

# Ratiometric GPS Iteration Localization Method Combined with the Angle of Arrival Measurement

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## **Abstract**

*Most of the localization algorithms have performed localization utilizing absolute point-to-point distance estimates. If the sensor network is not cooperative, there's no information about the strength of the original signal so that the location of a target node is hard to be found. It allows ratiometric location algorithm to be proposed. GPS algorithm give accurate location of unknown node, however it needs the absolute distance between the nodes. Ratiometric GPS iteration (RGPSi) localization method combines GPS and ratiometric algorithm for utilizing the benefits of two algorithms. RGPSi algorithm results in a little more position error and longer iteration than GPS algorithm. We introduce the method that helps ratiometric GPS iteration algorithm to be more accurate and reduce the iteration numbers through the angle of arrival localization algorithm. Not to add computation complexity to the RGPSi algorithm, nonlinear angle to location relationship is made linear. The proposed RGPSi combined with AOA algorithm shows more accurate position errors and less iteration numbers.*

**Keywords:** Localization, AoA, WSN, GPS, RVI, RSSI

## **1. Introduction**

It is important that the detection of target information or an event should be associated with the issue of determining its location. Therefore, the nodes in a sensor network are required to have ability to know their own physical locations. A network of a large number of densely distributed sensor nodes can apply localized algorithms and can also utilize location information for many network functions, to substantially reduce the complexity and processing requirements. That is, location aware network can allow location aided routing, collaborative signal processing and optimization of communication tasks in the network [1, 2].

Global Position System is representative in outdoor localization. However, constraints in size, form factor, and cost of construction of sensor nodes make traditional GPS receivers impractical to use in sensor nodes. Moreover, sensor networks may be deployed in regions where satellite signals may not be available. Hence, there has been a significant amount of work reported on algorithms for wireless nodes in a large ad hoc network to determine their locations without using GPS-like infrastructure [3-5]. Goals of these algorithms are to enable low cost, low complexity, small sensor nodes randomly deployed in a given target area to automatically determine their own positions with respect to some reference point.

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Most of the localization algorithms have performed localization utilizing absolute point-to-point distance estimate. Cooperative with the sensor network, the target is able to know the original signal (interested signal) strength at the target source as a pre-defined parameter or through communication. However, if the target is not a member of the sensor network or intruder, it is hard to get the original signal strength information, which prevents the use of absolute distance estimates. In the non-cooperative cases, instead, the original signal strength has to be estimated by collecting and analyzing a number of sensing data, often requiring non-linear optimization techniques [6, 7]. The high computation and communication overhead will be a tough challenge in small, low cost sensor nodes. In order to tackle this problem, a light-weight localization algorithm, dubbed Ratiometric Vector Iteration (RVI) [8] was proposed. RVI algorithm is based on relative distance ratio estimates rather than absolute distance estimates. Ratiometric algorithm concept was combined with GPS algorithm, called Ratiometric GPS Iteration (RGPSi) [9].

RVI or RGPSi utilizes relative distance ratios obtained from the relative signal strength ratio, which means they do need not the absolute distance estimation but the relative signal strength ratio. In Angle of Arrival (AoA), one of localization method, signal incident angles at a node can be measured by detecting the instant of peak strength. That means AoA also do not need the absolute incident signal strength. In this work, we present a localization technique by which the sensor nodes determine their position with respect to a set of fixed beacon nodes that are capable of covering the entire network area by powerful directional wireless transmissions. This localization method utilizes the Ratiometric GPS iteration and AOA algorithm together. Even though our technique requires costly implementation of the beacon nodes, we show that the sensor nodes do not need additional hardware complexity.

In this paper, we present a localization technique by which the sensor nodes determine their position with respect to a set of fixed beacon nodes that are capable of covering the entire network area by powerful directional wireless transmissions. This localization method utilizes the Ratiometric GPS iteration and AOA algorithm together. Even though our technique requires costly implementation of the beacon nodes, we show that the sensor nodes do not need additional hardware complexity. This paper is organized as follows. In Section 2 we discuss some of the main issues related to localization in sensor networks. We present our proposed technique in Section 3 with concepts of ratiometric algorithm. Performance of the proposed method, obtained from computer simulations, is presented in Section 4. Conclusions are presented in Section 5.

## **2. Localization in Wireless Sensor Networks**

There are several issues in making the location problem challenging. Size and construction costs prevent the use of complex hardware at sensor nodes. Densely deployed sensor nodes require fairly accurate estimation of sensor positions. Then, due the limited transmission range of sensor nodes can prevent direct communication with the beacon nodes which have their known position coordinates. Because sensor nodes will be deployed without a give localization pattern, they must determine their locations with respect to some fixed beacon nodes. Most of localization algorithm makes use of distance or angle measurements from a fixed set of reference points and apply multilateration or triangular techniques to determine the unknown location.

In Received signal strength (RSSI) measurements, the distance of the receiver from the transmitter can be determined with knowledge of the transmitter power, the path loss model, and the power of the received signal. A sensor node estimates the distances from three or more beacon nodes to compute its location. Multipath reflections, non-line-of-sight conditions, and other shadowing effects might prevent distance estimates reliable. To obtain reliable location estimates, there have been efforts using a combination of RSSI and other measurements proposed in [3-5]. However, nonuniform propagation environments are still challenging problem in RSSI methods.

Time-of-arrival and time-difference-of-arrival (ToA, TDoA) measurements estimate the distance among sensor nodes by measuring the propagation times or propagation time difference of the signals. The high propagation speed of wireless signals allow a small measurement error to cause a large error in the distance estimate. Hence, in a dense network such as a sensor network, Cricket [11] is the well-known solution based on localization techniques using ToA or TDoA measurements exploiting a signal such as ultrasound that has a smaller propagation speed. Fairly accurate results are given in virtue of additional hardware at the sensor nodes to receive the ultrasound signals.

Angle of arrival (AoA) measurements utilize special antenna configurations estimate the angle of arrival of the received signal from a beacon node. This concept is used for aircraft navigation systems. Beacon nodes transmit special omnidirectional signals and receiver can determine its bearings with respect to the stations [1]. There is the possibility of error in estimating the directions caused by multipath reflections.

Localization often utilizes absolute point-to-point distances estimated from RSS or time-of-arrival (TOA) or time-difference-of-arrival (TDOA) information. Centroid [12] is one of distributed localization algorithms which do not use the absolute distance estimates. Nodes localize themselves to the centroid of beacon node positions exploiting poor distance estimates using the RSS. The Weighted Centroid algorithm was proposed to improve Centroid [8]. However, all of the above solutions are suitable for localizing sensor network itself or cooperative targets these algorithms require communication between the target and sensors, impractical in non-cooperative scenarios.

### 3. System Model

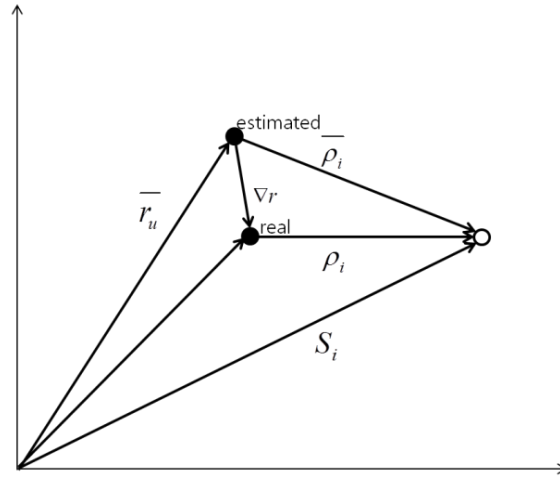
There are conventional, widely used RSS based sensing models: the log-normal model [10] and the simple exponential model [8]. In this paper, a simple exponential model is used as follows:

$$P_i = \frac{a}{\rho_i^\alpha} + n_i \quad (1)$$

where  $P_i$  is the measured RSS value in the  $i$ -th beacon from a target,  $a$  is the original signal strength at the target source.  $\rho_i = |r - S_i|$  means the Euclidean distance between  $r$  and  $S_i$  where  $r$  is the location of a target and  $S_i$  is the location of the  $i$ -th sensor in two dimensional coordinate system.  $\alpha$  is the path loss exponent and  $n_i$  is the White Gaussian noise. It is assumed  $P_i$ ,  $S_i$ , and  $\alpha$  are known values.

Because due to unknown  $a$ , it is difficult to estimate an absolute distance between nodes, in RVI, to determine the location of a target, a relative distance ratio  $P_i/P_j$  ( $i \neq j$ ) instead of absolute distances was used. Ignoring the noise, the distance ratio between the  $i$ -th sensor and  $j$ -th sensor can be expressed by the RSS ratio between them.

$$P_i^{-1/\alpha} : P_j^{-1/\alpha} = |\rho_i| : |\rho_j| \quad (2)$$



**Figure 1. Simplified GPS System. A filled circle is a target and empty circle is a sensor node (or beacon node)**

Ratiometric GPS Iteration algorithm is to combine the ratiometric concept and GPS algorithm. Ratiometric GPS Iteration uses the simplified version of GPS [11] shown in Figure 1. where  $\hat{r}_u$  and  $r_u$  is the estimated location and the real location of the target, respectively, and  $|\hat{\rho}_i|$  and  $|\rho_i|$  indicates the estimated and measured ranges to the  $i$ -th sensor. The algorithm makes the difference between  $|\hat{\rho}_i|$  and  $|\rho_i|$  smaller through updating the estimated location. Then, update vector is calculated as

$$\begin{aligned} \Delta r &= r_u - \hat{r}_u \\ \Delta \rho_i &= |\hat{\rho}_i| - |\rho_i| \cong \hat{I}_i \cdot \Delta r \end{aligned} \quad (3)$$

where  $\hat{I}_i$  is the unit vector of  $|\hat{\rho}_i|$ . The update vector  $\Delta r = [\Delta x \ \Delta y]$  is obtained from

$$\begin{bmatrix} \Delta \rho_1 \\ \Delta \rho_2 \\ \dots \\ \Delta \rho_n \end{bmatrix} = \begin{bmatrix} \hat{I}_{1x} & \hat{I}_{1y} \\ \hat{I}_{2x} & \hat{I}_{2y} \\ \dots & \dots \\ \hat{I}_{nx} & \hat{I}_{ny} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (4)$$

where  $n_i$  is the sensor nodes neighboring a target.

Ratiometric GPS Iteration algorithm is similar to GPS algorithm in calculation of the update vectors but doesn't require the absolute distances so it is similar to RVI. In Ratiometric GPS Iteration, instead of calculating  $\Delta \rho_i = |\hat{\rho}_i| - |\rho_i|$ , distance ratios will be calculated as follows:

$$\Delta u_i = \frac{|\hat{\rho}_i|}{\sum_{k=1}^n |\hat{\rho}_k|} - \frac{|\rho_i|}{\sum_{k=1}^n |\rho_k|} = \bar{u}_i - u_i \quad (5)$$

where  $\bar{u}_i$  and  $u_i$  are the normalized distance ratio of estimated and measured distances. The measured distance ratio can be modified with using Equation 2.

$$u_i = \frac{P_i^{-1/\alpha}}{\sum_{k=1}^n P_k^{-1/\alpha}} = \frac{1}{\sum_{k=1}^n (P_k/P_i)^{-1/\alpha}} \quad (6)$$

From Equation 5. and Equation 6.,

$$\Delta \rho_i \cong \Delta u_i \sum_{k=1}^n |\hat{\rho}_k| = |\hat{\rho}_i| - \frac{\sum_{k=1}^n |\hat{\rho}_k|}{\sum_{k=1}^n (P_k/P_i)^{-1/\alpha}} \quad (7)$$

We assumed that as the update continues repeatedly, the sum of estimated ranges,  $\sum_{k=1}^n |\hat{\rho}_k|$  will become equal to the sum of measured ones,  $\sum_{k=1}^n |\rho_k|$ . It is true when this algorithm converges and then through simulations, it is conformed.

Sensor nodes equipped with directional antennas can measure the angle of incident signal from a target.

$$\theta_i(r) = \tan^{-1} \frac{y_i - y}{x_i - x} + n_\theta = \bar{\theta}_i(r) + n_\theta \quad (8)$$

where,  $\theta_i$  is measured angle at the  $i$ -th sensor node and is related to the position of a target.  $r = [x, y]$  is the position vector of target,  $(x_i, y_i)$  is the coordinate of the  $i$ -th sensor and  $n_\theta$  is assumed to be the white Gaussian noise with zero mean and variance of  $\sigma_\theta^2$ . For simplicity,  $\bar{\theta}_i(r)$  can be linearized with Taylor series.

$$\bar{\theta}_i(r) = \bar{\theta}_i(r_0) + \Phi_i \Delta r + n \quad (9)$$

where  $\Phi_i$  is the Jacobian matrix, and  $\Delta r = (r - r_0)$ , where  $r_0$  is the initial estimated position. Considering the neighboring nodes around a target node and ignoring the higher order terms, Equation 9. can expressed

$$\begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \dots \\ \Delta \theta_n \end{bmatrix} = \begin{bmatrix} \bar{\theta}_1(r) - \bar{\theta}_1(r_0) \\ \bar{\theta}_2(r) - \bar{\theta}_2(r_0) \\ \dots \\ \bar{\theta}_n(r) - \bar{\theta}_n(r_0) \end{bmatrix} = \begin{bmatrix} \Phi_{1x} & \Phi_{1y} \\ \Phi_{2x} & \Phi_{2y} \\ \dots & \dots \\ \Phi_{nx} & \Phi_{ny} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (10)$$

In Equation 10.,  $\Phi_{ix}$  and  $\Phi_{iy}$  are the partial derivatives of  $\theta_i(r)$  with  $x$  and  $y$  respectively.

From Equation 4. and Equation 10., we can find final expression combining the Ratiometric GPS Iteration and AoA algorithm.

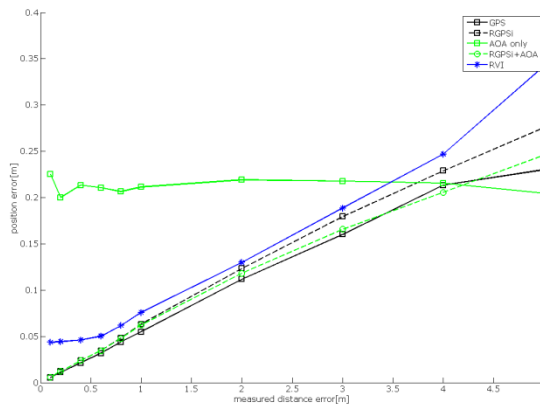
$$\begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \dots \\ \Delta\rho_n \\ \Delta\theta_1 \\ \Delta\theta_2 \\ \dots \\ \Delta\theta_n \end{bmatrix} = \begin{bmatrix} \hat{I}_{1,x} & \hat{I}_{1,y} \\ \hat{I}_{2,x} & \hat{I}_{2,y} \\ \dots & \dots \\ \hat{I}_{n,x} & \hat{I}_{n,y} \\ \Phi_{1,x} & \Phi_{1,y} \\ \Phi_{2,x} & \Phi_{2,y} \\ \dots & \dots \\ \Phi_{n,x} & \Phi_{n,y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} n_{\rho_1} \\ n_{\rho_2} \\ \dots \\ n_{\rho_n} \\ n_{\theta_1} \\ n_{\theta_2} \\ \dots \\ n_{\theta_n} \end{bmatrix} \quad (11)$$

We assume noise vector  $n$  to be Gaussian with zero mean and covariance matrix  $R$

$$R = \text{diag}(\sigma_{\rho_1}^2 \sigma_{\rho_2}^2 \dots \sigma_{\rho_n}^2 \sigma_{\theta_1}^2 \sigma_{\theta_2}^2 \dots \sigma_{\theta_n}^2) \quad (12)$$

#### 4. Simulation Results

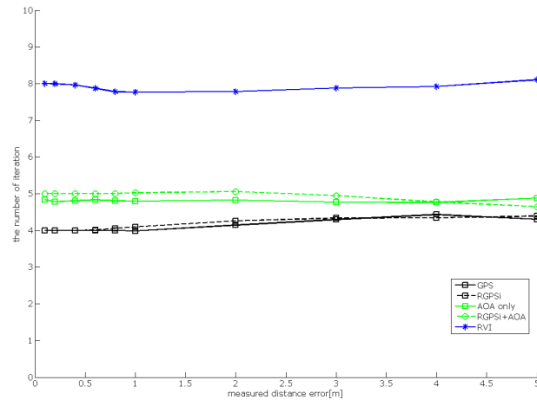
Localization algorithms using multilateration or triangulation need three or more beacon or sensor nodes which know their locations around a target of which location has to be found. We deployed three sensors ( $k=3$ ) which forms a triangular to contribute to find the target's location. Initial position of the target was found using weighted Centroid. Due to their property, algorithms adopting ratiometric concepts can accurately find the target's location when the target exists inside the triangle formed by three sensors. Generally, if the target is located inside the circumscribed circle passing through all three sensors, ratiometric algorithms accurately estimate the target location with no error.



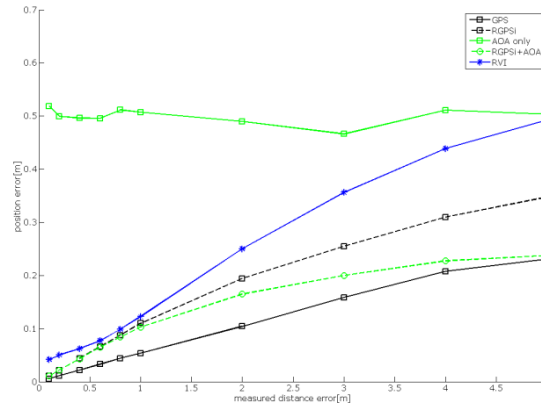
**Figure 2. Simulated position errors with angle error of 10° where the target is inside the triangle formed with sensor nodes.**

We have simulated several cases and then compared results from ratiometric GPS iteration assisted with AoA to the results from several algorithms, such as GPS, RVI, AoA-only, and ratiometric GPS iteration. For comparison, we perform the simulations with the measured distance standard deviation. Figure 2 shows the simulated position error when the target exists inside the triangle formed with sensors. For Calculation of RGPSi assisted with AoA, it is assumed that only one of sensor node of three sensors can measure the angle and other nodes

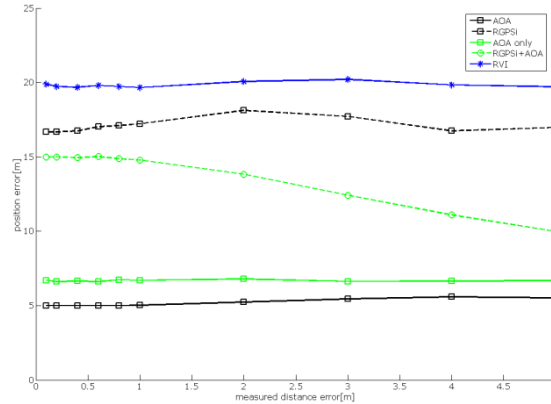
have ability to only measure signal strengths. Then, the angle standard deviation is  $10^\circ$ . As expected, because AOA algorithm doesn't depend on distance error, the position error shows nearly constant with respect to distance error. GPS, RVI, RGPSi and RGPSi assisted with AoA (RGPSi+AOA) show the nearly same position errors. However, entirely, GPS algorithm has the lowest position error and RVI does lowest. And RGPSi+AOA show better result than RGPSi. It can be known that AoA algorithm helps RGPSi improve accuracy. Figure 3 shows the iteration number to converge where RVI take more iterations than other algorithms.



**Figure 3. The number of iterations angle error of  $10^\circ$  where the target is inside the triangle formed with sensor nodes**

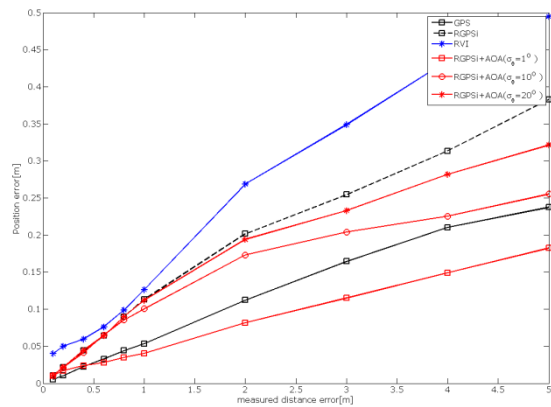


**Figure 4. Simulated position errors with angle error of  $10^\circ$  where the target is outside of the triangle formed with sensor nodes**



**Figure 5. The number of iteration with angle error of 10o where the target is outside of the triangle formed with sensor nodes**

Figure 4 is the result of position errors when the target is outside of the triangle but inside the circumscribed circle of three sensors. With comparison with Figure 3, errors of Figure 4 are greater than those of Figure 3. That means position accuracies depend on where the targets are. And then we can find angle assistance on RGPSi has more effect on accuracy. So when measure distance error gets more, errors of RGPSi+AOA algorithms approach those of GPS and show more accurate than only RGPSi. In Figure 5, the number of iteration is shown where we can find algorithms except AOA and GPS shows more iteration. Generally, Ratiometric algorithms cannot find the position of the target when it is out of a circumscribed circle formed with sensors. So, in the case of Figure 4 and Figure 5, the target is near the boundary of the circumscribed circle and the distance error makes the estimated position of the target out of circumscribed circle. This is the cause of the profound errors.



**Figure 6. Improvement of position errors with the angle measurements.**

Figure 6 shows the position errors with respect to measured angle error standard deviations of  $\sigma_{\theta}=1^{\circ}$ ,  $10^{\circ}$ , and  $20^{\circ}$ . It there is a possibility to measure an angle with  $\sigma_{\theta}=1^{\circ}$  accurately, the position error goes smaller than error of GPS algorithm. And then measured angle error gets more, the position error obtained by RGPSi+AOA get more and approaches to errors of RGPSi. We can conclude that due to angle assistance effect, RGPSi assisted with AOA improves their position errors.



## 5. Conclusion

Ratiometric algorithms are based on distance ratio estimates rather than absolute distance estimates which are often impossible to calculate. GPS algorithm is one of generally used localization algorithm. So, merits of GPS and RVI algorithm was combined to ratiometric GPS iteration. In this paper, to improve the accuracy of RGPSi algorithm, RGPSi assisted with AoA is proposed. This ideas result from angle measurement at sensor nodes can require not absolute signal strength but relative signal strength. Ratiometric algorithm also needs only the strength ratio. So, we can combine two algorithms and this method can improve the accuracy of the algorithm and reduce the iteration number with no increasing computation complex. And some simulation results show the proposed algorithm can improve the performance of ratiometric GPS iteration algorithm with the help of angle measurement. This method also can be implemented in various communication environments with low cost and high accuracy. And due to low computation complex, low performance processors can adopt this algorithm with no big burden.

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