New approach to MFR design with an alpha– beta filters

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Abstract

The probability of detection and the accuracy of a measurement are determined by the available SNR ratio and hence by the energy used for illuminating the target. Also, increasing the rate of illumination of a target allows obtaining less uncertain predictions. A high energy burst and a high illumination rate are desirable from the tracker's accuracy point of view. However they negatively affect the transmitter beam occupancy budget and therefore they are undesirable from the point of view of the radar's ability to deal with multiple targets. Hence, tracker parameters (including track illumination rate and dwell-time or burst energy) should be designed to satisfy a double objective such as minimum beam occupancy, which will give maximum efficiency in the use of radar's energy and maximum beam scheduling effectiveness (shorter pulses, reduced dwell-time)-desired tracking accuracy. In this paper we presented a new approach to optimally select both the energy of the tracking waveforms and the track sampling rates which jointly minimize occupancy and satisfy angular accuracy requirements for a MFR with an $\alpha - \beta$ *tracking filter.*

Keywords: *MFR, SNR, m*easurement accuracy, b*eamwidth, maximum linear acceleration, average energy*

1. Introduction

In summary, Multifunction Radar (MFR) single-target tracking is the set of operations aiming towards the calculation of an improved estimate of the target's actual position and dynamics and the production of an estimate of the target's future position (prediction), where the beam will be pointed and a range gate opened. The operational requirements to be satisfied by the tracker vary from radar system to radar system and from scenario to scenario. For a MFR sensing a potentially hostile environment, the objective should be i) to keep all the targets on their tracks with a high probability (which puts a requirement on the accuracy of the predicted position) and ii) to guarantee a given accuracy of the estimated (smoothed) position for a number of selected high priority targets. On the other side, the selection of the tracking parameters cannot be driven by individual track accuracy considerations only, but rather it is influenced also by other radar system aspects. In particular, the probability of detection and the accuracy of a measurement (which directly impact tracker's performance) are determined by the available SNR ratio and hence by the energy used for illuminating the target. Also, increasing the rate of illumination of a target allows obtaining less uncertain predictions. Thence a high energy burst and a high illumination rate are desirable from the tracker's accuracy point of view. However they negatively affect the transmitter beam occupancy budget and therefore they are undesirable from the point of view of the radar's ability to deal with multiple targets. Hence, tracker parameters (including track illumination rate and dwell-time or burst energy) should be designed to satisfy a double objective: minimum beam occupancy, which will give maximum efficiency in the use of radar's energy and maximum beam scheduling effectiveness (shorter pulses, reduced dwell-time) desired tracking accuracy. The basic MFR requirement is to keep the targets on track, which means that the error in predicted angle should be guaranteed to be within a fraction of the beamwidth, with a given confidence. In this contribution we will present a new procedure to optimally solve this compromise for a monopulse MFR with an $\alpha - \beta$ tracking filter.

2. Approach to tracker design

2.1. Basic Radar Tracking Svstem Characteristics

We will consider a monopulse radar with full ability i) to point its beam to any desired elevation-azimuth combination (within its coverage volume) and ii) to manage the energy of the transmitted waveform. Only the tracking function will be considered. When tracking a given target, the beam is pointed, at the next track illumination time, to the predicted position. If the target is not detected, the beam is assumed to be backscanned. Should the track be eventually lost (after N backscans), a reacquisition mode would be scheduled. To reacquire a target, a pattern of contiguous beams is assumed to be generated, each beam waveform having enough energy to guarantee a high probability of target's detection, provided the missing target is within the scanned volume. Three decoupled $\alpha - \beta$ filters with constant coefficients are assumed to operate in Radar Principal Cartesian Coordinates (principal axes of the error ellipsoid of the position measurements).

The objective of the design is to find the values of track updating rate and signal to noise ratio (hence, waveform energy for a given target size at a given distance), which give minimum beam occupancylradar's average energy, still satisfying the tracking accuracy requirements.

2.2. Minimax design of the α and β

The first step in the tracker design process is to find the values of the filter parameters $(\alpha - \beta)$ which minimize the maximum angle prediction error E, (minimax approach). This can be done numerically (following (2)) as it is described below:

With an x-sigma confidence, E_p can be written (for the two angle axes):

$$E_p = x\sigma_p(\alpha, \beta, \sigma_m) + B_p(\alpha, \beta, T, a) \quad .... \quad (1)$$

where, σ_p is the standard deviation of the prediction error due to measurement noise (which in turn depends on the measurement accuracy σ_m) and, B_p is the maximum bias due to a maneuver of size a (which in turn depends on the time between illuminations, T).

It is well known that:

$$\sigma_p = \sqrt{\frac{2\beta + \alpha\beta + 2\alpha^2}{\alpha(4 - 2\alpha - \beta)}}\sigma_m \qquad \ldots\ldots. \quad (2)$$

Also, the maximum bias, B_p, can be shown to be a function only of α, β and aT^2whose value in the range of interest can be closely approximated by the given in the Appendix. Using this function and (2). The normalized value of E,:

$$\frac{E_p}{x\sigma_m} = \frac{\sigma_p}{\sigma_m} + \frac{B_p}{x\sigma_m} = f(\alpha, \beta, \frac{aT^2}{x\sigma_m}) \qquad \ldots\ldots \quad (3)$$

appears as a function of, . α, β and $D = \frac{aT^2}{x\sigma_m}$

Now, the pairs ***a-B*** which minimize this function can be numerically found for given values of the “difficulty” parameter D Using in (3) these optimal values for α and β one can finally write the normalized minimum-maximum prediction error as a function (see Appendix):

$$\frac{E_p}{x\sigma_m} = f(\frac{aT^2}{x\sigma_m}) \qquad \ldots..(4)$$

2.3. Guaranteeing the error to be with in a fraction of the beamwidth

Now, the basic design condition (constraining the x-sigma peak angle error to be a fraction of the beamwidth) is written:

$$E_p \leq \gamma\Delta\theta R \qquad \ldots. \ (5)$$

where, E_p is measured in meters, R is the range of the target, $\Delta\theta$ is the beamwidth (systematically taken at boresight) and γ is a fraction.

For a maximum linear acceleration $a_{\max})$ (taken as the worst case), the design equation **(5)** can be written, using (4)**:**

$$\frac{E_p}{x\sigma_m} = f\left(\frac{a_{\max})T^2}{x\sigma_m}\right) \leq \gamma\frac{\Delta\theta R}{x\sigma_m} \qquad \ldots\ldots. \quad (6)$$

which for a given value of γ and range R, allows us to calculate the necessary time between illuminations T as a function of the measurement accuracy σ_m,

$$T = f(\sigma_m) \qquad \ldots\ldots \ (7)$$

Finally, using the monopulse measurement accuracy equation, T can be written as a function of the SNR (per illumination),

T = f(SNR) …… (8)

2.4. Optimizing the Energy-Occupancy Budget

The function in (8) (which turns out to depend on target's range) gives us pairs (T, SNR) which, for the optimal minimax tracking filter and the worst-case target's acceleration, satisfy the constraint **(5),** with equality. Now, among these pairs (T, SNR) we want to find the pair which absolutely minimizes the beam occupancy/radar's energy. The global necessary average energy can be written in terms of the single-illumination waveform energy E and the track updating rate l/T as:

$$E_{go} = \frac{1}{T}[E + \cdots + E(1-P_D)^N + ME_a(1-P_{DA})] \quad \ldots. \quad (9)$$

where it is assumed that whenever a target-on-track is not detected (with probability (1-P_D)) in the first illumination, N new attempts are successively made (beam backscan) until eventually the target is declared lost (with probability $(1-P_{DA})$])). In such a case, an acquisition pattern of M (adjacent) beams is generated, each beam transmitting a waveform of energy E, , in general different from E. Now, taking into account that there is a linear relationship between the energy E of the transmitted waveform and the SNR ratio (for given target range and size, E=K.SNR), the expression (9) for E, can be written:

$$E_{go} = K\frac{1}{T}[SNR + \cdots + SNR(1-P_D)^N + MSNR_a(1-P_{DA})] \quad \ldots\ldots \quad (10)$$

Further, taking (ideal detector, Swerling I target, given P_{FA}),

$$P_D = P_{FA}^{\left(\frac{1}{1+SNR}\right)}$$

$$P_{DA} = P_D + (1-P_D)P_D + \cdots + (1-P_D)^N P_D \quad \ldots\ldots. \quad (11)$$

the expression in (10) can be written as a function of T and SNR only.

Finally substituting (8) in (l0), a final expression for E_{go} as a function of SNR (actually a collection of functions, one for every range) can be obtained.

$$E_{go} = f(SNR) \quad \ldots\ldots\ldots \quad (12)$$

The position of the minimum of this function gives us the value of SNR (hence energy for given target size and range) which minimizes occupancy, with direct benefit in any track interleaving/task scheduling algorithm. For these optimal values of SNR (which turn out to be different for different values of N, but independent of target's range), the corresponding minimum necessary values of the sampling rates (which do depend on target's range) can be finally computed from (8). Thus, this method leads us to an optimal selection of both the energy of the tracking waveform and the track sampling rate for given targets at any range.

We numerically solved equation (6) (with equality sign, a_{max} =6g, x=2.5 and $\Delta\theta$ =21 milliradians) for a collection of ranges and found the interpolating functions (7) which give T versus measurement accuracy σ_m. These functions for R=50 Km. and R=5 Km. are included in the Appendix.

All of them have the form:

$$T = a_3(\frac{1}{\sigma_m})^3 + a_2(\frac{1}{\sigma_m})^2 + a_1(\frac{1}{\sigma_m})^1 + a_0 \qquad \ldots\ldots\ldots \qquad (13)$$

T can be further written in terms of the signal-to-noise ratio available to the detector, by substituting in (13) the monopulse measurement accuracy equation. We used the relationship (σ_m in meters):

$$\sigma_m = R\sqrt{[L\left(\frac{\Delta\theta}{1.5\sqrt{SNR}}\right)^2 + 1]} \qquad \ldots\ldots.. \quad (14)$$

Where L (we used L=4) is the term that accounts for the worst-case loss in SNR due to the target being off the beam peak. As expected, the profile of T versus SNR is monotonically increasing, so that one can interchange SNR (hence, energy) for sampling rate (see Fig. 1**).** The equation-10, which gives T as a function of SNR, we plotted (taking K=l**)** E_{go}**,** for M=6, SNR_a=25 dB and P_{DA}= 10^{-4} , as a function of SNR, for N=l**,** N=2 and N=3, for several ranges. Figures 2 and 3 show the results for ranges R=50 Km and 5 Km, respectively. It can be noted that E_{go} has a minimum at a given value of SNR. Although, **as** it could be expected, the required E_{go}, increases as the range decreases (due to the maneuver being larger when measured in angular units), the optimal value of SNR (SNR which gives minimum occupancy) turns out to be independent of the target's range.

The optimal values of *SNR* are approximately:

SNR*=16dB, for N=3

SNR*=17dB, for N=2

SNR*=20dB, for N=I

Now, having calculated the SNR*, we can compute σ_m**,** (from (14))**,** the time between illuminations T (from (13)) and α *and* β (from the functions in the Appendix). Tables 1 and 2 below summarize the results.

It is intresting to noyioce, from either Fig.2 or Fig.3, that, from the point of view of beam occupancy as measured by E_{go}, it is (slightly) preferable to transmit a tracking waveform to get an SNR of 16dB and backscan up to three times than to transmit a waveform to get 20dB and backscan once. Also, it should be noticed that the optimal values of *SNR* are such that, after N backscans, $P_{DA} \geq 0.99$, in accordance with the reasonable previous idea that the signal-to-noise ratio has to be large enough to avoid the necessity to frequently reacquire the target.

It should be also remarked that this one is a conservative design, since all tracks are updated as if all targets were maneuvering at 6g all the time. Thus a design more efficient from the point of view of resource utilization could be devised by using lower sampling rates for those targets known not to be maneuvering and have a maneuver detector trigger the scheduler to use higher sampling rates upon maneuver detection.

TABLE 1: Optimal Tracking Parameters

N	σ_m	α,	β	P_D	P_{DA}
1	5.81	0.5117	0.3082	0.9128	0.9924
2	8.16	0.4080	0.1730	0.8351	0.9955
3	9.14	0.3758	0.1422	0.7980	0.9983

TABLE2: Time between Illuminations (T) In Seconds

	N=1	N=2	N=3
R=50Km	1.210	0.850	0.750
R=25Km	0.855	0.601	0.532
R=10 Km	0.541	0.380	0.337
R=5 Km	0.382	0.269	0.238
R=1 Km	0.171	0.120	0.106

3. Results

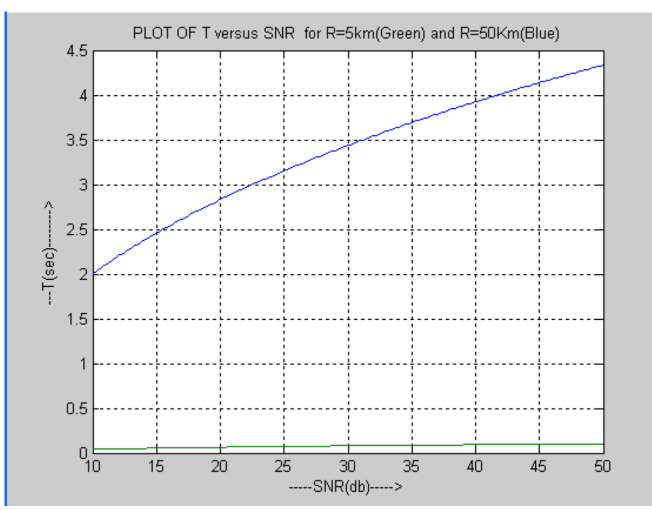

Fig.1: T = f(SNR):

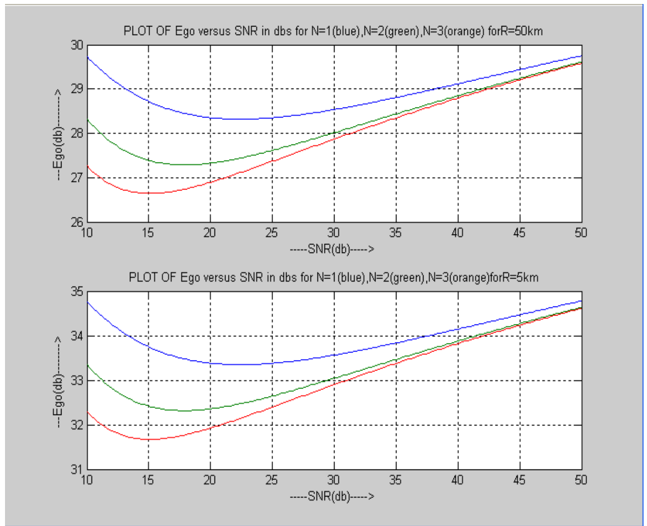

Fig.2. Occupancy E_{go} versus SNR:

4.Conclusion

A new approach to optimally select both the energy of the tracking waveforms and the track sampling rates which jointly minimize occupancy and satisfy angular accuracy requirements for a MFR has been presented. The design process has been illustrated for a particular case-study system. The results suggest that the radar's energy should be managed as a function of target's size and range as to obtain a SNR as close as possible to its optimal value SNR*. For every range, the value of T is automatically so given by equation (8).

References

[1] Blackman,S.S.., 1986, Multiple-Target Tracking with Radar Applications, Artech-House.

[2] Navarro, A.M., 1977, "Procedure for Tracking Manoeuvering Targets with a Multi-Purpose Phased-Array Radar System", Radar Conference, 150-154.

[3] Ethington, D.A., Kahrilas, P.J. and Wright, G.D., 1985, "Multifunction Rotating Electronically Scanned Radar (RESR) for Air Surveillance", Roc. IEEE.

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