Improve Method on DOA Estimation Accuracy of Signal Subspace Sensitivity Based on Mutual Coupling Matrix

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Abstract

The performance of the DOA algorithm and many other high resolution methods degrades severely with the high correlated multipath signals encountered in target low angle tracking. DOA estimation error is overcome by deriving new version of the effects model error and sensitivity analysis on subspace methods. An improved algorithm for DOA estimation of coherent signal is proposed. This algorithm has based on the structure of mutual coupling matrix of uniform linear array and effects model error sensitivity analysis. The performance of these two approaches is then compared with that of the well-known MUSIC algorithm using spatial smoothing. Through simulation, we show that the proposed method offers significantly improved estimation resolution and accuracy relative to existing method.

Keywords: *DOA, Sensitivity, Mutual Coupling, Coherent, Subspace*

1. Introduction

The estimation of direction of arrival(DOA) in multiple signals by using an smart array antenna very important in radar, sonar and wireless communication [1]. Smart antenna generally refers to any antenna array, terminated in a sophisticated signal processor, which can adjust or adapt its own beam pattern in order to emphasize signals of interest and to minimize interfering signals. Smart antennas generally encompass both switched beam and beam formed adaptive systems [2]. The rapid growth in demand for smart antennas is fueled by two major reasons [3]. First, the technology for high speed analog to digital converters and high speed digital signal processing is burgeoning at an alarming rate. Even though the concept of smart antennas has been around since the late years [4], the technology required in order to make the necessary rapid and computationally intense calculations has only emerged recently. Early smart antennas, or adaptive arrays, were limited in their capabilities because adaptive algorithms were usually implemented in analog hardware. Second, the global demand for all forms of wireless communication and sensing continues to grow at a rapid rate. Smart antennas are the practical realization of the subject of adaptive array signal processing and have a wide range of interesting applications.

Smart antennas have numerous important benefits in wireless applications as well as in sensors such as radar [5]. In the realm of mobile wireless applications, smart antennas can provide higher system capacities by directing narrow beams toward the users of interest, while nulling other users not of interest. Smart antenna can be used to enhance direction finding techniques by more accurately finding direction of arrival(DOA)at array of spectral estimation techniques can be incorporated, which are able to isolate the DOA with an angular precision that exceeds the resolution of the array [6]. Smart antenna direction finding capabilities also enhance geo location services enabling a wireless system to better determine the location of a particular mobile user. Additionally, smart antennas can direct the array main beam toward signals of interest even when no reference signal or training sequence is available. This capability is called blind adaptive beamforming [7]. Let us list some of the numerous potential benefits of smart antennas.

► Improved system capacities

► Higher permissible signal bandwidths

► Higher signal to interference ratios

► Sidelobe canceling or null steering

► Multipath mitigation

► Improved angle of arrival estimation and direction finding

► Improved array resolution

Many effective algorithm for simultaneously estimating DOA for multiple signals have been developed in the recent past [8]. Many high resolution eigen decomposition methods have been proposed, such as MUSIC, ESPRIT and other ML [9]. Most of these algorithms assume that the array manifold is known and the signals are non-coherent. DOA estimation has been known as spectral estimation, angle of arrival estimation, or bearing estimation [10]. Some of the earliest references refer to spectral estimation as the ability to select various frequency components out of a collection of signals. This concept was expanded to include frequency wavenumber problems and subsequently DOA estimation. Bearing estimation is a term more commonly used in the sonar community and is DOA estimation for acoustic problems. Much of the state of the art in DOA estimation has it root in time series analysis, spectrum analysis, eigen structure methods, parametric methods, linear prediction methods, beamforming, array processing, and adaptive array method. Most of them assume that the array response is completely known. However, many factors, such as mutual coupling among the different array sensors, will alter the array response in practical applications.

In this paper, a new DOA estimation considering minimum effects of model error uses sensitivity analysis on subspace method is proposed [11]. In the proposed DOA estimation method, the target DOA estimation can be minimize model errors from the angle in the target signal by sensitivity analysis on subspace method, which is a function of the target DOA. Since DOA resolution high increased from proposed method in this paper. Also the interference on the DOA estimation is reduced. In this way, the estimation resolution and accuracy of the proposed method are better than those for existing method.

2. Array Correlation Signal Analysis

MUSIC is an acronym which stands for MUltiple SIgnal Classification. This approach was first posed by Schmidt [12]. It is a popular high resolution eigen-structure method. MUSIC promises to provide unbiased estimates of the number of signals, the angles of arrival, and the strengths of the waveforms. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix diagonal. The incident signals may be somewhat correlated creating a non-diagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC method break down and other method must be implemented to correct this weakness. Many of the DOA algorithm rely on the array correlation matrix. In order to understand the array correlation matrix, let us begin with a description of the array, the received signal, and the additive noise. They are received by an array of M elements with Mpotential weights. Each received signal includes additive, zero mean, Gaussian noise. Time is represented by the k^{th} time sample. Thus, the array output can be given in the following form [13-14]

$$y(\mathrm{t}) = \mathrm{w}^H \; \mathrm{x(t)} \qquad \textbf{(1)}$$

Where

$$\mathrm{x(t)} = \mathrm{A\,s(t)} + \mathrm{N(t)} \qquad \textbf{(2)}$$

$$\mathrm{x}(t) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_D)] \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_D(t) \end{bmatrix} + \mathrm{N(t)} \qquad (3)$$

s(t) is signal vector, N(t) is noise vector, A is steering vectors, w is array weight, $(\ )^H$ is hermit matrix. Each of the D complex signals arrives at angles and is intercepted by the M antenna elements. It is initially assumed that the arriving signals are monochromatic and the number of arriving signals D < M. It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of the incoming signal. Obviously if the transmitters are moving, the matrix of steering vectors is changing with time and the corresponding arrival angles are changing. Unless otherwise stated, the time dependence will be suppressed in Eq(1) and (2). In order to simplify the notation let us define the array correlation matrix as follows [15-16]

$$R_{xx} = E[\mathrm{x(t)x(t)^H}] \qquad \textbf{(4)}$$

$$R_{xx} = E[\mathrm{x(t)x(t)^H}] = E[(A\,s + N)(s^H A^H + N^H)] \qquad \textbf{(5)}$$

$$= \mathrm{A\,E[s\,s}^H]A^H + E[N\,N^H] \qquad (6)$$

$$= \mathrm{A\,R}_{ss}\,A^H + R_{nn} \qquad (7)$$

Where R_{ss} is source correlation matrix, R_{nn} is nose correlation matrix. Consider an antenna composed of M antenna sensors arbitrarily located in spatial and assume that a signal impinges on the array. Then the output of m^{th} the sensor can be written as follow [5-6].

$$X(\mathrm{t}) = \mathrm{C}\;\Lambda\,\mathrm{A}(\theta)\,S(t) + N(t) \qquad \textbf{(8)}$$

where $\mathrm{X(t)}$ is the received signal vector, C is the mutual coupling matrix, Λ is complex diagonal matrix, $S(t)$ is the signal vector, $\mathrm{A}(\theta)$ is array manifold matrix, $a(\theta_m)$ is steering vector corresponding to the m^{th} source at direction θ_m, and $\mathrm{N(t)}$ is additive white noise. In particular, due to the effects of the array model response errors, the real array manifold is different form the ideal array manifold. Thus the accuracy and resolution of he DOA estimation will be degraded by the errors in the array manifold. We refer to the different between the ideal and real array parameters. The spatial spectrum of MUSIC and DOA estimation can be written as follow [17]

$$\mathrm{EF}(\theta,\mu) = \frac{1}{A(\theta,\mu)E_n^H(\mu)\,E_n(\mu)A^H(\theta,\mu)} \qquad \textbf{(9)}$$

Where μ is real array parameters, μ_0 is ideal array parameters, E_n is noise subspace. We can get the exact estimate of the true DOAs. When $\mu_0 \neq \mu$, the peaks of $\mathrm{EF}(\theta,\mu)$ will no longer be the true DOAs and the accuracy of DOA estimations can be degraded by even small model errors.

3. Sign Model Sensitivity on Subspace Method

We assume that the array correlation covariance matrix is exactly estimated, which means having an infinite number of independent samples, and the effect of model errors is sufficiently small so that the MUSIC spectrum has distinct peaks. Considering only the effect of variations in real parameters, we can see how much the peaks shift due to model error effects [18]. We can be written as follow

$$\overline{EF}(\theta,\mu) = \frac{1}{EF(\theta,\mu)} = A(\theta,\mu)E_n^H(\mu)\,E_n(\mu)A^H(\theta,\mu) \qquad \textbf{(10)}$$

In the presence of two sources with true DOA, θ_1 and θ_2, we perform the first-order Taylor expansion to see how much the peak locations shift due to model errors and get the angular deviation

$$\Delta\theta_i = \theta - \theta_i = -\frac{\left.\frac{\partial \overline{EF}_1(\theta_i,\mu)}{\partial \mu}\right|_{\mu=\mu_0}(\mu-\mu_0)}{\left.\frac{\partial \overline{EF}_1(\theta,\mu)}{\partial \theta}\right|_{\theta=\theta_i}} \qquad \textbf{(11)}$$

Where $\overline{EF}_1(\theta,\mu) = \frac{\partial \overline{\mathrm{EF}}(\theta,\mu)}{\partial\theta}$, $\mathrm{i} = 1,2$, $\mu - \mu_0$ is the model error. Let us assume that

$$\mu - \mu_0 = \sigma_\mu\,\gamma \qquad \textbf{(12)}$$

Where γ is a random vector with zero mean and unit variance and σ_u is a positive scalar. The mean can be written as follow

$$E[\mu - \mu_i] = 0 \qquad \textbf{(13)}$$

The standard deviation can be written as follow

$$SD(\theta-\theta_i)=\frac{\left|\frac{\partial\overline{EF}_1(\theta_i,\mu)}{\partial\mu}\Big|_{\mu=\mu_0}\right|}{\left|\frac{\partial\overline{EF}_1(\theta,\mu)}{\partial\theta}\Big|_{\theta=\theta_i}\right|}\sigma_\mu \quad (14)$$

Where σ_i and σ_{DOA} can be written as follow

$$\sigma_i=\frac{\left|\frac{\partial\overline{EF}_1(\theta_i,\mu)}{\partial\mu}\Big|_{\mu=\mu_0}\right|}{\left|\frac{\partial\overline{EF}_1(\theta,\mu)}{\partial\theta}\Big|_{\theta=\theta_i}\right|} \quad (15)$$

$$\sigma_{DOA}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2} \quad (16)$$

Where σ_{DOA} represents the mean square value of DOA errors caused by modeling errors and N is the number of signals. We can see that angular stander deviation will be increased with σ_μ. When the model errors are sufficiently large, the algorithm will fail to resolve two or more close sources, the model errors will affect the DOA resolution. In this case, as the model errors increase the valley between two peaks will become flattened and final the two peaks merge into one so that the algorithm loses angular resolution. To simplify the analysis, we assume that θ^* is constant and that $\theta^*=\theta_0$, where $\theta_0=(\theta_1+\theta_2)/2$. When $\theta^*=\theta_0$ and suing the first order Taylor expansion around μ_0, the smallest norm model error which will cause the MUSIC algorithm to fail is given by

$$\mu-\mu_i=-\frac{\overline{EF}_2(\theta_0,\mu_0)\frac{\partial\overline{EF}_0(\theta_0,\mu)}{\partial\mu}\Big|_{\mu=\mu_0}}{\left|\frac{\partial\overline{EF}_2(\theta_0,\mu)}{\partial\theta}\Big|_{\mu=\mu_0}\right|^2} \quad (17)$$

The failure threshold of the proposed algorithm is

$$\sigma_{fail}=\frac{1}{\sqrt{M}}\|(\mu-\mu_0)\| \quad (18)$$

4. Simulation

In this section, through simulation, we showed that we compare existing method with proposed method. Existing method used MUSIC method. Consider a uniform linear array with 9 sensors, each sensor separated by half wavelength. Figure 1 shows the Root Mean Square Error (RMSE) of the DOA estimation of a signal arriving using the MUSIC and proposed method for different SNR. The SNR is varied from 0dB to 30dB. In Figure 1, existing method showed $10^{-1.8}$ and proposed method $10^{-2.8}$ in the case of SNR=10dB, respectively. Proposal method improved 10dB than existing method in this paper. Figure 2 shows DOA estimation using existing method in angle[-10^o,10^o, 30^o]. The existing method increase DOA estimation resolution of a desired signal. Figure 3 shows DOA estimation using proposed method in angle[-10°, 10°, 30°]. The proposed method all estimated a desired signal, and DOA estimation improved resolution than existing method. Figure 4 shows DOA estimation using existing method in angle[0°, 3°, 5°]. Figure 4 is not estimation of 3 target signals. Figure 5 shows DOA estimation using proposed method in angle[0°, 3°, 5°]. Fig.5 is correctly estimation of 3 target signals

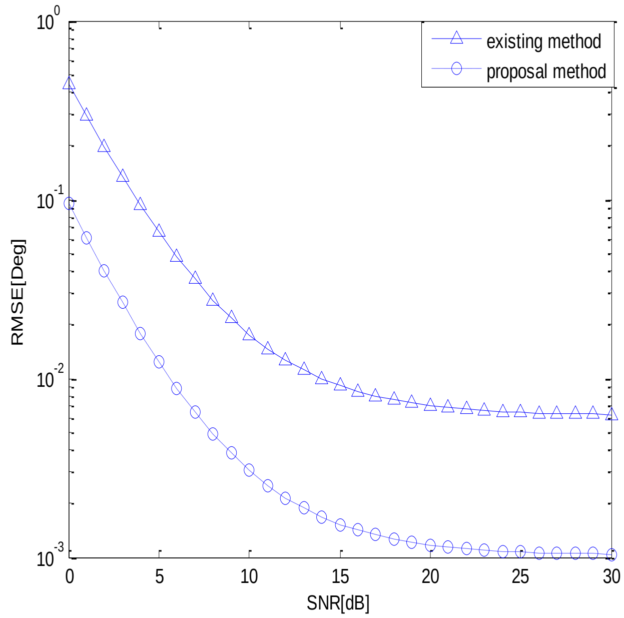

Figure 1. MMSE compare Existing method with Proposed Method

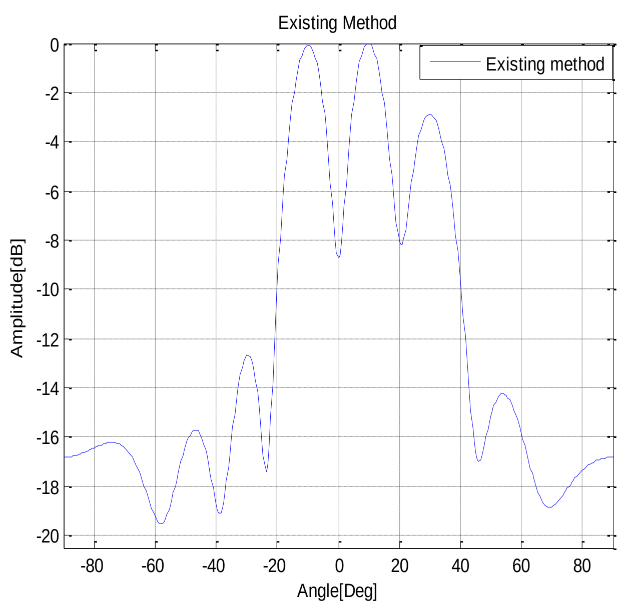

Figure 2. DOA of Existing method

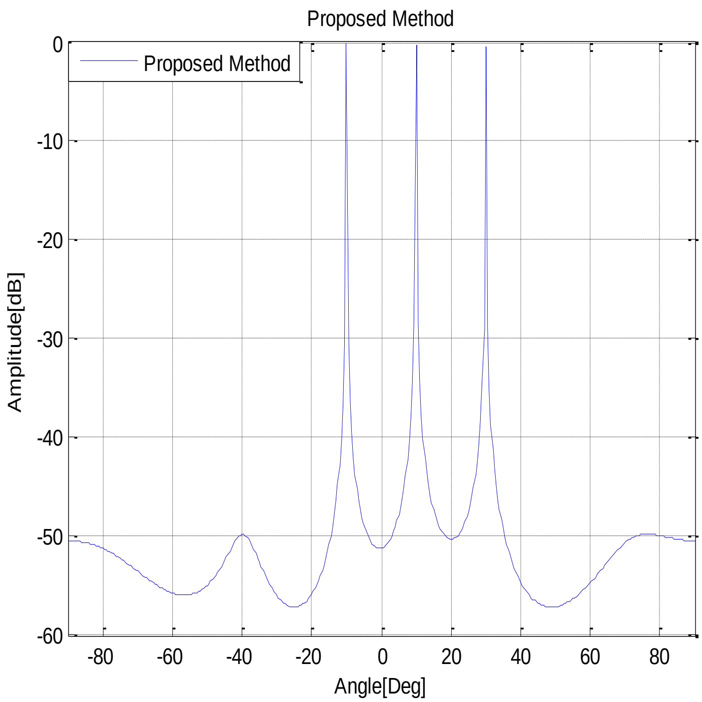

Figure 3. DOA of Proposed method

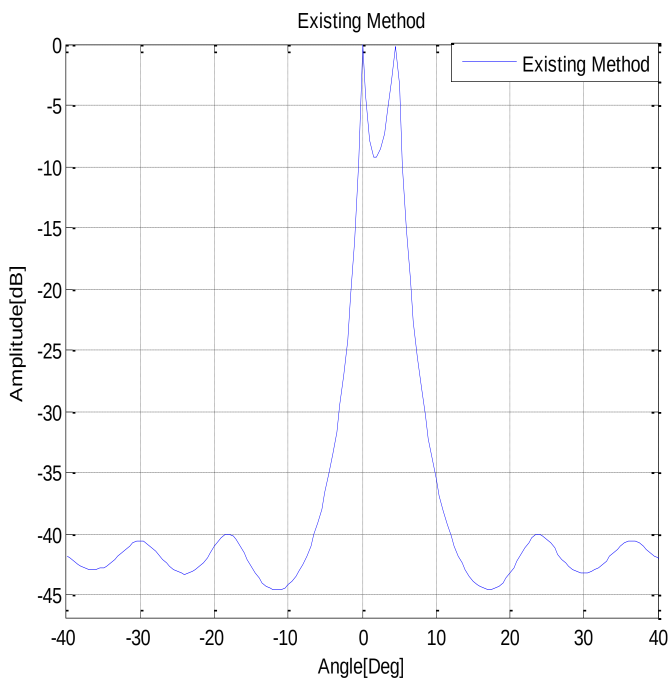

Figure 4. DOA of Proposed method

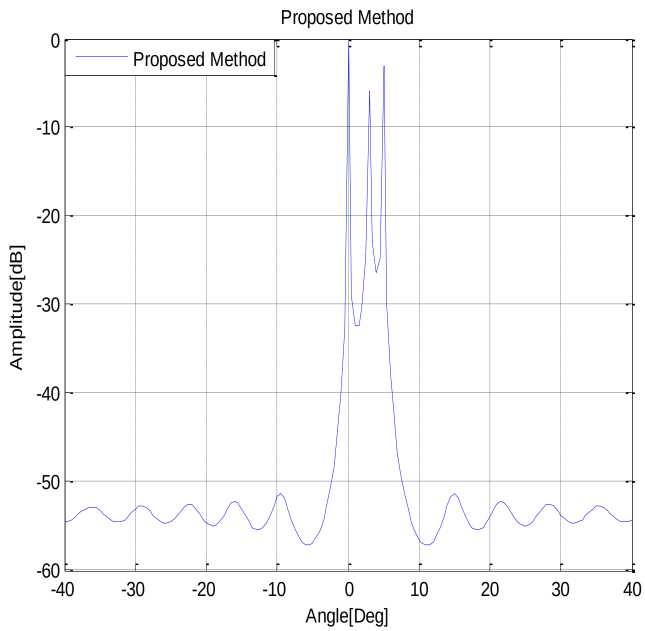

Figure 5. DOA of Proposed method

5. Conclusion

A method minimum angular resolution model errors base on sensitivity analysis on subspace methods has been proposed. In this paper, a new DOA estimation considering minimum effects of model error uses sensitivity analysis on subspace method is proposed. In the proposed DOA estimation method, DOA estimation can be minimum model errors from the angle in the desired signal by sensitivity analysis on subspace method. Since DOA resolution high increased from proposed method in this paper In the proposed method, sensitivity analysis on subspace methods are used to obtain an optimum estimation of the desired signal among received signal on antennal array system. Through simulation, the performance showed that the proposed method improved resolution and improved accuracy of DOA estimation relative to those achieved with existing method.

References

[1] F. B. Gross, “Smart Antennas for wireless Communication with Matlab”, McGraw-Hill Press, New York, **(2005).**

[2] T. K. Sarkar, M. C. Wicks, M. S. Palma and R. J. Bonneau, “Smart Antennas”, IEEE Press, New York, **(2003).**

[3] E. Nicolau, “Adaptive Array”, ELSEVIER Press, New York, **(1983).**

[4] J. E. Hudson, “Adaptive Array Principles”, IEE Press, London, **(1989).**

[5] S. Song, Yoo, M. K. Hyuck and M. B. Jun, “Computationally efficient smart antennas for CDMA wireless communications”, IEEE Transaction on Vehicular Technology, IEEE Press, New York, **v**ol. 50, no. 6, **(2001)**, pp. 1613-1628.

[6] T. S. G. Basha, G. Aloysius, B. R. Rajakumar, M. N. G. Prasad and P. V. Sridevi, “A constructive smart antenna beam forming technique with spatial diversity”, IET Microwaves Antennas & Propagation, IET Press New York, vol. 6, no. 7, **(2012)**, pp. 773-780.

[7] C. D. C. Chang and C. N. Hu, “Smart antennas for advanced communication systems”, IEEE Proceeding, IEEE Press, New York, vol. 100, no. 7, **(2012)**, pp. 2233-2249.

[8] R. K. Dandekar, H. Ling and G. Xu, “Experimental study of mutual coupling compensation in smart antenna applications”, IEEE Transactions on Wireless Communications, IEEE Press, New York, vol. 1, no. 3, **(2002)**, pp. 480-487.
[9] Y. Zhang, B. A. Obeidat and G. A. Moeness, “Spatial polarimetric time frequency distributions for direction of arrival estimations”, IEEE Transaction on Signal Processing, IEEE Press, New York, vol. 54, no. 4, **(2006)**, pp. 1327-1340.
[10] M. Jansson, A. Swindlehurst and B. Ottersten, “Weighted subspace fitting for general array error models”, IEEE Transaction on Signal Processing, IEEE Press, New York, vol. 46, no. 9, **(1998)**, pp. 2484-2498.
[11] F. M. Montoya, D. H. C. Rosales and C. A. L. Miranda, “DOA Estimation in Mobile Communications System using Subspace Tracking Methods”, IEEE Latin America Transactions , IEEE Press, New York, vol. 6, no. 2, **(2008)**, pp. 123-129.
[12] R. Schmidt, “Multiple Emitter Location and Signal Parameter estimation”, IEEE Transactions on Antenna propagation, IEEE Press, New York, vol. 34, no. 2, (**1986**), pp. 276-280.
[13] Y. T. Yu, S. L. Hoi-Shun, C. H. Niow and H. T. Hui, “Improved DOA esitmations using the receiving mutual impedances for mutual coupling compensation:an experimental sytdy”, IEEE Transaction on Wireless Communications, IEEE Press, New York, vol. 10, no. 7, **(2011)**, pp. 2228-2233.
[14] D. B. Rao, and K. V. S. Hari, “Effect of spatial smoothing on the performance of MUSIC and the minimum norm method”, IEE Proceedings Radar and Signal processing, IET Press, London, vol. 137, no. 6, **(1990)**, pp. 449-458.
[15] F. M. Montoya, D. H. Covarrubias and C. A. L. Miranda, “DOA estimation in mobile communications system using subspace tracking methods”, IEEE Latin America Transactions, IEEE Press, New York, vol. 6, no. 2, **(2008)**, pp. 123-129.
[16] L. M. Mccloud and L. L. Scharf, “A new subspace identification algorithm for high resolution DOA estimation”, IEEE Transactions on Antennas and Propagation, IEEE Press, New York, vol. 50, no. 10, **(2002)**, pp. 1382-1390.
[17] A. S. Lee and K. Thomaks, “A performance analysis of subspace based methods in the presence of model errors. I. The MUSIC algorithm”, IEEE Transactions on Signal Processing, IEEE Press, New York, vol. 40, no. 7, **(1992)**, pp. 1758-1774.
[18] A. Ferreol, P. Larazbal and viberg, “On the asymptotic performance analysis of subspace DOA estimation in the presence of modeling errors: case of MUSIC”, IEEE Transaction on Signal Processing, IEEE Press, New York, vol. 54, no. 3, **(2006)**, pp. 907-920.

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