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Boolean Control Network Based Modeling for Context-Aware System in Smart Home

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Abstract

Context-awareness is an important characteristic for smart home. Context-aware system reacts and adapts according to the changes in the domain environment. In this paper, we have presented a mathematical modeling used for a context-aware system based on Boolean control network with five nodes for smart home. This Boolean control network describes the relationship between the context elements (user, time, location, activity) and service states (morning call, normal, entertainment, sleeping, guarding). We expressed the dynamics of the state spaces by linear algebraic equation using semi-tensor product of matrices, which is effective to logical inference and control.

Keywords*: Boolean control networks, context-aware system, semi-tensor product (STP) of matrices, smart home*

1. Introduction

Researchers in different fields make a lot of contribution for context-aware system. The goal of the context-aware system in smart home is to collect the context information automatically and provides services that maximize the user's comfort, and safety, while minimizing energy consumption. Context-aware systems can be implemented using production rules (if-then relationships), neural networks, support vector machines, fuzzy logic, Bayesian networks, *etc* [1-5]. Boolean network (BN) was introduced by Kauffman to formulate the cellular networks [6]. A BN network consists of a set of Boolean variables whose state is determined by other variables in the network. BN also used for modeling some other complex systems such as neural networks, social and economic networks. The state of each variable of Boolean network can be determined by the state of its spatial neighbors. A BN network with n variables has 2^n possible states and the dynamics of BN is converted into an equivalent algebraic form as a standard discrete-time linear system using the matrix expression of logic. One possible way to enrich the dynamics of the BN is to consider BN with inputs and outputs called Boolean control network (BCN) [7].

If the context elements and services are considered as inputs and outputs respectively then the context-aware system can be assumed as a Boolean control networks. Smart home is deployed with sensor network to collect user information (user ID, location, and activity) and environmental (temperature, humidity, illumination, and CO_2) data. Context is formed with this data. Context-aware system offer different types of services to the user based on the available context information as shown in figure 1. The set of service rules are involved for this purpose. Usually these service rules are represented by First Order Predicate Logic (FOPL) statement. These FOPL statements can be converted to Boolean expressions. Our aim is to create a mathematical model that represents the effects of

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different control inputs and to make predictions about state of service. Firstly these services rules are converted in to Boolean expressions then using semi-tensor product we can derive linear algebraic equation of these Boolean expression. These linear algebraic equations express state-spaces for smart home. Reasoning is an important terms used in context-aware system. Through reasoning inference can be done to deduce new knowledge form available knowledge. In this paper, we introduced matrix expression of logic with semi-tensor matrix product which is convenient for logical inference.

Figure 1. Context-Aware Service Environment in Smart Home

The remaining of the paper is organized as follows: A brief review of related topics for Boolean control network is presented in section 2. Section 3 provides information about state-space description of the system. Section 4 presents mathematical modeling of the state-space followed by mathematical modeling of service space in section 5. In section 6, we illustrate the methods of inference. Finally the conclusion and future work are drawn in section 7.

2. Theoretical Background

2.1. Boolean Control Network

A Boolean control network (BCN) is a discrete-time logical dynamic control system. Its dynamics can be expressed as

$$\begin{cases} x_1(t+1) = f_1(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t)) \\ \quad \vdots \\ x_n(t+1) = f_n(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t)) \end{cases} \quad (1)$$

where, f_i $:D^{n+m}\rightarrow D=\{T,F\},or\{1,0\}$, $i=1,\ldots,n$, are $n+m$-ary logical functions, $x_j\in D$, $j=1,2,\ldots n$, are states, $u_l\in D$, $l=1,2,\ldots m$ are control inputs.

In [8], authors proposed the concept of semi-tensor product and used it to represent BCNs in a linear algebraic state-space form. A briefly review on some topics are represented, which is useful for studying BCNs in a control-theoretic framework.

2.2. Matrix Expression of Logic

A logical variable represents value from a set D={T,F},or{1,0}. For matrix expression we identify truth "T" and false "F", with the vectors

$$T := 1 \sim \begin{bmatrix}1\\0\end{bmatrix} \quad or \quad \delta_2^1,\ F := 0 \sim \begin{bmatrix}0\\1\end{bmatrix} \quad or \quad \delta_2^2$$

where, δ_k^i is the i[th] column of the identity matrix I_k and $\Delta_k := \{\delta_k^i | i = 1,2,\ldots,k\}$

Definition 1: A matrix $L\in M_{n\times m}$ is called a logical matrix if $Col(L)\subset\Delta_n$. The set of $n\times m$ logical matrices is denoted by $\mathcal{L}_{n\times m}$. If $L\in\mathcal{L}_{n\times m}$, then it has the form $L = [\delta_n^{i_1} \quad \delta_n^{i_2} \quad \cdots \quad \delta_n^{i_m}]$ or in compact form $L=\delta_n\,[i_1\, i_2 \ldots i_m]$.

2.3. Semi-tensor Matrix Product

Definition 2: Unlike Kronecker product (⊗), the semi-tensor product (⋉) is a generalization of the conventional matrix product that allows multiplying two matrices of arbitrary dimensions. The semi-tensor product of two matrices $A\in M_{m\times n}$ and $B\in M_{p\times q}$ is

$$A \ltimes B=(A \otimes I_{\alpha/n})(B \otimes I_{\alpha/p}) \tag{2}$$

where α is equal to least common multiple of n and p $(lcm(n,p))$.

2.4. Algebraic Representation of Boolean Functions

A Boolean function can be converted into an algebraic form using the semi-tensor matrix product. Any Boolean function of n variables f: $\{F,T\}^n\rightarrow\{F,T\}$ can be equivalently represented as a mapping $\int = \{\partial_2^1, \partial_2^2\}^n \rightarrow \{\partial_2^1, \partial_2^2\}$

Definition 3: A 2×2^r matrix M_σ is said to be the structure matrix of the r-ary logical operator σ if

$$\sigma(p_1, \ldots, p_r) = M_\sigma \ltimes p_1 \ltimes \ldots \ltimes p_r := M_\sigma \ltimes_{i=1}^{r} p_i \tag{3}$$

Table 1. Listed Some of the Structure Matrices Used in BCN.

Table 1. Structure Matrix for Basic Logical Operators

Logical Operator	Structure Matrix	Logical Operator	Structure Matrix
Negation(¬)	$MN = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \delta_2[2 \quad 1]$	Disjunction(∨)	$MD = \begin{bmatrix}1 & 1 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} = \delta_2[1 \quad 1 \quad 1 \quad 2]$

Conjunction(∧)	$MC = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \delta_2[1\ 2\ 2\ 2]$	Conditional(→)	$MI = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \delta_2[1\ 2\ 1\ 1]$
Biconditional(↔)	$ME = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \delta_2[1\ 2\ 2\ 1]$	Exclusive Or	$MP = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \delta_2[2\ 1\ 1\ 2]$
Dummy(σd)	$ED = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \delta_2[1\ 2\ 1\ 2]$	Power reduced	$MR = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \delta_2[1\ 4]$

Definition 4: A swap matrix $W_{[m,n]}$is an mn×mn matrix, defined as follows. Its rows and columns are labeled by double index (i, j), the columns are arranged by the ordered multi-index Id(i, j: m, n), and the rows are arranged by the ordered multi-index Id(j, i: m, n). The element at position [(I, J), (i, j)] is then

$$w_{(I,J),(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1, & I = i \text{ and } J = j, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

2.5. Procedure to Make Structure Matrix

We can make structure matrix of the function $\sigma(p_1, p_2,\ldots,p_r)$ in the following three steps.

Step 1. Using the fact that $pM = (I_2 \otimes M)p$, all factors of structure matrices M_j or $I_2 \otimes M_j$ can be move to the front and move all the variables, p_i, to the rear of the product.

$$\sigma(p_1, \ldots, p_r) = \ltimes_i \xi_i = \ltimes_j N_j \ltimes_k p_{i_k} \tag{5}$$

where $N_j \in \{I_{2^s} \otimes MN, I_{2^s} \otimes MD, I_{2^s} \otimes MC \mid s = 0,1,2,\ldots\}, i_k \in \{1, 2, \ldots, r\}$

Step 2. Using a swap matrix the order of two logical variables can be changed $W_{[2]} p_i p_j = p_j\, p_i$

$$\ltimes_k p_{i_k} = M \ltimes p_1^{k_1} \ltimes p_2^{k_2} \ltimes \ldots \ltimes p_r^{k_r} \tag{6}$$

Step 3. Using a power-reducing matrix, the power of the p_i's can all be reduced to 1. The coefficient matrices, generated by reducing orders, can be moved to the front part.

2.6. Algebraic Representation of BCNs

The dynamics of BCNs can be represented by a set of Boolean functions, so a linear algebraic state-space representation can be possible using semi-tensor matrix product.

Theorem 1: Consider a BCN with state variables $x_1,\ldots\ldots,x_n$ and inputs $u_1,\ldots\ldots u_m$ with x_i, $u_i \in \{\partial_2^1, \partial_2^2\}$. Then the dynamics of equation (1) can be expressed as

$$\mathbf{x}(t+1)=L\ltimes \boldsymbol{u}(t) \ltimes \boldsymbol{x}(t) \tag{7}$$

where the matrix L is called the transition matrix of the BCN and $L \in \mathcal{L}_{2^m \times 2^{n+m}}$, $x(t+1) = \ltimes_{i=1}^{n} x_i(t+1)$, $u(t) = \ltimes_{j=1}^{m} u_j(t)$, $x(t) = \ltimes_{k=1}^{n} x_k(t)$

2.7. Solution of a Logical Equation

Using the following algorithm, scalar form of the logical unknowns can be easily calculated.

Algorithm 1 : Let, $x = \ltimes_{j=1}^{n} p_j = \delta_{2^n}^{i}$, where $p_j \in \Delta$ are in vector form. Then:

The scalar form of $\{p_j\}$ can be calculated from i inductively as follows:

Step 1. Set $q_0 := 2^n - i$

Step 2. Calculated p_j and q_j, $j=1,2,\ldots,n$, recursively by

$$\begin{cases} p_j = \left\lceil \dfrac{q_{j-1}}{2^{n-j}} \right\rceil \\ q_j = q_{j-1} - p_j \times 2^{n-j}, \; j = 1,2,\ldots,n \end{cases}$$

3. State-Space Description of the System

We consider five states for this system: Morning call state, Normal state, Entertainment state, Sleeping state and Guard state. Five different services will be offered by these states. We can define the states by FOPL (First Order Predicate Logic) statement.

Morning Call State:

For all users, if user is available, and the last state is “Morning call”, or “Sleeping”, and time is “Morning”, and user location in “Bedroom”, and user activity is “Lying” then next state is “Morning call”.

∀X,user(X)∧last_state(“Morningcall”)∨last_state(“Sleeping”)∧last_time(“Mornin g”) ∧ location(X, “Bed-room”)∧activity(X, “Lying”) →next_state(“Morning call”)

Normal State:

For all users, if user is available, and the last state is “Morning call”, or “Normal”,or “Entertainment”, or “Guard”, and user location in inside of house then next state is “Normal”.

∀X,user(X)∧last_state(“Morningcall”)∨last_state(“Normal”)∨last_state(“Entertai n-ment”)∨last_state(“Guard”)∧¬state(“Sleeping”)∧¬location(X,“outside”)→next_stat e(“Normal”)

Entertainment State:

For all users, if user is available, and the last state is “Normal”, or “Entertainment”, or not “Sleeping”, and time is “Evening” and user location in “Sofa” then next state is “Entertainment”.

∀X,user(X)∧(last_state(“Normal”)∨last_state(“Entertainment”)∧¬last_state (“Sleep-ing”)∧time(“Evening”)∧location(X, “Sofa”)→next_state(“Entertainment”)

Sleeping State:

For all users, if user is available, and the last state is "Normal" or "Entertainment", or "Sleeping", and not "Morning", and time is "Night", and person location in "Bedroom" then next state is "Sleeping".

∀X,user(X)∧last_state ("Normal")∨last_state("Entertainment")∨last_state("Sleep") ∧¬last_state("Morningcall")∧time("Night")∧location(X,"Bed-room")∧activity(X, "Ly-ing")) →next_state("Sleeping")

Guard State:

For all users, if user is available, and the last state is "Normal", or "Guard", or not "Morning", or "Entertainment", or "Sleeping" and user location in "Outside", and user activity is "Lying" then next state is "Guard".

∀X,user(X)∧last_state("Normal")∨last_state("Guard")∨¬last_state("Morning") ∨last_state("Entertainment")∨last_state("Sleep")∧location(X,"Outside")∧activity(X , "Lying") →next_state("Guard")

Every home user has tagId, name and relation. We can define home user by, X is a home user if has tagId, name and relation. Home user information is listed in table II.

tnr(Y,X,Z)→user(X).

We can illustrate tnr(Y,X,Z) by combining three one-argument predicates tagId(Y), name(X), relation(Z).

tagId(Y)∧name(X)∧relation(Z)→tnr(Y,X,Z)

Table 2. Home User Information

tagId(Y)	name(X)	relation(Z)		user(X)
1001	Mr. Hye	Father		Mr. Hye
1002	Mrs. Soo	Mother		Mrs. Soo
1003	Mr. Hyo	Son	→	Mr. Hyo
1004	Ms. Ha	Daughter		Ms. Ha
1005	Mr. Hyun	Grandfather		Mr. Hyun

We convert FOPL (First Order Predicate Logic) to propositional Logic in two-steps: First, replacing universal quantification by conjunctions, second, defining propositional variables from the predicates [9].

4. Mathematical Modeling of the State-Space

We can represent each state as a logical function of context elements and states. These context elements and states are Boolean logical variables with value either true (1) or false (0) at any given time *t*. Then at time *t+1* the states of this network are updated by context elements and last state values according to some desired logical rules logical equation of smart home context-aware system. We can represent five states by logic variables, such as morning call state = x_1, normal state = x_2, entertainment state = x_3, sleeping state = x_4, and guarding state = x_5. For input it uses eight context elements with Boolean logic values true and false. These context elements are categorized as user u_1 (father(p), mother(q), son(r), daughter(s), grandfather(t)), time (morning(u_2), evening(u_7), night(u_5)), location (bedroom(u_3), sofa(u_8), outside(u_6)) and activity (lying(u_4)). Each state can be defined by logical relation between the context elements and states.

$Morningcall\ state(t+1)=((Morningcall\ state(t)\vee Sleeping\ state(t))\wedge\neg Entertainment\ State(t))\wedge user(t)\wedge Time\_morning(t)\wedge Location\_bedroom(t)\wedge Activity\_lying(t)$

$Normal\ state(t+1)=(Morningcall\ state(t)\vee Normal\ state(t)\vee Entertainment\ State(t)\vee Guard\ state(t))\wedge(\neg Sleep\ state(t))\wedge user(t)\wedge(\neg Location\_outside(t))$

$Entertainment\ state(t+1)=(Normal\ state(t)\vee Entertainment\ state(t))\wedge(\neg Sleeping\ state(t))\wedge user(t)\wedge Time\_evening(t)\wedge Location\_sofa(t)$

$Sleeping\ state(t+1)=((Normal\ state(t)\vee Entertainment\ state(t)\vee Sleeping\ state(t))\wedge(\neg Morning\ call\ state)\wedge user(t)\wedge Time\_night(t)\wedge Location\_bedroom(t)\wedge Activity\_lying(t)$

$$Guarding\ state(t+1)=((Normal\ state(t)\vee Guard\ state(t)\vee(\neg (Morning\ call\ state(t))\vee Entertainment\ state(t)\vee Sleeping\ state(t)\wedge(\neg user(t))\wedge Location\_outside(t) \quad (8)$$

Using logic variables we can express equation (8) as

$$x_1(t+1) = ((x_1(t) \vee x_4 (t))\wedge\neg x_3(t)) \wedge u_1(t) \wedge u_2 (t) \wedge u_3(t) \wedge u_4(t)$$

$$x_2(t+1) = ((x_1(t) \vee x_2(t) \vee x_3(t)\vee x_5(t))\wedge\neg x_4(t)) \wedge u_1 (t) \wedge (\neg u_6(t))$$

$$x_3(t+1) =(x_2(t) \vee x_3(t)) \wedge (\neg x_4(t)) \wedge (u_1 (t)) \wedge (u_7(t) \wedge u_8(t))$$

$$x_4(t+1) = ((x_2(t) \vee x_3(t) \vee x_4(t))\wedge\neg x_1) \wedge u_1 (t) \wedge u_5(t) \wedge u_3(t) \wedge u_4(t)$$

$$x_5(t+1) = ((x_2(t) \vee x_5(t)\vee(\neg x_1(t))\vee x_3(t) \vee x_4(t)) \wedge(\neg u_1(t)) \wedge u_6(t) \quad (9)$$

Figure 2. shows the state-transition diagram of this system. Every state depends on the values of others state. When system is in normal state then next step can be entertainment state or sleeping state or guard state. When system is in guard state then next step will be only normal state. When system is in entertainment state then next state will be normal state or sleeping state. When system is in sleeping state then next step will be morning call. When morning call state is true then next state will be normal state. So every state depends on another state's last value. Every state has fixed number of context information, when the context information satisfied the logic equation then that state is ready to provide specific service. Equation (9) shows the relation between state variables (x_1,x_2,x_3,x_4,x_5) and input variables ($u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8$).

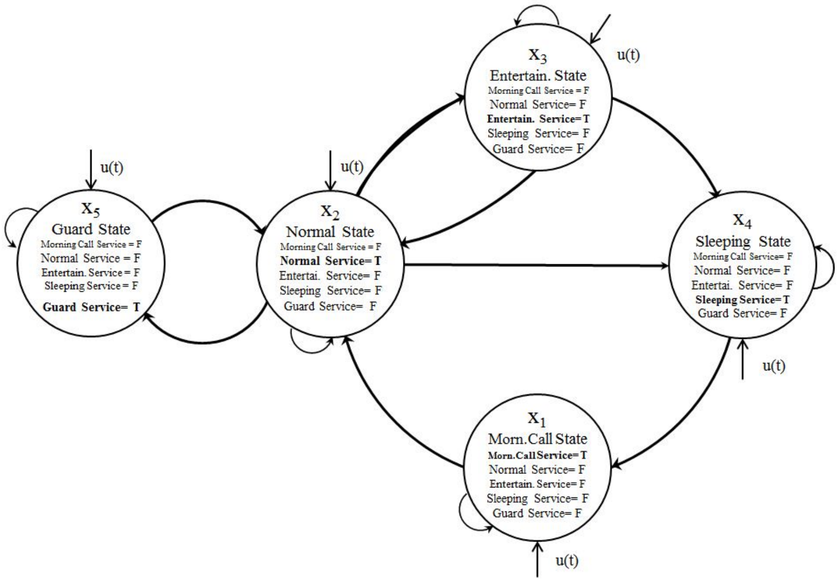

Figure 2. State-Transition Diagram

User u_1 can be defined logically as

$$user=(Father \vee Mother \vee Son \vee Daughter \vee Grandfather) \qquad (10)$$

This logical function (10) can be represented by logic variables, and structure matrix of logical operator, then converted into an algebraic function using semi-tensor matrix product ($\ltimes$), definition and theorems as presented in section 2.

$$u_1=p \vee q \vee r \vee s \vee t=MD \ltimes MD \ltimes MD \ltimes MD \ltimes p \ltimes q \ltimes r \ltimes s \ltimes t=L_p \ltimes x \qquad (11)$$

where, $x=p \ltimes q \ltimes r \ltimes s$, $MD=$ structure matrix of logical disjunction operation , $L \in \mathcal{L}_{2\times 2^5}= \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2]$= transition matrix. Structure matrix is unique and it is a matrix representation of sum of product (SOP) of logic variable in logic expressions. It describes the dynamics of the system. Semi-tensor product (STP) toolbox based on Matlab is used for related computations [10].

Similarly by converting logical equation (9) into an algebraic function as follows

$$x_1(t+1)= MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MD \ltimes x_1 \ltimes x_4 \ltimes MN \ltimes x_3 \ltimes u_1 \ltimes u_2 \ltimes u_3 \ltimes u_4$$
$$=MC^5 \ltimes MD \ltimes (I_2 \otimes (I_2 \otimes MN)) \ltimes (I_4 \otimes W_{[2]}) \ltimes (I_2 \otimes W_{[2]}) \ltimes W_{[2]} \ltimes (I_8 \otimes W_{[2]}) \ltimes (I_4 \otimes W_{[2]}) \ltimes (I_2 \otimes$$
$$W_{[2]}) \ltimes (I_{16} \otimes W_{[2]}) \ltimes (I_8 \otimes W_{[2]}) \ltimes (I_4 \otimes W_{[2]}) \ltimes (I_{32} \otimes W_{[2]}) \ltimes (I_{16} \otimes W_{[2]}) \ltimes (I_8 \otimes W_{[2]}) \ltimes (I_{32} \otimes W_{[2]}) \ltimes x_1 \ltimes x_3 \ltimes x_4 \ltimes u_1 \ltimes u_2 \ltimes u_3 \ltimes u_4$$

$$x_2(t+1)= MC \ltimes MC \ltimes MC \ltimes MD \ltimes MD \ltimes MD \ltimes x_1 \ltimes x_2 \ltimes x_3 \ltimes x_5 \ltimes MN \ltimes x_4 \ltimes u_1 \ltimes MN \ltimes u_6$$
$$=MC^3 \ltimes MD^3 \ltimes (I_2 \otimes (I_2 \otimes (I_2 \otimes (I_2 \otimes MN \ltimes (I_2 \otimes (I_2 \otimes MN)))))) \ltimes (I_{16} \otimes W_{[2]}) \ltimes (I_8 \otimes W_{[2]}) \ltimes$$
$$(I_4 \otimes W_{[2]}) \ltimes (I_2 \otimes W_{[2]}) \ltimes W_{[2]} \ltimes (I_{32} \otimes W_{[2]}) \ltimes (I_{16} \otimes W_{[2]}) \ltimes (I_8 \otimes W_{[2]}) \ltimes (I_4 \otimes W_{[2]}) \ltimes (I_2 \otimes W_{[2]})$$
$$\ltimes (I_{32} \otimes W_{[2]}) \ltimes x_1 \ltimes x_2 \ltimes x_3 \ltimes x_4 \ltimes x_5 \ltimes u_1 \ltimes u_6$$

$x_3(t+1)=MC\ltimes MC\ltimes MC\ltimes MC\ltimes MD\ltimes x_2\ltimes x_3\ltimes MN\ltimes x_4\ltimes u_1\ltimes u_7\ltimes u_8$

$=MC^4\ltimes MD\ltimes(I_2\otimes(I_2\otimes MN))\ltimes(I_4\otimes W_{[2]})\ltimes(I_2\otimes W_{[2]})\ltimes W_{[2]}\ltimes(I_8\otimes W_{[2]})\ltimes(I_4+W_{[2]})\ltimes$

$(I_2\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_8\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})\ltimes x_2\ltimes x_3\ltimes x_4\ltimes u_1\ltimes u_7\ltimes u_8$

$x_4(t+1)=MC\ltimes MC\ltimes MC\ltimes MC\ltimes MC\ltimes MD\ltimes MD\ltimes x_2\ltimes x_3\ltimes x_4\ltimes MN\ltimes x_1\ltimes u_1\ltimes u_3\ltimes u_4\ltimes u_5$

$=MC^5\ltimes MD^2\ltimes(I_2\otimes(I_2\otimes(I_2\otimes MN)))\ltimes(I_8\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})\ltimes(I_2\otimes W_{[2]})\ltimes W_{[2]}\ltimes(I_{16}\otimes W_{[2]})$

$\ltimes(I_8\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})\ltimes(I_2\otimes W_{[2]})\ltimes(I_{32}\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_8\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})\ltimes(I_{64}\otimes W_{[2]})\ltimes(I_{32}\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_8\otimes W_{[2]})\ltimes(I_{64}\otimes W_{[2]})\ltimes(I_{32}\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes x_1\ltimes x_2\ltimes x_3\ltimes x_4\ltimes u_1\ltimes u_3\ltimes u_4\ltimes u_5$

$x_5(t+1)=MC\ltimes MC\ltimes MD\ltimes MD\ltimes MD\ltimes MD\ltimes x_2\ltimes x_5\ltimes MN\ltimes x_1\ltimes x_3\ltimes x_4\ltimes MN\ltimes u_1\ltimes u_6$

$=MC^2\ltimes MD^4\ltimes(I_2\otimes(I_2\otimes MN\ltimes(I_2\otimes(I_2\otimes(I_2+MN)))))\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})$

$\ltimes(I_2\otimes W_{[2]})\ltimes W_{[2]}\ltimes(I_{32}\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_8\otimes W_{[2]})\ltimes(I_4\otimes W_{[2]})\ltimes(I_2\otimes W_{[2]})\ltimes(I_8\otimes W_{[2]})$

$\ltimes(I_4\otimes W_{[2]})\ltimes(I_{16}\otimes W_{[2]})\ltimes(I_{32}\otimes W_{[2]})\ltimes x_1\ltimes x_2\ltimes x_3\ltimes x_4\ltimes x_5\ltimes x_2\ltimes u_1\ltimes u_6$ (12)

where, MC = structure matrix of logical conjunction operation, MD= structure matrix of logical disjunction operation, I_2= identity matrix, $W_{[2]}$=swap matrix.

Following the procedure described in section 2.5 and 2.6, algebraic equation of (12) can be expressed as

$$\boldsymbol{x}(t+1)=L\ltimes\boldsymbol{u}(t)\ltimes\boldsymbol{x}(t) \quad (13)$$

where, $L\in\mathcal{L}_{2^5\times 2^{5+8}}$, $x(t+1)=\ltimes_{i=1}^{5} x_i(t+1)$, $u(t)=\ltimes_{j=1}^{8} u_j(t)$, $x(t)=\ltimes_{k=1}^{5} x_k(t)$

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5. Mathematical Modeling of Service Space

Each service state will provide separate services when certain logic relations are satisfied. We can define services by First Order Predicate Logic (FOPL) as

state(x)=service(x)→appliance (Name, status)

Morning Call Service

state("Morningcall")=service("Morningcall")→appliance("Alarm","on")*AND* appliance("Light","on")*AND*appliance("Blind","on")*AND* appliance("Coffee_maker","on")*AND*appliance("Water_heater","on")*AND* appliance("Aircondition", "on") *AND* appliance(" TV", "on")

Normal service

state("Normal")=service("Normal")→appliance("Alarm","off")*AND* appliance("Light","on")*AND*appliance("Blind","on")*AND* appliance("Coffee_maker","off")*AND*appliance("Water_heater","off")*AND* appliance("Aircondition","on")*AND*appliance(" TV","off")

Entertainment service

state("Entertainment")=service("Entertainment")→appliance("Alarm","off")*AND* appliance("Light","on")*AND*appliance("Blind","off")*AND* appliance("Coffee_maker","off")*AND*appliance("Water_heater","off")*AND* appliance("Aircondition","on")*AND* appliance("TV", "on")

Sleeping service

state("Sleeping")=service("Sleeping")→appliance("Alarm","off")*AND* appliance("Light","off")*AND*appliance("Blind","off")*AND* appliance("Coffee_maker","off")*AND*appliance("Water_heater","off")*AND*applianc e(" Aircondition","on")*AND* appliance(" TV","off")

Guard service

state("Guard") = service("Guard") →∀X appliance(" X","off")

Similarly, like section 4 we can convert above FOPL to propositional logic. We can represent five output services by logic variables, such as morning call service = y_1, normal service = y_2, entertainment service = y_3, sleeping service= y_4, guarding service = y_5. Every service is composed of some controlling facilities to turn on and off of home appliances (coffee_maker(e), water_heater(f), airconditioner(g), TV(h)) and devices(alarm(b), light(c), blind(d)). We can describe the service by the output function of home appliances and device.

$y_1 = b \wedge c \wedge d \wedge e \wedge f \wedge g \wedge \neg h$

$= MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes MN \ltimes h$

$= MC^6 \ltimes (I_2 \otimes I_2 \otimes I_2 \otimes I_2 \otimes I_2 \otimes I_2 \otimes MN)))))) b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h$

$y_2 = \neg b \wedge c \wedge d \wedge \neg e \wedge \neg f \wedge g \wedge \neg h$

$= MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MN \ltimes b \ltimes c \ltimes d \ltimes MN \ltimes e \ltimes MN \ltimes f \ltimes g \ltimes MN \ltimes h$

$= MC^6 \ltimes MN \ltimes (I_2 \otimes (I_2 \otimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes (I_2 \otimes MN)))))) \ltimes b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h$

$y_3 = \neg b \wedge c \wedge \neg d \wedge \neg e \wedge \neg f \wedge g \wedge h$

$= MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MN \ltimes b \ltimes c \ltimes MN \ltimes d \ltimes MN \ltimes e \ltimes MN \ltimes f \ltimes g \ltimes h$

$=MC^6 \ltimes MN \ltimes (I_2 \otimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN)))) \ltimes b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h$

$y_4 = \neg b \wedge \neg c \wedge \neg d \wedge \neg e \wedge \neg f \wedge g \wedge \neg h$

$=MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MN \ltimes b \ltimes MN \ltimes c \ltimes MN \ltimes d \ltimes MN \ltimes e \ltimes MN \ltimes f \ltimes g \ltimes MN \ltimes h$

$=MC^6 \ltimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes (I_2 \otimes MN)))))) \ltimes b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h$

$y_5 = \neg b \wedge \neg c \wedge \neg d \wedge \neg e \wedge \neg f \wedge g \wedge \neg h$

$= MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MC \ltimes MN \ltimes b \ltimes MN \ltimes c \ltimes MN \ltimes d \ltimes MN \ltimes e \ltimes MN \ltimes f \ltimes MN \ltimes g \ltimes MN \ltimes h$

$$=MC^6 \ltimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN \ltimes (I_2 \otimes MN)))))) \ltimes b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h \quad (14)$$

Following the procedure described in section 2.5 and 2.6, algebraic equation of (14) can be expressed as

$$\mathbf{y} = L_s \ltimes \mathbf{x_s} \quad (15)$$

where, $y = \ltimes_{j=1}^{5} y_i$, $x_s = b \ltimes c \ltimes d \ltimes e \ltimes f \ltimes g \ltimes h$, L_s is a transition matrix of dimension $2^5 \times 2^7$

The columns L_s are δ_{32}[32 16 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 23 2323 23 23 23 23 23 23 23 2 32 32 32 32 32 32 323232323232 3232 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 24 32 32 32 32 32 32 32 32 32 32 32 32 32 28 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 32 3232 32 32 32 32 32 32 32 32 32 32 32 32 30 32 31].

The vector value of service output is

$$x_s = L_s^T \ltimes y \quad (16)$$

6. Methods of Inference

This system will check the context information periodically and update the context information. Once the context information is updated then it will be used for calculation of current state of the system. When the current state is changed then the service is selected in the next step. According to algorithm in section 2.7, we can calculate the scalar form of different logical variables from the logical vector values. The scalar form of logical variables used in system is tabulated in Table 3.

Table 3. Scalar Form of Logical Variables

service	x(t+1)	$y_1\ y_2\ y_3\ y_4\ y_5$	x_s	b c d e f g h
Morning call	δ_{32}^{16}	1 0 0 0 0	δ_{128}^{2}	1 1 1 1 1 1 0
Normal	δ_{32}^{24}	0 1 0 0 0	δ_{128}^{79}	0 1 1 0 0 1 0

Entertainment	δ_{32}^{29}	0 0 1 0 0	δ_{128}^{93}	0 1 0 0 0 1 1
Sleeping	δ_{32}^{30}	0 0 0 1 0	δ_{128}^{126}	0 0 0 0 0 1 0
Guard	δ_{32}^{31}	0 0 0 0 1	δ_{128}^{129}	0 0 0 0 0 0 0

7. Conclusion

In this paper, we have tried to focus on realization of Boolean control network based context-aware system in smart home. We have defined the logical relation between context information and state variables. Then these logical relations are converted to matrix expression using semi-tensor matrix product. State transition matrix is a matrix expression of sum of product (SOP) representation of logic variables which describes the dynamic of the system. With this matrix expression we can control and do inference easily. Based on the observation result, we can conclude that Boolean control network is realizable for context-aware system in smart home. All the logical variables represented by values 1 or 0 in this system. It can be expressed more efficiently by using k-valued logic. At present we have applied the network with 5 nodes, number of nodes can be extended based on the state of the service. We can use Bayesian networks to make a probabilistic decision about the state changes.

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