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Research on the Correlation between Urban Scale and Congestion Level and Computer Simulation Based on CA Model

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Abstract

With the constantly expanding of the scale of the city, urban traffic congestion has aroused people's attention. As a result, traffic congestion can lead to excessive energy consumption and the environmental pollution, which restrict the sustainable development of the city. This paper aimed to build a mathematical model and a traffic flow simulation based on CA model to study the intrinsic reasons of the congestion dilemma in the city especially in the big city, and to explore the correlation between urban scale and congestion effect. Research results showed that, if the existing traffic density was constant and the destination was always random, the continuous expansion of the scale of the city would inevitably leaded to increased congestion. And finally getting the conclusion that the traffic congestion state was easier to accept in the city within 20km x 20km, and this study would provide reference for the government to formulate reasonable urban and transportation planning.

***Keywords**: Urban scale; Traffic flow; Congestion level; Grid model; Cellular automaton*

1. Introduction

Urban traffic is an important part of the modern city, it maintains the normal operation of the whole city. With the continuous expansion of the scale of the city, a series of problems caused by urban traffic has caused people's attention [1-3]. As an example, urban traffic congestion is one of the most concerned problems in the process of urban development. The essence of congestion is the accumulation of large traffic flow in a limited space in a certain period of time, and it usually happened in the holiday or rush hour. The main phenomenon is that the traffic flow on the road waiting too long or moving slowly [4-5], especially in the larger cities. Traffic congestion can lead to excessive energy consumption and urban environmental pollution, it has become a bottleneck restricting the sustainable development of the city. This paper intended to explore the relationship between the urban scale and the congestion level by using the mathematical model and the computer simulation of the automaton, though this to find the optimal city size, and provided a reference for the scientific urban planning and transportation policy.

The essence of urban congestion was studied by some scholars at home and abroad. The studies always included three parts, namely the essence, quantitative standard and control measures of the traffic congestion. Tai Zhong [6] gave a definition of the city traffic congestion, this paper pointed out that the essence of the phenomenon of urban traffic congestion was that the traffic demand exceeded the traffic capacity that can be accommodated. The excess traffic could not be met and stranded on the current road. Tong Bingxun [7] investigated the traffic congestion and road service level, and used it to define the road traffic congestion level. Gan Xuezhen [8] proposed the relevant attributes

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of urban traffic congestion, which was the location, time, category, reason and the time spent on the evacuation. Shao Zufeng [9] studied the basic theory of the urban traffic congestion, proposed four theories of urban traffic congestion governance, which were total traffic capacity reduction, average, continuous and separation principle. Meyer, Michael, Byrne [10] had built some congestion indicators in some major cities, but still can't use these indicators to compare the congestion level in different places or time. Francois and Schwartz, Lomax, Willis [11] quantified the congestion level, and built up the congestion performance indicators on this basis. Meng Lu [12], established the fuzzy mathematical model to identify the congestion level of urban road network, and used this to analyze the level of road congestion. Lee Kang [13] proposed some specific measures to reduce urban traffic congestion, including: 1.limit the number of cars;2.restricted access in the center area of the city, using levying high parking fees, congestion charges and other measures to control;3. Optimized the layout of the distribution and the attraction place of the traffic flow, which was to establish a multi-center resource distribution city model.

The essence of urban traffic congestion, the generation mechanism and control measures were studied by the scholars. But in the past, the study of indicators was always using a qualitative way, and lack of quantitative analysis. Therefore, this paper intended to construct a mathematical model of the relationship between urban scale and congestion level based on the original research, and used the automaton computer model to simulate the traffic flow. The purpose was to explore the inner theoretical basis of urban congestion.

2. Mathematical Derivation of the Correlation between Urban Scale and Traffic Congestion

Generally speaking, for different cities, if the space, density and time distribution parameters of the traffic demand in cities persisted, urban expansion would continue to aggravate the state of the city's traffic congestion [14]. Firstly, this paper intended to study the mathematical essence of urban congestion by building mathematical model. Based on literature researches, urban traffic congestion was defined as the vehicle travel slowly in the peak period of time and the travel time was severely stretched. When the road vehicles increased to a certain extent, as the vehicle traveling spacing decreases, the deriver would slow down and even stop, so there was the traffic congestion. Therefore, the study of urban traffic congestion mainly included two aspects, which were the average vehicle traffic flowrate and evacuation time during peak hours. These two aspects were interrelated, the growth of the average traffic flowrate in the unit of the road leading to the more average evacuation time.

In this paper, firstly we studied the average traffic flowrate of different scale cities in the peak period of time. Different cities had different road network information, but it could be transformed into a square road network by means of topological equivalence [15-17]. This paper intended to establish the most basic city grid network and a mathematical model to demonstrate the correlation between urban scale and urban congestion level. The unit of the grid type road was rectangular. The arterial roads of the city were set at a certain distance, other residential roads were arranged between the main roads. These roads divided the city into different plots. Ordinary cities, such as Beijing and Jinan, could be seen as the city of grid layout. The city grid model of the road network was shown in Figure 1. Firstly we established an $n \times n$ Network, black dotted line represented the arterial road network in the city, the box in the road network represented the work area of the city. The red square points represented the source and the attraction of urban traffic flow. Assuming that the traffic demand was uniformly random, that meant each working area had the same probability of traffic demand and each working area was the same as the destination. At the same time, it was assumed that the density of the traffic demand in the city must be kept as the same within the time period of the study. Then in the urban area shown in Figure 1, the total traffic volume of each working area was the total demand of the traffic flow.

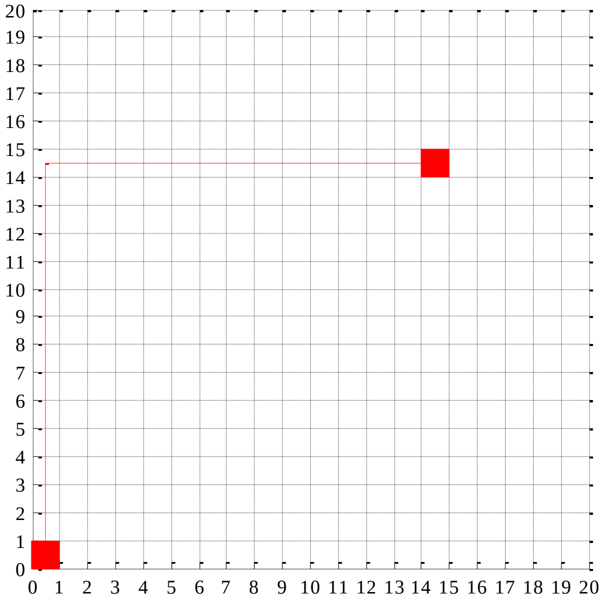


Figure 1. A n×n（20×20）City Model Built with Square Road Network

Assumed that the congestion of urban traffic was proportional to the average demand of the traffic flow on each road, That is, the more traffic flow on each road in the region the more congestion level happened. The average congestion on the city's roads can be expressed as (1):

$$Crowd = \text{Potential aggregate demand(Q)/Total road mileage} \tag{1}$$

The total length of the city road was the sum of the length of the road of the four directions, the calculation was 2n (n-1). The calculation of the total potential aggregate demand in the region was based on the method of subsection demand calculation. Supposed that D_{11} was the sum of the distance between the first urban area of the first rows to the other urban areas in the first rows, D_{12} was the sum of the distance between the second urban area of the first rows to the other urban areas in the first rows, D_{1k} was the sum of the distance between the kth urban area of the first rows to the other urban areas in the first rows. By calculation, we could get that:

$$D_{1\mathrm{k}} = 1+2+3+\cdots+(k\text{-}1)+(1+2+3+\cdots+n\text{-}k) \tag{2}$$

Simplify the equation:

$$D_{1\mathrm{k}} = \frac{1}{2}\left(k(k-1)+(n-k)(n-k+1)\right)$$

(3)

Therefore, the potential travel distance between the urban areas in the first line could be expressed as:

$$TD_1 = \sum_{\mathrm{k}=1}^{\mathrm{n}}\left[\frac{1}{2}\left(k(k-1)+(n-k)(n-k+1)\right)\right] = \sum_{k=1}^{n}\frac{1}{2}\left(n^2-(2k-1)n\right)+\sum_{k=1}^{n}\left(k^2-k\right) \tag{4}$$

After simplification:

$$TD_1=\sum_{k=1}^{n}k^2-\sum_{k=1}^{n}k=$$
$$\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)=$$
$$\frac{1}{3}n^3-\frac{1}{3}n$$
(5)

The same procedure might be easily adapted to obtain that the sum of the potential travel distance between urban areas in the first line and urban areas in the second line could be expressed as $TD_1+\mathrm{n}^2$, the sum of the potential travel distance between the urban areas in the first line and urban areas in the 3th line could be expressed as $TD_1+2\mathrm{n}^2$, the sum of the potential travel distance between the urban areas in the first line and urban areas in the kth line could be expressed as $TD_1+(k-1)\mathrm{n}^2$, so the total potential traffic demand between the urban areas in the first line and urban areas in the other place could be expressed as:

$$\sum_{k=1}^{n}\left(TD_1+(k-1)n^2\right)=nTD_1+\frac{1}{2}n(n-1)n^2$$
(6)

Therefore:

$$T_1=nTD_1+\frac{1}{2}n^3(n-1)$$
(7)

$$T_2=TD_1+n^2+\left(TD_1+n^2\right)+\cdots+\left(TD_1+(n-2)n^2\right) \tag{8}$$

$$T_k=TD_1+n^2+\left(TD_1+2n^2\right)+\cdots+\left(TD_1+(k-1)n^2\right)+ \\ TD_1+n^2+\cdots+\left(TD_1+(n-k)n^2\right) \tag{9}$$

After simplification:

$$T_k=n^2k^2-\left(n^2+n^3\right)k+\frac{1}{2}\left(n^4+n^3\right)+nTD_1 \tag{10}$$

All the potential traffic demand in the region could be calculated as:

$$\begin{aligned}T_{total}&=\sum_{k=1}^{n}\left(n^2k^2-\left(n^2+n^3\right)k+\frac{1}{2}\left(n^4+n^3\right)+nTD_1\right)\\&=n^2\sum_{k=1}^{n}k^2-\left(n^2+n^3\right)\sum_{k=1}^{n}k+\\&\quad n\left(\frac{1}{2}\left(n^4+n^3\right)+n\left(\frac{1}{3}n^3-\frac{1}{3}n\right)\right)\\&=\frac{2}{3}n^3\left(n^2-1\right)\end{aligned} \tag{11}$$

At the same time, because of the assumption that the traffic demand in urban areas was uniform in the peak time, the probability of any area in the city as destination was the same, either one was $1/\left(n^2-1\right)$, n^2-1 was the number of potential destinations. Therefore, the expectation of all the potential traffic demand in the region was:

$$\frac{2n^3\left(n^2-1\right)}{3\left(n^2-1\right)}=\frac{2}{3}n^3$$

(12)

The physical meaning of the formula (12) represented although the scale of the city continued to expand, but the probability of each area being a destination would decrease with the expansion of the city, the traffic demand in a unit area was not significantly improved. The above results could be drawn into the formula (13).

$$\text{Crowd}=\frac{2}{3}n^3/\left(2n\left(n-1\right)\right)=\frac{1}{3}\frac{n^2}{n-1}$$

(13)

The relationship was shown in Figure 2.

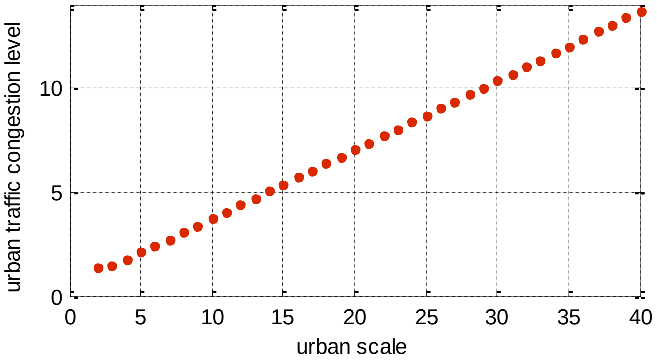


Figure 2. Relationship between Urban Scale and Urban Traffic Congestion Level

Figure 2 showed that the average urban congestion level was proportional to the size of the city. With the continuous expansion of the scale of the city, the congestion level of the city would be increased with the assumption of this paper. In addition, because of the random characteristics of the traffic flow and no uniformity characteristic of the trip, the actual congestion level would be greater than the results of this paper. The above theory demonstrated the relationship between urban scale and traffic congestion semi quantitatively. But urban traffic flow was a complex process, which had the characteristics of randomness and dynamics. In order to verify the rationality of the above mathematical model, and to make the evaluation more random and dynamic. Based on the above research, a computer simulation was carried out based on the automaton model. The average time of traffic flow evacuation in urban rush hour was used to reflect the level of traffic congestion. Using the computer to simulate the traffic flow and to calculate the average evacuation time in different scale cities during rush hours. And then got the relationship between different urban scale and traffic congestion level.

3. Numerical Simulation of Urban Traffic Flow Evacuation Process based on CA Model

The Automaton Cellular (CA) model could be used to approximately simulate the model of urban traffic flow [18-20]. This paper used the scattered cells to express the traffic flow in urban area. Each scattered cell took "traffic occupy” and "no" two states, and used the same rule for local updates. In this way, the traffic cells within the urban area could form a dynamic traffic flow system. It was also assumed that the density of traffic demand in the city was constant. By using this model, the state of traffic flow in different cities with different size and the average time of evacuation could be obtained by computer simulation. And this would reflect the relationship between different urban scale and traffic congestion level.

3.1. Dynamic Traffic Flow Simulation in Different Scale Cities

In order to simulate real urban traffic flow, each city area took the size of 0.5km×0.5km, and the entire city space was divided into a uniform distribution of the square grid. Each cell represented a city block area, each cell took "traffic occupy “and "no" two states. "Traffic occupy" represented the traffic flow was through the area, "No" represented the area was empty. Using X to represented the state of each cell in the urban area, "1" means the traffic occupy, "0" means No. Figure 3 was the generated two-dimensional plane grid using computer, the length and width of each grid were 0.5. The central coordinates of the urban area could be calculated by equation (14).

$$x(i,j)=0.5i-0.25; y(i,j)=0.5j-0.25$$

(14)

Supposed that all the cell states were updated according to the same rules, and each cell's destination was generated randomly. In the process of each evolution, the distance between the center and the target of the cell was calculated. The state of the cell itself and the surrounding cells were updated by the above principle, and all the cell changed in the same time. The state of the evolution of each cell was determined by the current state of the cell and the state of the neighboring cells. The neighborhood here referred to the Neumann Von neighborhood of the cell, as shown in Figure 3.

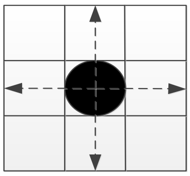


Figure 3. The Von Neumann Neighborhood of the Cellular

For each cell, the destination of the next step could be only in the Neumann Von neighborhood. The evolution rule was to calculate the distance between the cell itself and the four Neumann Von neighborhood and the target grid of the cell, the nearest grid was chosen as the candidate for the following grid. Assuming the coordinates of the target grid was (x,y), and the distance between A_{ij} and destination (x,y) was represented by D_{ij}, the cellular distances could be expressed as:

$$D(i\quad,j)=\sqrt{(0.5i-x-0.25)^2+(0.5j-y-0.25)^2} \tag{15}$$

$$D(i+1,j)=\sqrt{(0.5i-x+0.25)^2+(0.5j-y-0.25)^2} \tag{16}$$

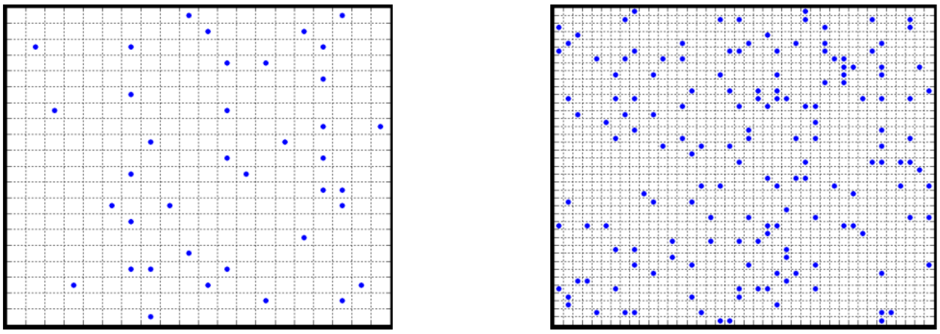
$$D(i,j-1)=\sqrt{(0.5i-x-0.25)^2+(0.5j-y-0.75)^2} \tag{17}$$

$$D(i,j+1)=\sqrt{(0.5i-x-0.25)^2+(0.5j-y+0.25)^2} \tag{18}$$

For the next step of the grid cell A_{ij}, we should firstly computed the $D(i,j)$, $D(i-1,j)$, $D(i+1,j)$, $D(i,j-1)$, $D(i,j+1)$ values, and then chosen the grid with the minimum value as candidate grid of the next step grid. The candidate grid still needed to verify whether the status of its possession was "no". Otherwise the selection of the candidate grid was to choose grid with the distance close to the candidate grid. And so on, until all the grids were eligible. The whole process was called an iteration, and repeated the process until all the cells reach their target grid.

3.2. Simulation Results and Analysis

In order to compare the congestion level of different scale cities during the peak period. In this paper, we simulated two different urban scale. The research objects were a city area of 10km×10km and a city area of 20km×20km, respectively. The cities of the two scales were divided into 20×20=400 and 40×40=1600 grids, each grid represented a 500m×500m urban area. Assuming the initial state of traffic flow average density was 0.4, the traffic demand was randomly generated, that was, a cell. 40 and 160 cells were randomly distributed in the urban grid in two scales of cities. Modeling results were shown in Figure 4. Each of cells had its own specific destination, and the destination was randomly distributed within the cities. The program would simulate the operation of the urban traffic flow in the morning rush hour or evening rush hour. The average evacuation time of traffic flow during peak period was used to reflect the city's congestion level. That is, the more time the total evacuation of the time, the more serious traffic congestion level in the time period. Assuming that the normal speed of the traffic flow was 60km/hour, so the time step was 0.5km/ (1km/min) =30s.

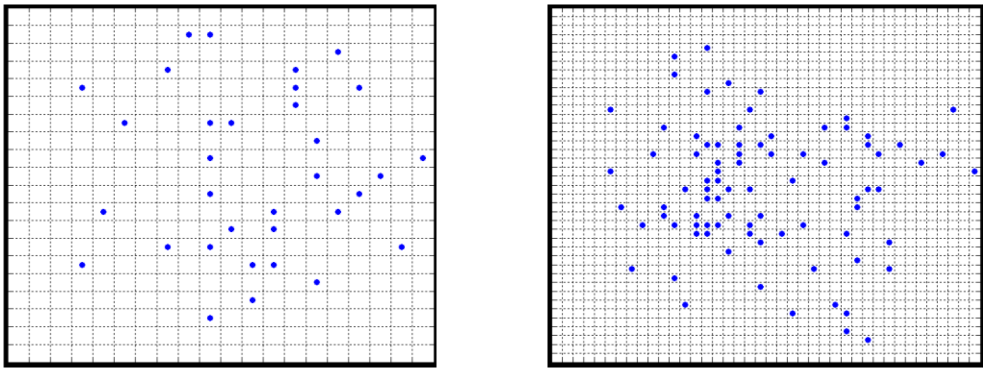


Initial state of traffic flow during rush period in an area of 10km×10km

Initial state of traffic flow during rush period in an area of 20km×20km

Figure 4. Initial State of Urban Traffic Flow during Rush Period in Different Scale of Cities

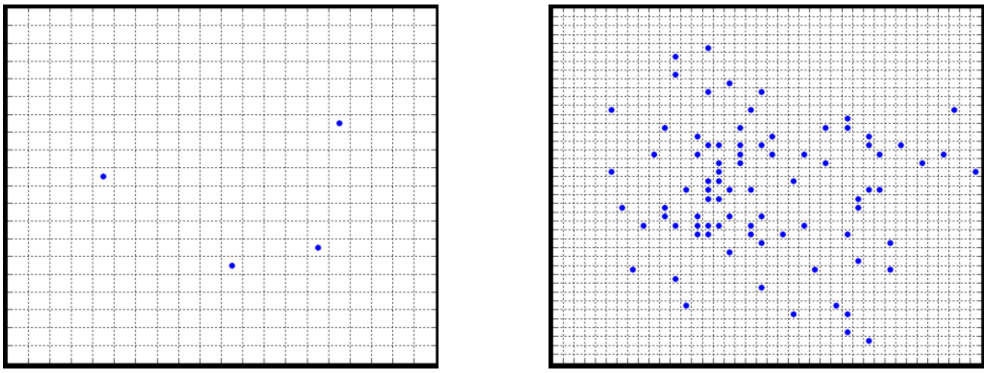
In this paper, the influence of traffic light, obstacle, sudden accident and distribution of urban resources on traffic flow was not considered, only considered the impact of traffic flow itself on the congestion. Through multiple simulation, the simulation process was shown in Figure 5, Figure 6. A 10km×10km urban cells needed to be iterated for 23 times on average, and the time step was 30s, so the evacuation time was 12min. A 20km×20km urban cells needed to be iterated for 73 times on average, and the evacuation time was 37min.



state of urban traffic flow in 20mins during peak periods in an area of 10km×10km

state of urban traffic flow in 20mins during peak periods in an area of 20km×20km

Figure 5. State of Urban Traffic in 20mins during Peak Periods in Different Scale of Cities



state of urban traffic flow in 35mins during peak periods in an area of 10km×10km

state of urban traffic flow in 35mins during peak periods in an area of 20km×20km

Figure 6. State of Urban Traffic in 35mins during Peak Periods in Different Scale of Cities

Some typical time points were analyzed in the simulation process. In the first case, the state of 2.5, 5, 7.5, 10min and the state of 10, 20, 30, 35min in the second case were observed and analyzed, See Figure 5, Figure 6. As a result of analysis, because the traffic demand was uniform. So the initial state of the traffic flow distribution was uniform, the "No" space was more, most of the region had no obvious congestion. After 5min in the 10km×10km urban cells and after 5min in the 20km x 20km urban cells the traffic flow congested in urban centers. The gap between the vehicle and the vehicle was very dense, and the vehicle was moving slowly, this was due to the different purposes of each traffic cell, but most of the traffic flow must through the center area of the city, so caused the congestion. After this, the congestion situation began to gradually improve, most of the cells traveled significantly faster after that. And after 37min and 12min, respectively, the evacuation process was completed, the simulation ended.

4. Conclusions

In this paper, we studied the relationship between urban scale and traffic congestion level by building a mathematical model and simulation based on CA model. The same results were attained, that were: 1) Urban scale was one of the internal factors causing traffic congestion, and the level of traffic congestion was proportional to the city scale.

With the increase in the size of the city, if we did not change the layout of urban resources allocation, the level of traffic congestion would increase.2) Due to the resource constraints of road, the simulated congestion level was slightly higher than the theoretical derivation. The two conclusions were basically the same. At the same time, because the traffic flow considered in this paper was ideal, so the actual congestion would be more intense than the simulation. Developing a more scientific city and transportation planning would be an important issue in big cities.

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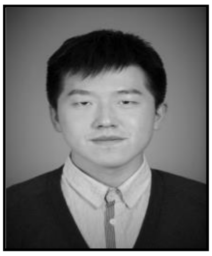

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