure 1.

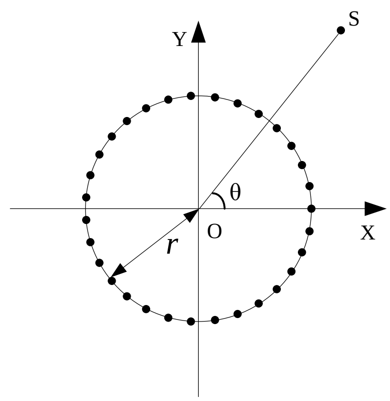

Figure 1. The UCA and the signals

The steering vector of the UCA with regard to the center of the array could be expressed as

$$\mathbf{a}(\theta) = \left[e^{j\xi\cos\theta}, e^{j\xi\cos(\theta-2\pi/N)}, \cdots, e^{j\xi\cos(\theta-2\pi(N-1)/N)} \right]^T \tag{1}$$

where $\xi = 2\pi r / \lambda$, λ is the wavelength, and $(\cdot)^T$ represents the transpose of $(\cdot)$. Denote the nth antenna data in the ith sample as $x_n(i)$, we have the array data vector in the ith sample

$$\mathbf{X}(i) = [x_1(i), x_2(i), \cdots, x_N(i)]^T . \tag{2}$$

Generally, in the spatial spectrum estimation technique of UCA, the forward-backward smoothing algorithms are needed to solve the coherent sources [10]. Meanwhile, the UCA data is needed to be transformed to the interpolated ULA data [11-14].

To design the interpolated array, the field of view of the array is divided into sectors. The lth sector is defined by the interval $[\theta_l^{(1)}, \theta_l^{(2)}]$. And define a set of angles [10]

$$\boldsymbol{\theta}_l = [\theta_l^{(1)}, \theta_l^{(1)} + \Delta\theta, \theta_l^{(1)} + 2\Delta\theta, \cdots, \theta_l^{(2)}] \tag{3}$$

for each sector, where $\Delta\theta$ is the angle interval. These angles are used only in the design of the interpolation matrix. Compute the steering vectors associated with the set θ_l for the given array and arrange them in a matrix as follows:

$$\mathbf{A}_l = [\mathbf{a}(\theta_l^{(1)}), \cdots, \mathbf{a}(\theta_l^{(2)})]\,. \tag{4}$$

In other words $\mathbf{A}_l$ is a section of the array manifold of the real array. Denote $\overline{\mathbf{A}}_l$ as the section of the interpolated virtual array manifold computed for the set of angles θ_l:

$$\overline{\mathbf{A}}_l = \left[\bar{\mathbf{a}}(\theta_l^{(1)}), \cdots, \bar{\mathbf{a}}(\theta_l^{(2)})\right] \tag{5}$$

where $\bar{\mathbf{a}}(\theta_l)$ is the response of the interpolated virtual array to the same signals of θ_l. Assume that there exists a interpolated transformation matrix $\mathbf{B}_l$ such that

$$\mathbf{B}_l \mathbf{A}_l = \overline{\mathbf{A}}_l\,. \tag{6}$$

Of course, the interpolation is not exact and therefore the equality above not really holds. The "best" interpolation matrix is the one which will give the best fit between the interpolated response $\mathbf{B}_l \mathbf{A}_l$ and the desired response $\overline{\mathbf{A}}_l$ [10]. Given these interpolation matrices we can compute the array data vector $\mathbf{X}$ of the real array:

$$\overline{\mathbf{X}}_l = \mathbf{B}_l \mathbf{X} \tag{7}$$

where

$$\mathbf{B}_l = \overline{\mathbf{A}}_l \mathbf{A}_l^H (\mathbf{A}_l \mathbf{A}_l^H)^{-1} \tag{8}$$

where $(\cdot)^H$ represents the conjugate transpose of $(\cdot)$. The spatial smoothing method can now be applied to the virtual array (i.e., to $\overline{\mathbf{X}}_l$) rather than to the real array (i.e., to $\mathbf{X}$). Denotes the antenna element number of the virtual ULA array as $\bar{N}$. Divide the virtual array into P subarrays with the equal antenna element number M. Let K be the number of the coherent sources, according to the restraint $K < M \le \bar{N}$, there are $P = \bar{N} - M + 1$ subarrays. We can express the pth virtual subarray data vector as

$$\overline{\mathbf{X}}_{l,p}(i) = [\bar{x}_{l,p}(i), \bar{x}_{l,p+1}(i), \cdots, \bar{x}_{l,p+\mathrm{M}-1}(i)]^T \tag{9}$$

where $\bar{x}_{l,n}(i)$ is the nth virtual antenna data in the ith sample of the lth sector, $n = p, \ldots, p + M - 1$, and $p = 1, \ldots, P$. The average $M \times M$ virtual array covariance matrix is expressed as

$$\overline{\mathbf{R}}_{l,p} = \frac{1}{I} \sum_{i=1}^{I} \overline{\mathbf{X}}_{l,p}(i) \overline{\mathbf{X}}_{l,p}{}^H(i) \tag{10}$$

where I is the sample number.

From (10), the virtual array covariance matrix estimate using the forward-only smoothing can be expressed as

$$\overline{\mathbf{R}}_l^f = \frac{1}{P} \sum_{p=1}^{P} \overline{\mathbf{R}}_{l,p}\,. \tag{11}$$

Also, the forward-only smoothing virtual signal covariance matrix estimate of the coherent sources can be obtained by

$$\begin{aligned}\bar{\mathbf{R}}_{lS}^{f} &= \frac{1}{P}\sum_{p=1}^{P}\bar{\mathbf{D}}^{(p-1)}\left(\frac{1}{I}\sum_{i=1}^{I}\bar{\mathbf{S}}(i)\bar{\mathbf{S}}^{H}(i)\right)\bar{\mathbf{D}}^{-(p-1)} \\ \bar{\mathbf{D}} &= diag[e^{j\bar{\beta}_1}, e^{j\bar{\beta}_2}, \cdots, e^{j\bar{\beta}_K}] \\ \bar{\mathbf{S}}(i) &= [s_1(i), \cdots, s_K(i)]^T\end{aligned} \tag{12}$$

where $\bar{\beta}_k = 2\pi\bar{d}\sin\theta_k/\lambda$, $s_k(i)$ is the complex envelopes of the *k*th coherent signal in the *i*th sample, $k = 1,\ldots,K$, θ_k is the azimuth of the *k*th coherent source, λ is the wavelength. $\bar{d}$ is the adjacent antenna element space of the virtual ULA. In like manner, the estimation matrix using backward-only smoothing are given by

$$\bar{\mathbf{R}}_l^b = \mathbf{J}(\bar{\mathbf{R}}_l^f)^*\mathbf{J} \tag{13}$$

where $(\cdot)^*$ represents the conjugate of $(\cdot)$, and $\mathbf{J}$ is a $M \times M$ exchange matrix with 1's on the anti-diagonal. Finally, the forward-backward smoothing virtual covariance matrix estimate $\bar{\mathbf{R}}_l^{bf}$ can be expressed as the mean of the forward-only smoothing and the backward-only smoothing estimate matrices. We can compute the signal subspace $\bar{\mathbf{E}}_s$ of the interpolated array by the eigenvalues decomposition. It spans the signal subspace of the interpolated array. The noise subspace $\bar{\mathbf{E}}_n$ of the interpolated array can be found by computing the subspace orthogonal to $\bar{\mathbf{E}}_s$. The peaks of $1/\left|\bar{\mathbf{a}}(\theta)^H\bar{\mathbf{E}}_n\right|^2$ provide the MUSIC DOA estimates based on the interpolated array, rather than the real array.

3. A Modified Denoise Approach

In general, the "best" interpolation matrix $\hat{\mathbf{B}}_l$ is obtained by solving the following minimization problem

$$\hat{\mathbf{B}}_l = \arg\min_{\mathbf{B}_l}\frac{\left\|\bar{\mathbf{A}}_l - \mathbf{B}_l\mathbf{A}_l\right\|_F}{\left\|\bar{\mathbf{A}}_l\right\|_F} \tag{14}$$

where $\|\cdot\|_F$ is Frobenius norm. For convenience, this minimization problem is named the minimization error criterion (MEC). Nevertheless, if the SNR is quite low, the MUSIC DOA estimation might fail by using MEC. Thus, it is necessary to maximize the SNR in the process of the interpolated transformation [15-18]. To solve this problem, a new modified denoise approach is presented in the following.

In fact, the real SNR of the signals is constant in one experiment data batch. Thus, the pseudo signal to noise ratio (PSNR) obtained from the eigenvalues of $\bar{\mathbf{R}}_l^{bf}$ is used as the variable for this optimization problem. In ideal case, the signal and the noise are the stationary and ergodic complex-valued random processes with zero mean, and they have the infinite data length. The noise is assumed to be uncorrelated with the signals and it is uncorrelated from element to element. Consequently, the variance of the noise may be equal to the minimum eigenvalue of the forward-backward smoothing virtual covariance matrix estimate $\bar{\mathbf{R}}_l^{bf}$. Moreover, the variance of the signal is equal to its maximum eigenvalue. Nevertheless, in the practical application, the data length is finite and the noise might correlate with the signals and correlated from element to element. Consequently, denotes the eigenvalues of $\bar{\mathbf{R}}_l^{bf}$ as $\bar{\lambda}_n$, and $\bar{\lambda}_1 > \bar{\lambda}_2 > \cdots > \bar{\lambda}_M$, where $n = 1, \ldots, M$ and M is the antenna element number of the virtual subarray. We define the pseudo noise variance estimate by

$$\hat{\sigma}_{noise}^2 = \frac{1}{M-K} \sum_{n=K+1}^{M} \bar{\lambda}_n \tag{15}$$

where K is the number of the coherent sources. And the pseudo signal variance estimate is denoted as

$$\hat{\sigma}_{signal}^2 = \frac{1}{K} \sum_{n=1}^{K} \bar{\lambda}_n \text{-} \hat{\sigma}_{noise}^2 . \tag{16}$$

Thus, the variable of PSNR is written as

$$PSNR = 10 \log_{10} \left(\frac{\hat{\sigma}_{signal}^2}{\hat{\sigma}_{noise}^2} \right). \tag{17}$$

And a denoise method to obtain the interpolation matrix $\hat{\mathbf{B}}_l$ is available by solving the following maximization problem

$$\hat{\mathbf{B}}_l == \arg \max_{\mathbf{B}_l} (PSNR) . \tag{18}$$

This denoise method could largely improve the resolution ability of UCA DOA estimation. To improve its estimate accuracy, a modification is needed to be introduced. Thus, a modified denoise approach is obtained by solving the following modified maximization problem

$$\hat{\mathbf{B}}_l == \arg \max_{\mathbf{B}_l} (PSNR) \text{ subject to } \frac{\left\| \bar{\mathbf{A}}_l - \mathbf{B}_l \mathbf{A}_l \right\|_F}{\left\| \bar{\mathbf{A}}_l \right\|_F} < \varepsilon \tag{19}$$

where ε is the error which is limited in some ranges for the practical application. When the DOA is estimated in an interested area, the azimuth angle range of the area is often fixed. Thus, the interpolated transformation matrix $\mathbf{B}_l$ might only be obtained by changing the parameters of the virtual array antenna. Besides, the number of the virtual array elements is equal to the number of the real array elements. Consequently, let the ratio of the space between the adjacent elements $\bar{d}$ to the wavelength λ denotes the virtual array parameter variables, we have

$$vratio = \bar{d} / \lambda . \tag{20}$$

And the (19) could be rewritten as

$$vratio_{opt} = \arg \max_{vratio} (PSNR) \text{ subject to } \frac{\left\| \bar{\mathbf{A}}_l - \mathbf{B}_l \mathbf{A}_l \right\|_F}{\left\| \bar{\mathbf{A}}_l \right\|_F} < \varepsilon \tag{21}$$

where $vratio_{opt}$ is the “best” $vratio$. The (18) is rewritten as

$$vratio_{snr} = \arg \max_{vratio} (PSNR) . \tag{22}$$

And the (14) is rewritten as

$$vratio_{error} = \arg \min_{vratio} \frac{\left\| \bar{\mathbf{A}}_l - \mathbf{B}_l \mathbf{A}_l \right\|_F}{\left\| \bar{\mathbf{A}}_l \right\|_F} \tag{23}$$

where $vratio_{snr}$ is the virtual array parameter obtained by (22) and $vratio_{error}$ is the virtual array parameter obtained by (23).

As the practical SNR is different in the different received data batch and the subspace decomposition for equation (22) is nonlinear, it is difficult to solve the optimization problem in (21) by the mathematical solution exactly. It needs an approximate solution. From (22) and (23), we could obtained the solution of (21) with the following equation

$$vratio_{modified} = w_{snr} \cdot vratio_{snr} + w_{error} \cdot vratio_{error} \tag{24}$$

where w_{snr} and w_{error} is the weighting coefficients of $vratio_{snr}$ and $vratio_{error}$. Assuming the 0% ineffective probability of MEC is SNR_{error0}, and the 100% ineffective probability of MEC is SNR_{error100}, we have the weighting coefficient

$$w_{snr} = \begin{cases} 0 & SNR > SNR_{\text{error0}} \\ w' & SNR_{\text{error0}} > SNR > SNR_{\text{error100}} \\ 1 & SNR < SNR_{\text{error100}} \end{cases} \tag{25}$$

where SNR is the variable of the SNR of the received data, and

$$w' = \frac{SNR_{\text{error0}} - SNR}{SNR_{\text{error0}} - SNR_{\text{error100}}} \tag{26}$$

and

$$w_{error} = 1 - w_{snr} . \tag{27}$$

Although the SNR could not be obtained exactly, it could be approximately estimated from the DFT spectrum of the received data. Consequently, the weighting coefficient in the equation (25) could be obtained.

4. Simulation and Analysis

In this section we evaluate the performance of the modified denoise approach mentioned above. Consider a UCA with 64 identical and omni directional sensors in a radar system, and the radius of the array is 20m, the circumferential space between the UCA elements d=1.96m, the frequency investigated is 400MHz. The antenna element number of the virtual ULA array $\bar{N}$ is equal to N. The adjacent antenna element space of the virtual ULA $\bar{d}$ ranges from 0.01λ to 0.5λ with the span 0.01λ. And the number of these parameters is 50. We assume that there are two interfering signal sources. And they have the same frequency. Thus they are the coherent sources. The azimuth angles of the interfering signal sources are -15.0º and 30.0º measured in the plane containing the elements. The snapshots are 256. We assume for simplicity that the interfering signal sources are in the same plane as the UCA. The noise is the additive white Gaussian noise. We might obtain the estimate of the SNR by investigate the difference of the spectra amplitude to the noise power level based on the DFT spectra of the received signal. In this simulation, with the above simulation parameters, we have SNR_{error0}=3.098dB, SNR_{error100}=0dB. We selected the SNR that is larger than the SNR_{error100} and smaller than the SNR_{error0}.

Simulation 1: In this simulation, we selected the input signals with SNR which is larger than $\text{SNR}_{\text{error0}}$, such as 4dB. Figure 2 depicts the MUSIC spectra of the 50 different virtual ULA $\bar{d}$ in this case. The $\bar{d}$ correlated to the denoise method is 0.01λ, and the $\bar{d}$ correlated to the MEC is 0.44λ, both of which are invariant with the different SNR.

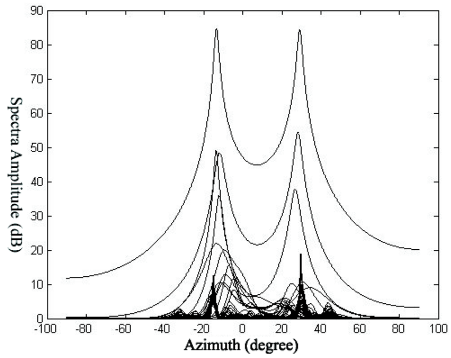

Figure 2. The MUSIC spectra of the 50 different virtual ULA $\bar{d}$ (SNR = 4dB)

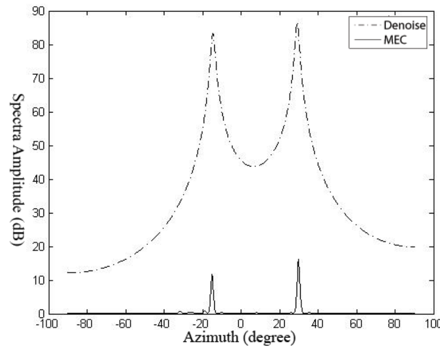

Figure 3. The MUSIC spectra of the denoise method and MEC (SNR = 4dB)

Figure 3 depicts the MUSIC spectra of the denoise method and the MEC. The chain line represents the result based on the denoise method. The active line represents the result based on the MEC. The estimate results based on the denoise method is -14.6º and 30.8º, and the estimate results based on the MEC is -15.1º and 29.9º. In this circumstance, the MEC is much more accurate than the denoise method. Table 1 depicts the performance of the 20 times simulation for each method. From Table 1, we might see that in the circumstance of $SNR > SNR_{\text{error0}}$, the MEC is much better than the denoise method because its bias and variance of estimates is much smaller. In this circumstance, the estimate results of the modified denoise approach equals those of the MEC rather than those of the denoise method.

Table 1. The performance of the 20 simulations (SNR = 4dB)

Method	Bias(degree)	Variance	Failed Times
MEC	0.13	0.0015	0
Denoise	1.01	0.8500	0
Modified Denoise	0.13	0.0015	0

Simulation 2: In this simulation, we selected the input signals with SNR which is lower than SNR_{error0} and larger than SNR_{error100}, such as 1.4dB. Figure 4 depicts the MUSIC spectra of 50 different virtual ULA $\bar{d}$ in this case.

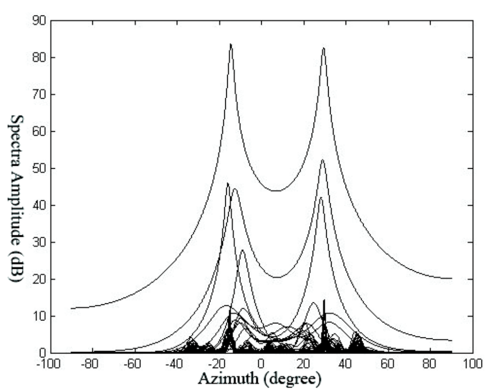

Figure 4. The MUSIC spectra of 50 different virtual ULA $\bar{d}$ (SNR = 1.4dB)

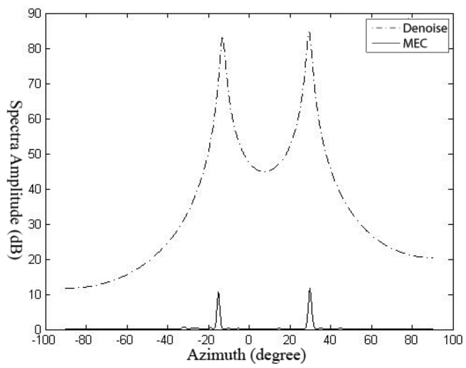

Figure 5. The MUSIC spectra of the denoise method and MEC while the latter is effective (SNR = 1.4dB)

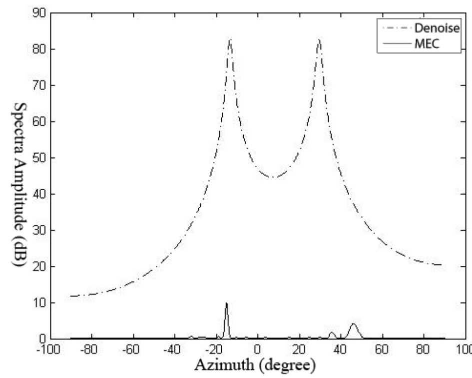

Figure 6. The MUSIC spectra of the denoise method and MEC when the latter is ineffective (SNR = 1.4dB)

Figure 5 depicts the MUSIC spectra of the denoise method and MEC while the latter is effective. The estimate result based on the denoise method is -14.6º and 30.3º, and the estimate result based on the MEC is -15.1º and 29.8º. In this circumstance, the MEC is still more accurate than the denoise method. Figure 6 depicts the MUSIC spectra of the denoise method and MEC when the latter fails. The estimate result based on the denoise method is -13.6º and 30.0º, and the estimate result based on the MEC is -15.1º and 46.7º. Apparently, 46.7º is beyond the margin of error. We can see it from Figure 6 that sometimes the MEC might fail to estimate the azimuth angles correctly, but the denoise method is more robust. Table 2 depicts the performance of the 20 times simulations. From Table 2, we could see that in the case of $SNR_{\text{error0}} > SNR > SNR_{\text{error100}}$, the MEC is still much more accurate than the denoise method while it is effective, but sometimes it might fail. The modified denoise approach is a weighted fusion method, which takes the weighted sum of the denoise method and the MEC results. Thus, the modified denoise approach fails no times in the simulations, which is more robust than MEC. Moreover, its bias is 0.26 degree and its variance is 0.14, which are smaller than the denoise method.

Table 2. The performance of the 20 simulations (SNR = 1.4dB)

Method	Bias(degree)	Variance	Failed Times
MEC	0.1125	0.0029	4
Denoise	1.13	0.3625	0
Modified Denoise	0.26	0.14	0

Simulation 3: In this simulation, we selected the input signals with SNR which is lower than $\text{SNR}_{\text{error100}}$, such as -3.5dB. Figure 7 depicts the MUSIC spectra of 50 different virtual ULA $\bar{d}$ in this case.

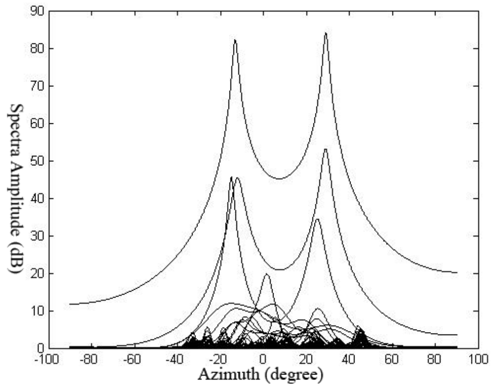

Figure 7. The MUSIC spectra of the 50 different virtual ULA $\bar{d}$ (SNR =-3.5dB)

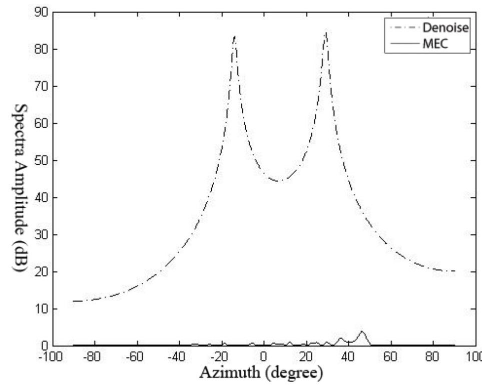

Figure 8. The MUSIC spectra of the denoise method and MEC (SNR = -3.5dB)

Figure 8 depicts the MUSIC spectra of the denoise method and MEC. The estimate results based on the denoise method are -13.8º and 28.6º, and the estimate results based on the MEC are beyond margin of error. We can see it in Fig. 8 that in this circumstance, the MEC has completely failed. In another words, the MEC might not be used for the estimation the DOA anymore when the investigated $SNR < SNR_{\text{error}100}$. Table 3 depicts the performance of the 20 times simulation for each criterion. From Table 3, we can see that in the circumstances of $SNR < SNR_{\text{error}100}$, the minimum error criterion is completely ineffective. The hybrid criterion takes the estimate result based on the denoise method. It might still be effective at a very low SNR while keep accurate at the same time.

Table 3. The performance of the 20 simulations (SNR = -3.5dB)

Method	Bias(degree)	Variance	Failed Times
MEC	\	\	20
Denoise	4.45	0.5995	0
Modified Denoise	4.45	0.5995	0

5. Conclusion

We present a modified denoise approach by selecting a more robust interpolated transformation matrix for UCA DOA estimation in low SNR case. It could be applied to the spatial smoothing technique to handle the DOA estimation of the coherent signal sources. We compared the performance of the MEC and the denoise method, both of which have advantages and disadvantages. The denoise method is more robust, but the MEC is more accurate. The modified denoise approach is the organic fusion results of them. It is more robust and accurate even if the SNR is very low. The statistics of bias, variance and failed times of the simulation results demonstrate the validity of the proposed method.

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References

[1] F. Belloni and V. Koivunen, "Reducing bias in beamspace methods for uniform circular array", IEEE International Conference on Acoustics, Speech, and Signal Processing, **(2005)**, pp. 973-976.
[2] P. Panagiotou, A. Polydoros and P. Thiennviboon, "Multi-path direction-of-arrival estimation for cognitive radio sensing", IEEE 21st International Symposium on Personal Indoor and Mobile Radio Communications, **(2010)**, pp. 791-796.
[3] B. Liao, Y. -T. Wu and S.-C. Chan, "A Generalized Algorithm for Fast Two-Dimensional Angle Estimation of a Single Source with Uniform Circular Arrays", IEEE Antennas and Wireless Propagation Letters, vol. 11, **(2012)**, pp. 984-986.
[4] B. H. Wang and H. T. Hui, "Investigation on the FFT-Based Antenna Selection for Compact Uniform Circular Arrays in Correlated MIMO Channels", IEEE Transaction on signal processing, vol. 59, no. 2, **(2011)**, pp. 739-746.
[5] R. Verghese, S. T. Subha and V. Bhaskar, "Ergodic capacity analysis and asymptotic tightness for multi-element antenna array with spatial correlation and transmit water-filling", IET Commun, vol. 6, **(2012)**, pp. 22-28.
[6] M. S. Mohammadi and M. M. Taheri, "Blind source separation and tracking of multiple frequency hopping signals for cognitive radio", Wireless Days 2008, WD08, 1st IFIP, **(2008)**, pp. 1-5.
[7] A. H. Tewfik and W. Hong, "On the application of uniform linear array bearing estimation techniques to uniform circular array", IEEE Trans. Signal Processing, **(1992)**, pp. 1008-1011.
[8] K. M. Reddy and V. U. Reddy, "Analysis of spatial smoothing with uniform circular arrays", Signal Processing, IEEE Transactions, vol. 47, no. 6, **(1999)**, pp. 1726-1730.
[9] J. Ping and C. Xu, "Location of interfering signal source in cognitive radio", Journal of Yanshan University, vol. 34, no. 6, **(2010)**, pp. 542-545.
[10] B. Friedlander and A. J. Weiss, "Direction finding using spatial smoothing with interpolated arrays", IEEE Transactions on Aerospace and Electronic Systems, vol. 28, no. 2, **(1992)**, pp. 574-587.
[11] M. Buhren, M. Pesavento and J. F. Bohme, "A new approach to array interpolation by generation of artificial shift invariances: interpolated esprit", IEEE International Conference on Acoustics, Speech and Signal Processing, vo. 5, **(2003)**, pp. 205-208.
[12] A. B. Gershman and J. F. Bohme, "Eigenstructure beamspace root estimator bank with interpolated array", IEEE International Conference on Acoustics, Speech and Signal Processing, vol. 4, **(1998)**, pp. 2017-2020.
[13] Y. W. Wu, "Estimating directions of arrival with generalized MUSIC using a linear-interpolated array manifold", International Conference on Acoustics, Speech, and Signal Processing, **(1991)**, pp. 3333-3336.
[14] A. J. Weiss and B. Friedlander, "Performance analysis of spatial smoothing with interpolated arrays", IEEE Transactions on Signal Processing, vol. 41, no. 5, **(1993)**, pp. 1881-1892.
[15] H. S. Yazdi and F. Homayouni, "Impulsive Noise Suppression of Images Using Adaptive Median Filter", IJSIP, vol. 3, no. 3, **(2009)**, pp. 1-12.
[16] V. JaganNaveen, T. prabakar, J. V. Suman and P. D. Pradeep, "Noise Suppression in Speech Signals using Adaptive Algorithms, IJSIP, vol. 3, no. 3, **(2009)**, pp. 87-96.
[17] G. Tsolis and T. D. Xenos, "Signal Denoising Using Empirical Mode Decomposition and Higher Order Statistics", IJSIP, vol. 4, no. 2, **(2011)**, pp. 91-106.
[18] M. Hossein-Abad Hadi, K. Morteza and S. Y. Hadi, "Least mean square algorithm tuned by fuzzy c-means for impulsive noise suppression of gray-level images", IJSIP, vol. 3, no. 4, **(2009)**, pp. 55-66.

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