International Journal of Smart Home
Vol. 10, No. 4 (2016), pp. 143-152
http://dx.doi.org/10.14257/ijsh.2016.10.4.14

Production Rate Determination for Linear Construction Projects Based on Linear Scheduling Method

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Abstract

Traditional scheduling methods such as CPM or PERT have been found to be inappropriate for scheduling the linear construction projects. According to the limitations of Traditional scheduling methods, Linear Scheduling Method (LSM) is proposed specifically for linear construction projects. Aiming at overcoming the limitations of network technology in deciding the production rate of linear construction projects, basing on LSM with its mathematical principle, this paper proposes a new method to decide the production rate of linear construction projects. This paper first analyses the parameters of linear scheduling method and their interrelationships, such as distance interval, time interval, activity, CAP determination of LSM. Then it gives determination method of production rate and the rate float. The determination method can increase the effectiveness of production rate input and output data for the linear projects, which will significantly improve the resource allocation. It also draws the effect of production rate float in the construction schedule control.

Keywords*: linear project, production Rate, production Rate float, linear scheduling method*

1. Introduction

Currently, typical linear construction projects, including railway, road, tunnel, and pipeline, mainly apply Critical Path Method (CPM) to schedule the process. However, studies have found the limitations of CPM in scheduling linear projects [1-2]. In contrast to the limitations of CPM, a new scheduling method known as Linear Scheduling Method (LSM) has gradually obtained attention of academic circles, which maintains the continuity of the resource when applied in linear projects. Furthermore, this method is simple, clear and comprehensible.

Production rate is one of the most important parameters in construction scheduling and management, which is particularly significant in process scheduling. For linear construction project, production rates of activities are beneficial for the development and accuracy of linear schedules. LSM has the advantage in that the production rate of each work process can be easily identified by the extent of the gradient [3]. The nature of linear projects dictates an assortment of variables that affect the production rate of each activity.

CPM involves no production rate, and can be hardly optimized.

According to Mubarak [4], one of the obstacles hindering LSM's popularity is its limited computer programs. When computer programs are applied to achieve automatic linear scheduling, production rate and rate float are particularly crucial, because of its direct influence to the efficiency and accuracy of the result. Until now, production rate is

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ISSN: 1975-4094 IJSH

always treated as a constant, which results in the neglect of dynamic change in process scheduling and limits the acceptance of LSM.

This paper explores the production rate determination in the LSM, and applies mathematical principles to analyze the production rate in the time-distance two-dimensional coordinate system. The study presents a new method for calculating production rate on basis of LSM, and proposes an algorithm for production rate in calculating critical activity sequences. The effects of these equations in calculating critical activity path of linear scheduling are tested.

The layout of the rest of the article is as follows. Section 2 reviews the literature with basic theories and research status. Section 3 presents the problem and its mathematical programming formulations of production rate. Section 4 presents mathematical programming of production rate float with the critical activities path. Section 5 summaries the paper with some short conclusions.

2. Literature Review

2.1. Linear Scheduling Method

LSM is a new planning method proposed specifically for linear construction projects [5-7]. According to the characteristics of linear construction projects, LSM uses rectangular coordinate to describe the construction schedule plan in a linear project. The horizontal axis is used to represent the location of the project, and the vertical axis is used to represent time progress. According to its construction time and location, any project can be showcased by certain icons in a two-dimensional coordinate system. Jafari and Moslehi consider a problem of scheduling a single machine under linear deterioration which aims at minimizing the number of tardy jobs [9]. It is proved that the problem is NP-hard. Computational results for 1,800 sample problems demonstrate that the B&B method can solve problems with 28 jobs quickly and in some other groups larger problems are also solved. Linear scheduling method and constraint programming techniques used for solving schedule control problems faced during railroad construction[10]. A duration optimization model while introducing the concept of multi-skilling to integrate single/multiple-skilled crews to improve work performance also proposed. Moreover, to enhance the efficiency of problem solving, constraint programming (CP) is used to handle complicated combinatorial scheduling problems, and several heuristic rules involving schedules are engaged [11].

The construction activities of linear project can be divided into three types: Linear activities, Bar Activities and Block Activities. The three types of activities can be further divided according to whether they are continuous constructions or whether they are throughout the whole project [8].

As presented in Figure 1, in the scheduling, progress of the five activities in terms of time and space are drawn at a time-distance two-dimensional coordinate system, and the slope in the diagram represents the production rate of the activities. The user can clearly understand the provided information, including production rate, progress of the project, and constraining relations with adjacent activities at a certain time or location point. For example, A, C and E are linear activities; B is a block activity; D is a bar activity. Minimum time constraint is imposed on A and C, and the minimum distance constraint is imposed on C and E. The bold line in Figure 1 is the critical path of the entire construction project.

In the study of the critical path methods, [8] proposed a critical path method to determine LSM program. However, this approach ignores a number of predecessors and successors. To solve this problem, Harris proposed another method to determine CAP of LSM for linear projects, basing on working procedure logic and production rate [12]; The researcher considered the maximum time constraints and maximum distance constraints between activities on the basis of the minimum time and distance constraints[13]. Compared to Rowings & Harmelink, this is a more comprehensible approach with more precise and accurate results. by comparing it with five related previous studies, RCPM evaluates the performance of the resource-constrained critical path method. This comparison shows that RCPM performs well in identifying resource links and alternative schedules, compared to other methods[14].

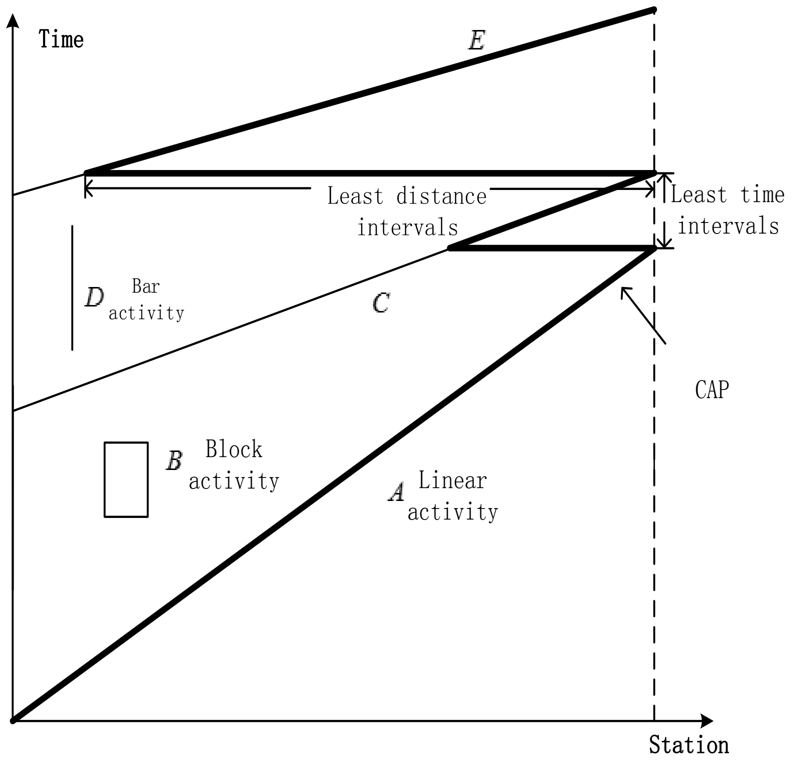

Figure 1. Schematic Diagram of a Simple Linear Scheduling Project

2.2. Production Rate and Rate Float

Production rate is one of the most important features of LSM, and an important variable involved in process scheduling and optimization. The diversification of rate will also influence the description of the former and the later construction activities' constrains. Some scholars have established theirs studies of the production rate of linear projects based on LSM. On the basis of control path, the researcher then introduced the concept of rate float, which laid the foundations of LSM's further development. A linear scheduling model with varying production rates (LSMVPR) is presented,it introducing an activity performance index (API) to indicate the variance of predicted production rates [15]. It presented the concept and calculation methods of work window, related the methods to LSM, and achieved automatic program of production rate process scheduling on the basis of minimum time constraint. Lee and Diekmann proposed a delay analysis method that can rationally apportion the concurrent delay with consideration of the nonlinear production rates of activities in construction projects [16].

According to the ratio between gradient and productivity in LSM, Many researchers have studied the production rate of the construction projects, but seldom use LSM to acquire the production rate. These studies limited to thoroughly discuss the definition of production rate, and weak in considering the geometric meaning of the rate. A general algorithm developed for data storage, an approach that facilitates the storage of activity-specific production and contextual information in an integrated fashion within project models [17].

Researchers did provide the concept of rate float and mathematical formulas, however only the subsequent construction activity rate is considered, and preceding construction activity rate is neglected. Besides, previous research cannot dynamically optimize the process scheduling, or only have the ability to schedule the process under the premises of variable rate, which is also hard to optimize.

3. Production Rate Determination

The production rate refers to the average velocity at which the linear construction activities progress, and is used to describe the construction mileage of linear activities completed within unit time. In the following, a hypothesis associated with the production rate is proposed on combining the characteristics of linear construction projects.

In the use of LSM for making linear project scheduling, according to the actual operation of linear project, assume a linear activity in different segments may correspond to different production rates, and for the same linear construction activity in the same zone the production rate remains unchanged.

3.1. Production Rate Initialization

When designing construction program, the rate is always presumed as a constant, which determines that these models lack the dynamic capability to optimize the result based on the variable rate. And the rate is usually programmed by programmers according to their experience.

Initial rate is obtained from programmers' empirical estimation. Basing on the characteristics of linear projects, this study proposes a hypothesis relating to production rate, which applies for linear production process.

A linear project contains several linear construction activities which are performed by each construction groups. According to practical tasks of linear projects, presume that linear activities correspond to different production rates during different sections, and the production rate of the same linear activity in the same section is constant, the rate can be represented as follows:

$$Rate_{t,i} = Q_{t,i} / D_{t,i} \tag{1}$$

$D_{t,i}$ is the duration of repetitive activity type i in section j, $Q_{t,i}$ is the quantity of work for repetitive activity type i in section j, $Rate_{t,i}$ is production rate of crew formation for repetitive activity type i in section j.

3.2. Production Rate Graphic

In the linear scheduling diagram, a linear construction activity reflects its rate by the cotangent the bevel derived from its corresponding diagonal in the first quadrant. Figure 2 describes the geometrical meaning of the linear activity rate. In the diagram, for Activity A in the first segment, the A-1 section corresponds to the slash angle α, the rate presented as $Cot\alpha$. Without loss of generality, the production rate of Activity t in section i is denoted as:

$$Rate_{t,i} = Cot\theta_{t,i} = W_{t,i} / (Comp_{t,i} - Comm_{t,i}) \tag{2}$$

$\theta_{t,i}$ is the angle of activity t corresponding to section i in the first quadrant; $W_{t,i}$ is the amount of activity t in section i usually expressed by the mileage in this section. $Comm_{t,i}$ represents the starting time of activity t in section i. $Comm_{t,i}$ represents the finish time of activity t in section i. As showed in Figure 2, Activity A has different production rates in three different sections $A-1$, $A-2$, $A-3$, wherein, and the rate of $A-1$ has is the highest rate.

3.3. Dynamic Adjustment Process of Production Rate

According to the actual operation of linear project scheduling, the schedules can be divided into two types, namely the initial schedule and the adjusted schedule. For the two types of construction plans, we can divide the determination methods for linear project production rate into two different types of methods:

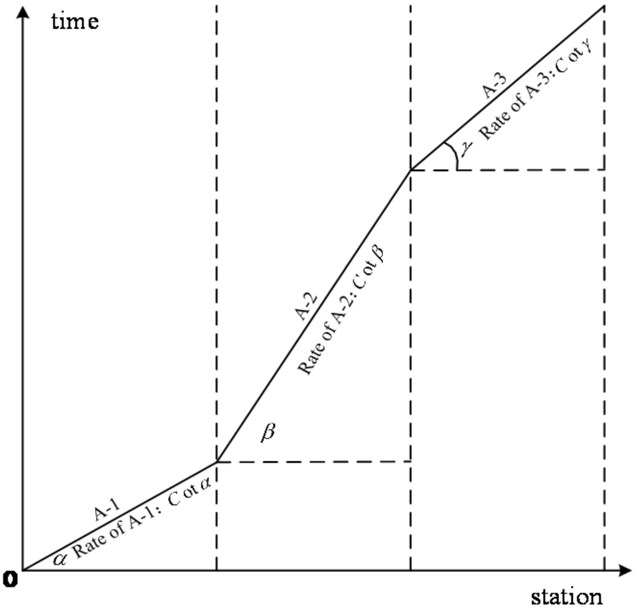

Figure 2. Relationship of Production Rate Based on LSM

Determination of production rate during schedule making: Assuming the production rate is given based on availability of all kinds of resources and the actual situation on the site when the schedule is being made. The production rate during schedule making for activity t in the segment i is $Rate_{t,i}$

Determination of production rate when the schedule is adjusted: when the schedule has been adjusted, the planner should first determine the start and end time of the construction, and calculate the adjusted rate base on the new time parameter. The rate of activity t in the section i after adjustment should be $ARate_{t,i}$.

Next, the calculation method for $ARate_{t,i}$ is illustrated. In Figure 3, Linear Activity A is divided into three sections $A-1$, $A-2$, $A-3$. The three broken lines located in the diagram below represents original construction schedule of Activity A. The first two broken lines located in the upper area represent the real construction situation during the former two sections. We can see that the construction has lagged behind in the first two segments. Presuming that at this moment the construction activity of $A-3$ section is re-scheduled to equal the end time to the original plan, the rate R_{A-3} corresponding Activity $A-3$ after adjustment can be marked as:

$$R_{A-3} = W_{A-3} / (Comp_{A-3} - Cpmm_{A-3}) \tag{3}$$

Where $Comp_{A-3}$ is the completion date for $A-3$ after adjustment; $Cpmm_{A-3}$ is the beginning date of adjusted $A-3$; and W_{A-3} is the construction amount of $A-3$. Without loss of generality, the adjusted rate of activity t in segment i is recorded as:

$$ARate_{t,i} = W_{t,i} / (AComp_{t,i} - AComm_{t,i}) \quad (4)$$

In this section, $W_{t,i}$ is the construction amount of activity t in segment i, $AComp_{t,i}$ is the adjusted end time of Activity *t* in Segment i; $AComm_{t,i}$ is the adjusted start date of Activity *t* in Segment i

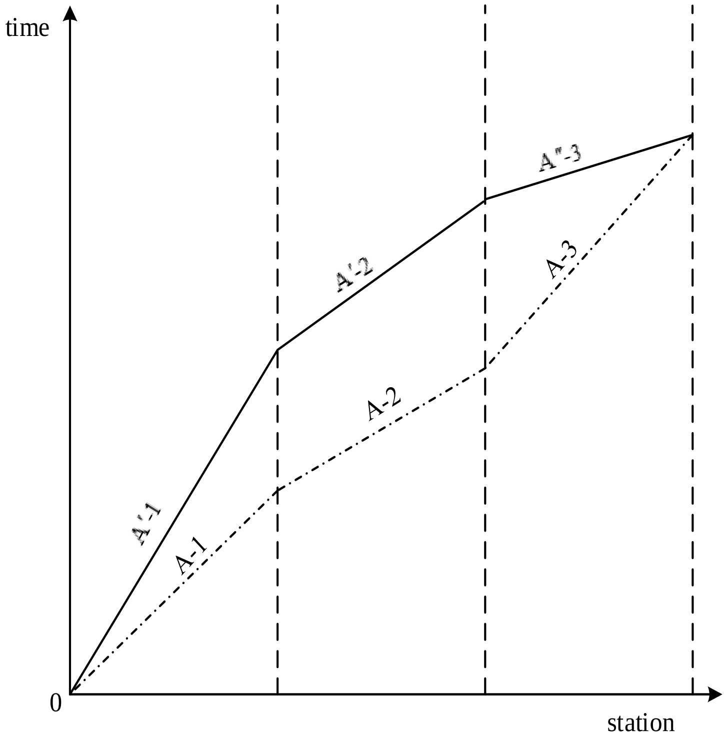

Figure 3. Schematic Diagram of the Determination Method of Production Rate

4. Production Rate Float

4.1. Concept of Production Rate Float

Based on the critical activity path in linear scheduling method, the concept of the production rate float can be given under LSM method of construction activity.

The concept of the production rate float is proposed by Harmelink, which refers to the change value when a certain activity is about to become non-critical activity segment [8]. Mattila and Abraham define the construction float rates as: without affecting the minimum spacing between two adjacent activities, the float value of an activity between the minimum production rate and the planned production rate [18].

In Linear Scheduling Method, the production rate float can be found only in linear activities, and only in non-critical linear activities or non-critical parts of linear activities. As for the rate float of the key linear activities and the key part of linear activities is 0.

4.2. Calculation of Production Rate Float

The calculation of production rate float is given by Harmelink[19], and we further developed it in this paper. The following is the calculation steps of production rate.

First, use LSM to create linear project schedules, and use the critical path calculation method to present the critical path of this schedule; Figure 4 shows a LSM example and displays the critical path in this schedule, which is marked as bold.

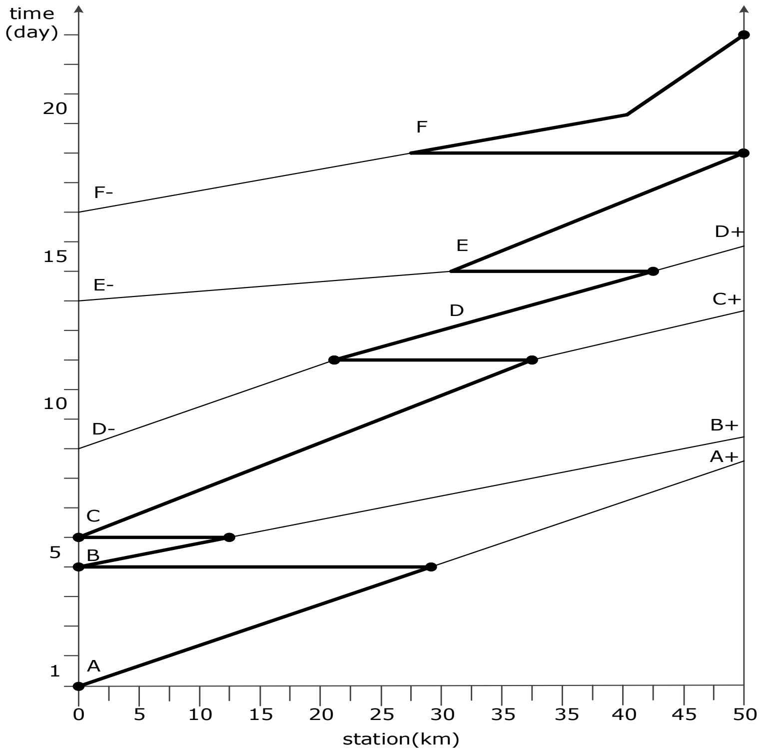

Figure 4. An Example for LSM with Critical Activity Path Marked

Second, discern the Beginning Non-controlling Activities segment in the LSM schedule and the Ending Non-controlling segment; After determination of the critical path of the LSM schedule, the activities in this schedule can be divided into critical activities (when the only part of the activities is on the critical path, the part is called Controlling Segment) and non-critical activities (when the only part of the activities is situated on the non-critical path, that part is called Non-controlling Segment). Wherein, according to its location (on the critical path), the Non-controlling segments can be divided into Beginning Non-controlling Segment and Ending Non-controlling segment. Two types of non-critical activities segments are given as examples in Figure 4. Take Activity D for instance, the left part is Beginning Non-controlling Segment; the middle part is the Controlling Segment; and the right part is Ending Non-controlling Segment.

Third, determine the lowest possible production rate of Beginning Ending Non-controlling Segment (determine the earliest possible start time of Beginning Non-controlling Segment, and the latest possible end time of Ending Non-controlling Segment).

Fourth, calculate the production rate float between of non-controlling activities (segments).

This paper gives detailed explanation for determination of the lowest production rate of non-controlling segment and the calculation of production rate float.

As showed in Figure 5, the calculation method for production rate float is given for Beginning Non-controlling Segment of the linear activities. Activity D- in the diagram is the Beginning Non-controlling Segment of Activity D.

The start point of activity D- is (x_{1D-}, y_{1D-}), and the end point is (x_{2D-}, y_{2D-}). Without violating the time constraint between Activity D- and Activities C-, the earliest possible start time of Activitiey D- is (x'_{1D-}, y'_{1D-}) . Thus, the lowest possible production rate of activity D- is $LRate_D$:

$$LRate_{D-} = (x_{2D-} - x'_{1D-})/(y_{2D-} - y'_{1D-}) \tag{5}$$

According to the definition of production rate float, Activity D- can be described as:

$$Float_{D-} = (x_{2D-} - x_{1D-})/(y_{2D-} - y_{1D-}) - (x_{2D-} - x'_{1D-})/(y_{2D-} - y'_{1D-}) \tag{6}$$

Without loss of generality, the production rate float of any Beginning Non-controlling linear activity α is described as:

$$Float_{\alpha} = (x_{2\alpha} - x_{1\alpha})/(y_{2\alpha} - y_{1\alpha}) - (x_{2\alpha} - x'_{1\alpha})/(y_{2\alpha} - y'_{1\alpha}) \tag{7}$$

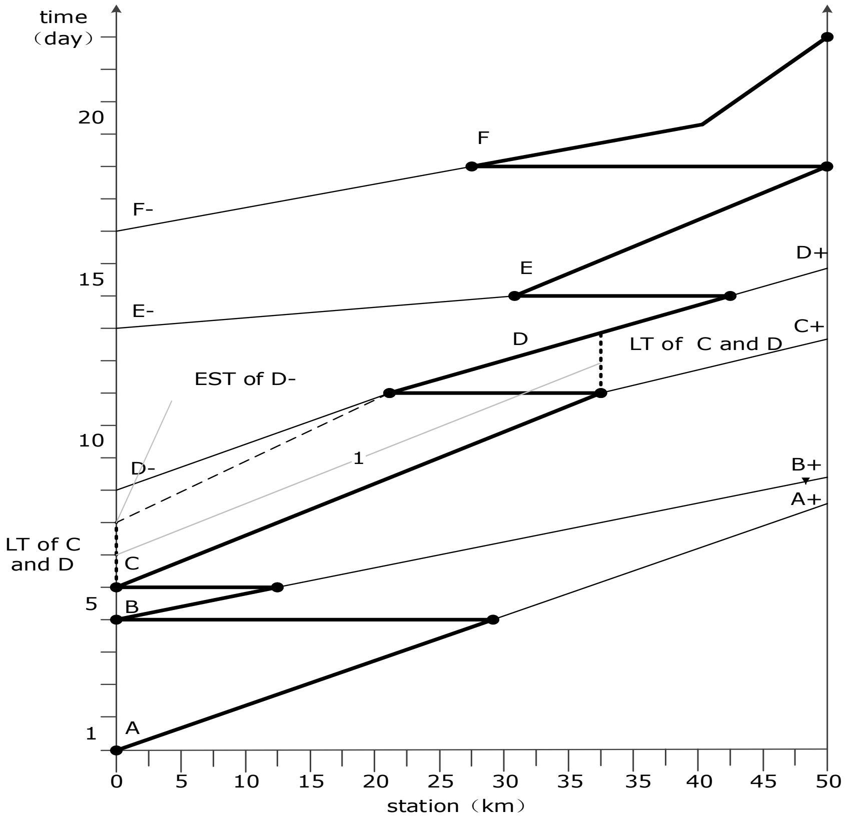

Figure 5. Schematic Diagram of Front Non-controlling Activity

Figure 6 provides an explanation for calculation of production rate float of front Non-controlling Segment. The activity D+ in the Diagram is the Ending Non-controlling Segment of activity D.

The start point of activity D+ is (x_{1D+}, y_{1D+}), the end point is (x_{2D+}, y_{2D+}), without violating the time constraint between D+ and C+, the earliest possible start time of activity D+ is (x'_{2D+}, y'_{2D+}). Thus, the lowest possible production rate of Activity D+ is $LRate_{D+}$:

$$LRate_{D-} = (x'_{2D+} - x_{1D+})/(y'_{2D+} - y_{1D+}) \tag{8}$$

According to the definition of production rate float, the production rate float of activity D+ can be described as follows:

$$Float_{D-} = (x_{2D+} - x_{1D+})/(y_{2D+} - y_{1D+}) - (x'_{2D+} - x_{1D+})/(y'_{2D+} - y_{1D+}) \tag{9}$$

Without loss of generality, the production rate float of any Beginning Non-controlling Activity β is described as:

$$Float_{\beta} = (x_{2\beta} - x_{1\beta})/(y_{2\beta} - y_{1\beta}) - (x'_{2\beta} - x_{1\beta})/(y'_{2\beta} - y_{1\beta}) \tag{10}$$

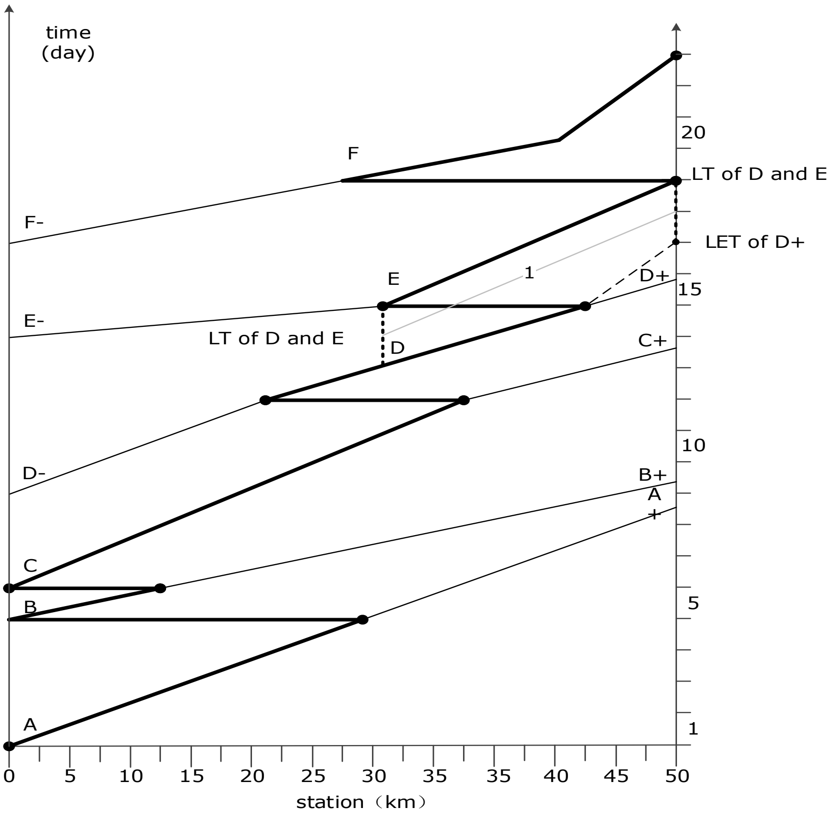

Figure 6. Schematic Diagram of Ending Non-controlling Activity

5. Conclusions

In the adjustment process of the schedule, the adjusted parameters can be obtained firstly, including the new start and end time of the adjusted schedule. On the basis of these time parameters, production rate after adjustment is obtained by means of the calculation method of production rate presented above. On the basis of the new rate, managers can recombine the sources and process adjustment accordingly, which is useful to make the subsequent activities meet the needs of new production rate in the further construction process. That is, for the optimization and adjustment of the schedules, the role of the production rate is to determine the allocation of the resource.

This study constructs a new method to present the production rate within the frame of LSM, which can achieve dynamic adjustment of the rate. This method is based on the linear feature of the linear production activities in the time-distance two-dimensional coordinate system, and the proportional characteristics between the slope and the production rate. The study presents a geometric algorithm to calculate production rate float in the critical sequence. The method can be illustrated by independent graph, and easy to achieve LSM automatic illustration by programming, which will provide mathematical supports for dynamic adjustment in LSM critical path.

Acknowledgements

This research was supported by the Fundamental Research Funds for the Central Universities under grant number O15JB00020.

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