

Applying Multifractal Formalism Combined with Wavelet Transform to Wood Defects Detection

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Abstract

Nowadays, X-ray computed tomography (CT) wood nondestructive testing technology has been applied to detection of internal defects in log for the purpose of obtaining the optimal wood cutting plan. Multifractal spectrum and wavelet transform are usually used for analyzing, modeling, and extracting different complex features of signals and images. A novel CT image edge detection method which using multifractal spectrum theory combined with wavelet transform is applied in this paper. The new method can be divided into the following main steps: (1) Calculating the wavelet module values of wood defect image. (2) Combining wavelet transform module values with multifractal theory. (3) Calculating the multifractal spectrum from the wavelet transform. (4) Selecting the appropriate threshold to wood defects detection. A large number of experimental results show that the new method to recognize the wood defects is effective.

Keywords: *Computer tomography; Multifractal spectrum; wavelet transform; wood defects detection.*

1. Introduction

During the past decades, a large number of effective nondestructive log testing methods have been applied in laboratory experiments and in industrial production. Such as acoustic emission testing, electromagnetic testing, ultrasonic testing, liquid penetrant testing, eddy current testing, radiographic testing, nuclear magnetic resonance (NMR) and laser testing etc [1]. Among all the nondestructive testing methods radiographic testing is the most widely used, especially X-ray testing method. In this paper, we chosen X-ray computed tomography scanning technology for nondestructive log testing. X-ray computed tomography can reduce the effect of physical factors and provide three dimensional (3D) information of the sample compared with the conventional radiography method [2-3]. X-ray computed tomography has provided a better method for observing intact wood structure. By the method of X-ray computed tomography, a large number of relevant experiments (include wood defects, wood moisture, wood density et al) have been completed in the past decades by the scientists from all over the world [4-9]. Among all of these, defects detection is the most important application of X-ray computed tomography.

After acquiring the X-ray computed tomography image, image edge detection is a key step in the defects detection of log images. A large number of edge detection methods have been proposed for different purposes in different applications.

But most of them can be grouped into two categories. Among the earliest works of edge detection are Roberts, Sobel, and Prewitt edge detectors, all of which detect edges use convolution masks to approximate the first derivative. Other methods, such as

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Laplacian of Gaussian (LOG) edge detector, usually search for zero crossing in a second derivative of an image [10]. In contrast, canny edge detector is a relatively new operator. Three performance criteria defined for the optimality edge detection were good detection, good localization, and unique answer to a true edge [11].

Both wavelet transform and multifractal spectrum theory have been widely used in signal analysis, object modeling, and image processing, *etc.* [12-18]. Due to the image signal mutation point and noise have different sensitivity on wavelet coefficients, make the wavelet transform has good image denoising effect. And multifractal spectrum theory is can not only outstanding global image features, but also better response the local image characteristics compared with traditional image processing methods. In recent years, the wavelet local maxima modulus based method is received widespread attentions because it can calculate the multifractal spectrum simply and effectively [19-22]. A promising edge detection method combined wavelet transform with multifractal spectrum theory was provides in this paper.

2. Multifractal Formalism

Multifractal also called complex fractal or multiple scale fractal, often used to summarized a fractal system which cannot fully describe by using a single exponent (the fractal dimension), it is from the local property to study the features of the shape. Multifractal spectrum theory is can not only outstanding integral fractal features, but also better response the local fractal characteristics compared with other single exponent methods.

2.1. Lipschitz-Hölder Exponent

Assume that $f(x)$ is the function of $f: R \rightarrow R$, where $|f(x) - P(x - x_0)| < C|x - x_0|^\alpha$, $\alpha > 0$, P is a polynomial which is not more than α , $x \in B[x_0, \delta]$, $C > 0$, and C is a constant.

Let

$$\alpha(x_0) = \sup(\alpha : f \in C_{x_0}^\alpha) \quad (1)$$

$$\alpha(x) = \sup(\alpha : f \in C_x^\alpha) \quad (2)$$

Then $\alpha(x_0)$ is the Hölder exponent of $f(x)$ at point x_0 and $\alpha(x)$ is the Hölder exponent of $f(x)$ in the domain of definition. The Hölder exponent α can describe the local characteristics well of function $f(x)$.

2.2. Legendre Spectrum

First of all, we defined

$$M_n(x, y) = \sum_{k=0}^{v_n} [c_n(I_n^k)^x \mu(I_n)^{-y}] \quad (3)$$

$$M(x, y) = \lim_{n \rightarrow \infty} \left[\sup \frac{\log M_n(x, y)}{n} \right] \quad (4)$$

$$\Omega = \{(x, y) : X(x, y) < 0\} \quad (5)$$

If $c_n(I_n^k)\mu(I_n^k) \neq 0$, then there exists a concave function τ as:

$$\Omega = \{(x, y) \in R^2 : y < \tau(x - 0)\} \quad (6)$$

Let τ^* be the Legendre spectrum of τ , then the Legendre transformation of τ is determined by:

$$f_l(\alpha) = \tau^*(\alpha) = \inf_q [q\alpha - \tau(q)] \quad (7)$$

The function $f_l(\alpha)$ is called Legendre spectrum of Hölder exponent α , where q is a weighting factor.

3. Wavelet Transform and Its Module Value

The wavelet transform is to zoom and translate a basis wavelet, a scaling function with variable amplitude, and integrate the product which consists in expanding signals based wavelets. The basis wavelet (also called mother wavelet) is denoted by $\psi(x)$. The zoom and translation operations are performed by the following two parameters: the scale parameter s and the location parameter b . The wavelet transform of the signal x is defined as:

$$Wf(s, b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \psi\left(\frac{x-b}{s}\right) dx \quad (8)$$

We choose the two dimensional Gaussian function $\theta(x) = \exp(-x^2/2)$ as the smoothing kernel, and choose the second order partial derivatives of Gaussian function in the x, y direction as the basis wavelet.

$$\psi^1 = \frac{\partial \theta}{\partial x_1} = \left(-\frac{1}{2\pi\sigma^4}\right) \left(1 - \frac{x_1^2}{\sigma^2}\right) e^{-\frac{x_1^2+x_2^2}{2\sigma^2}} \quad (9)$$

$$\psi^2 = \frac{\partial \theta}{\partial x_2} = \left(-\frac{1}{2\pi\sigma^4}\right) \left(1 - \frac{x_2^2}{\sigma^2}\right) e^{-\frac{x_1^2+x_2^2}{2\sigma^2}} \quad (10)$$

Then, the wavelet transform in the point x_1 is defined by:

$$\begin{aligned} WT_f^1(\vec{b}, s) &= \frac{1}{s} \iint f(x_1, x_2) \psi\left(\frac{x_1-b_1}{s}, \frac{x_2-b_2}{s}\right) dx_1 dx_2 \\ &= \frac{1}{s} \left(\frac{1}{2\pi\sigma^4}\right) \iint f(x_1, x_2) \left(1 - \frac{x_1-b_1}{(s\sigma)^2}\right) e^{-\frac{(x_1-b_1)^2+(x_2-b_2)^2}{2(s\sigma)^2}} dx_1 dx_2 \end{aligned} \quad (11)$$

The wavelet transform in the point y_2 is defined by:

$$\begin{aligned}
 WT_f^2(\vec{b}, s) &= \frac{1}{s} \iint f(x_1, x_2) \psi\left(\frac{x_1 - b_1}{s}, \frac{x_2 - b_2}{s}\right) dx_1 dx_2 \\
 &= \frac{1}{s} \left(\frac{1}{2\pi\sigma^4}\right) \iint f(x_1, x_2) \left(1 - \frac{x_2 - b_1}{(s\sigma)^2}\right) e^{-\frac{(x_1 - b_1)^2 + (x_2 - b_2)^2}{2(s\sigma)^2}} dx_1 dx_2
 \end{aligned} \tag{12}$$

For the calculation of two dimensional wavelet transform modulus maxima needs a certain direction. Thus, the partial derivative of Gaussian function in the x, y direction is usually defined as a basis wavelet vector.

$$WT_f(\vec{b}, s) = \begin{cases} WT_f^1(\vec{b}, s) \\ WT_f^2(\vec{b}, s) \end{cases}, \vec{b} = (b_1, b_2) \tag{13}$$

The module of this wavelet transform is determined by:

$$Mf(\vec{b}, s) = \sqrt{|WT_f^1(\vec{b}, s)|^2 + |WT_f^2(\vec{b}, s)|^2} \tag{14}$$

The direction of the wavelet transform vector is defined as:

$$Af(\vec{b}, s) = \begin{cases} s(\vec{b}) \\ \pi - s(\vec{b}) \end{cases}, \text{ where } s(\vec{b}) = \tan^{-1}\left(\frac{WT_f^2(\vec{b}, s)}{WT_f^1(\vec{b}, s)}\right) \tag{15}$$

When $s \in 2^m, m \in Z$, the Reference [23] point out the wavelet transform module value $Mf(\vec{b}, s)$ and the Hölder exponent α have the close link as follows:

$$|Mf(\vec{b}, 2^m)| \leq K(2^m)^\alpha \tag{16}$$

Where the index K is a constant.

4. Image Multifractal Analysis Method Based on Wavelet Transform

Multifractal can provide statistical information on fractal scale, but can't describe the local fractal properties and kinetic features adequately. The wavelet transform theory is a powerful tool to reveal the fractal properties of the local scale. We can see the rich details of fractal by using wavelet transform. Therefore, we introduce the theory of multifractal spectrum combined with wavelet transform. The partition function based on wavelet transform is defined as follows:

$$Z(q, s) = \sum_{b \in L_s} |Mf(\vec{b}, s)|^q \tag{17}$$

Where L_s is the set of all wavelet transform modulus maxima at the scale s , satisfying their local derivative constraint in scale [24].

For each weighting factor $q \in R$, the scaling function $\tau(q)$ with the partition function $Z(q, s)$ asymptotic decay on scale s :

$$\tau(q) = \liminf_{s \rightarrow 0} \frac{\log Z(q, s)}{\log s} \tag{18}$$

This means that its scaling function $\tau(q)$ and the partition function $Z(q, s)$ has the following relationship:

$$Z(q, s) \sim s^{\tau(q)} \quad (19)$$

The major realization process of image edge detection by using the multifractal spectrum calculation based wavelet transform modulus maxima method can be summarized as follows:

- (1) Calculate the wavelet transform module value and its direction of each point on the original image by using formula (13) and (14) at multiple scales.
- (2) Compute the Hölder exponent α of each point using function (16).
- (3) Compute the scaling function $\tau(q)$ based the partition function $Z(q, s)$ using formula (17) and (18).
- (4) Calculate the Legendre multifractal spectrum $f_l(\alpha)$ of each point using function (7).
- (5) Select the appropriate multifractal spectrum threshold for image edge detection.

5. Experiment and Results

The image used in the experiment is a Korean pine X-ray CT cross section image with crack and knot. In order to make valuable timber resource used reasonably and ensure the quality of wood processing, the exact position and accurate edge detection of several wood defects must be determined. Then, the optimal sawing line to cut the log is given.

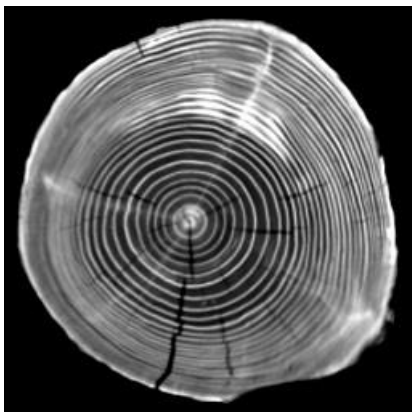


Figure 1. The Original Wood CT Image

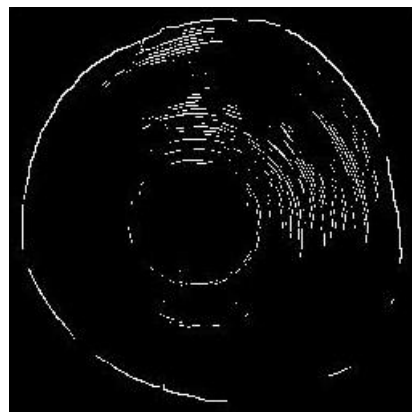


Figure 2. Image after Sobel Operator Processing

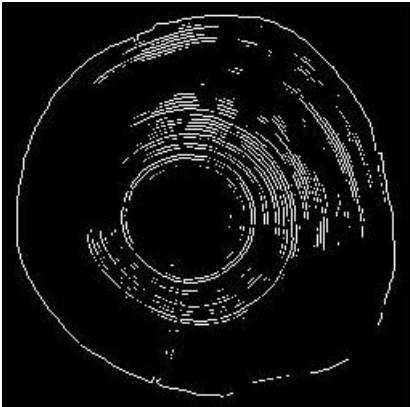


Figure 3. Image after Roberts Operator Processing



Figure 4. Image after Prewitt Operator Processing

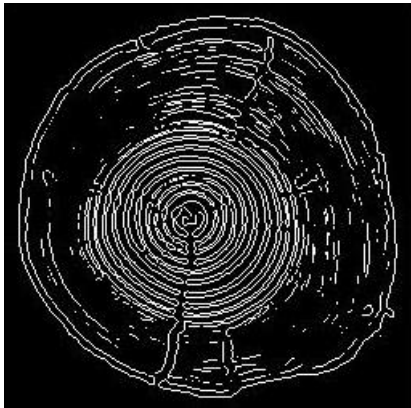


Figure 5. Image after Log Operator Processing.

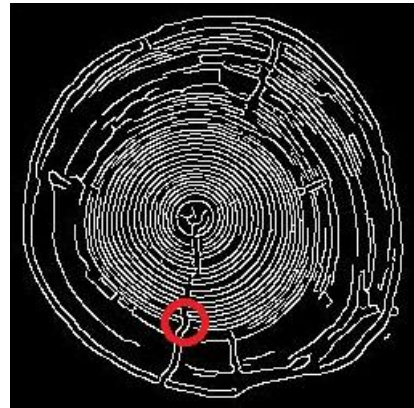


Figure 6. Image of Legendre Spectrum Edge Detection.

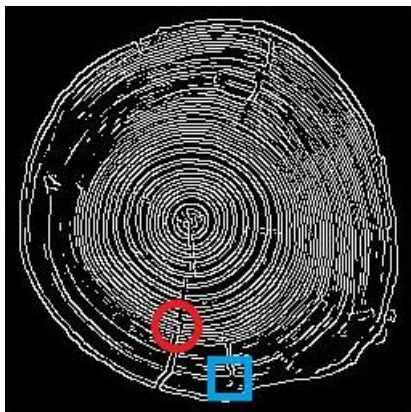


Figure 7. Image of Wavelet Transform Edge Detection.

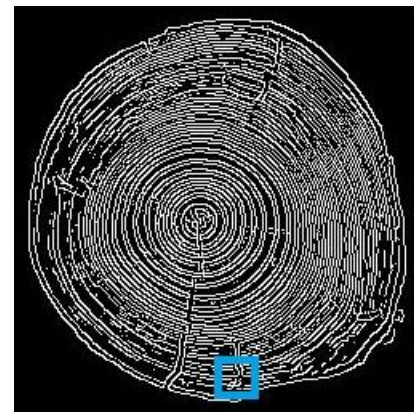


Figure 8. Image of Multifractal Combined with Wavelet Transform Edge Detection

Figure 1 is the original log CT image with crack and knot defects. Figures 2-5 are the images processed by using traditional Sobel operator, Robert operator, Prewitt operator, and Log operator edge detection methods with adaptive threshold. Figures 6-7 are the edge detection images processed by using Legendre spectrum and wavelet transform, respectively. Figure 8 is the detail edge detection image by using multifractal combined with wavelet transform method.

As can be seen in Figures 2-5, the result images after conventional edge detection operators cannot well depict detailed edge information in original log CT image. The detected edge information by the classical Sobel, Robert, Prewitt and Log operators is imperfect and uncontinuous. It cannot reflect the shape and position coordinates of log defects exactly. As can be seen in Figure 6, the detected edge image processed by Legendre spectrum can well reflect the global information of log defects, but the local information is not complete. From the crack defect in the red circle in Figure 6 and Figure 7, it can be seen that the wavelet transform can better reflect the local characteristics of defect images compared with the Legendre spectrum method. And from the crack defect in the blue square in Figure 7 and Figure 8, it can be seen that the novel multifractal combined with wavelet transform method can better shown the local and global characteristics compared with the wavelet transform method. In conclusion, compared with the traditional operators, the multifractal combined with wavelet transform is a better log defect edge detection method. This method can better reflect the local and global characteristics of log defect images at the same time.

6. Conclusion

Log nondestructive testing technology is a newly emerged synthetical technology, and has been developed very fast in recent years. X-ray computed tomography has many unique advantages such as high resolution, easy operating, quick testing and visual results compared with other conventional nondestructive testing methods. A new multifractal formalism combined with wavelet transform log CT image edge detection method is applied in this paper. By calculating the wavelet transform values, the Hölder exponent α can be calculated. And by calculating the partition function $Z(q,s)$, the scaling function $\tau(q)$ can be computed. After the calculation of the scaling function $\tau(q)$ and the Hölder exponent α , the wavelet-based multifractal spectrum $f_i(\alpha)$ can be computed. By setting a suitable threshold of $f_i(\alpha)$, the intact defects edge of log CT images can be extracted accurately. Then the better log sawing line can be obtained according to the practical need. Wavelet-based multifractal spectrum method is an excellent edge detection method, which can display the intact edge information of log CT defect images from local to global. A promising edge detection method for identify the log interior defects by using multifractal combined with wavelet transform is provided.

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References

- [1] L. Yu and D.W. Qi, "Analysis and processing of decayed log CT image based on multifractal theory", *Computers and electronics in agriculture*, vol. 63, (2008), pp. 147-154.
- [2] V. Rasiyah and L. Aylmore, "Estimating microscale spatial distribution of conductivity and pore continuity using computed tomography", *Soil Sci. Soc. Am. J.*, vol. 62, (1998a), pp. 1197-1202.
- [3] V. Rasiyah and L. Aylmore, "Characterizing the changes in soil porosity by computed tomography and fractal dimension", *Soil Sci.*, vol. 163, (1998b), pp. 203-211.
- [4] F. Longuetaud, J.M. Leban, F. Mothe, E. Kerrien and M.O. Berger, "Automatic detection of pith on CT images of spruce logs" *Computers and electronics in agriculture*, vol. 44, (2004), pp. 107-119.
- [5] S.M. Bhandarkar, X.Z. Luo, R. Daniels and E.W. Tollner EW, "Detection of cracks in computer tomography images of logs", *Pattern recognition letters*, vol. 26, (2005), pp. 2282-2294.
- [6] S. Alkan, Y. Zhang and F. Lam, "Moisture distribution changes and wetwood behavior in subalpine fir wood during drying using high X-ray energy industrial CT scanner", *DRYING TECHNOLOGY*, vol. 25, (2007), pp. 483-488.
- [7] X. He and D.W. Qi, "Density and moisture content forecasting based on X-ray computed tomography", *European journal of wood and wood products*, vol. 71, (2013), pp. 647-652.
- [8] G.Y. Peng, Z.H. Jiang, X.G. Liu, Y. Yu, S.M. Yang, B. Deng, T.Q. Xiao and X.H. Wang, "Measuring the Density of Wood and Bamboo Using Computed tomography", *Spectroscopy and spectral analysis*, vol. 32, (2012), pp. 1935-1938.
- [9] S. Gernot, K. Andreja, P. Bernhard, P. Alexander, A.K. Frederick and D. Manfred, "Microstructure of viscoelastic thermal compressed (VTC) wood using computed microtomography", *Wood science and technology*, vol. 47, (2013), pp. 121-139.
- [10] D. Ziou and S. Tabbone, "Edge detection techniques: An overview", *International Journal of Pattern Recognition and Image Analysis*, vol. 4, (1998), pp. 537-559.
- [11] J. Canny, "A computational approach to edge detection", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 8, (1986), pp. 679-698.
- [12] T. Chang and C.J. Kuo, "Texture analysis and classification with tree-structured wavelet transform", *IEEE transactions on image processing: a publication of the IEEE Signal Processing Society*, vol. 2, (1993), pp. 429-470.
- [13] P. C. Ivanov, M. G. Rosenblum, C.K. Peng, J. Mietus, S. Havlin, H.E. Stanley, A.L. Goldberger, "Scaling behaviour of heartbeat intervals obtained by wavelet-based time-series analysis", *Nature*, vol. 383, (1996), pp. 323-330.
- [14] H. Adeli, Z. Zhou and N. Dadmehr, "Analysis of EEG records in an epileptic patient using wavelet transform", *Journal of neuroscience methods*, vol. 123, (2003), pp. 69-87.
- [15] N. Hajime, B.K.T Dang, M. Tsutomu and B. Barnabás, "Max-plus algebra-based wavelet transforms and their FPGA implementation for image coding", *Information Sciences*, vol. 180, (2010), pp. 3232-3247.
- [16] D. Grech and G. Pamula, "Multifractal Background Noise of Monofractal Signals", *Acta physica polonica a*, vol. 121, (2012), pp. B34-B39.
- [17] Y. ZG, Y. Leung, Y.D. Chen, Q. Zhang, V. Anh and Y. Zhou, "Multifractal analyses of daily rainfall time series in Pearl River basin of China", *Physica a-statistical mechanics and its applications*, vol. 405, (2014), pp. 193-202.
- [18] L. Yu and D.W. Qi, "Applying multifractal spectrum combined with fractal discrete Brownian motion model to wood defects recognition", *Wood science and technology*, vol. 45, (2011), pp. 511-519.
- [19] R.S. Zbigniew and P.J.M.S. Arno, "Wavelet transform based multifractal formalism in outlier detection and localization for financial time series", *Physica A*, vol. 309, (2002), pp. 388-402.
- [20] O. Abdeldjalil and F. Smain, "Wavelet-based multifractal analysis of 1-D and 2-D signals: New results", *Analog integrated circuits and signal processing*, vol. 69, (2011), pp. 3-15.
- [21] S. Yu, B. Markus, W. Christian, M. Ewald and T. Stefan, "Wavelet-based multifractal analysis of fMRI time series", *NeuroImage*, vol. 22, (2004), pp. 1195-1202.
- [22] W. Herwig, G.R. Stéphane, J. Stéphane and A. Patrice, "Wavelet leaders and bootstrap for multifractal analysis of images", *Signal Processing*, vol. 89, (2009), pp. 1100-1114.
- [23] S. Mallat and W.L. Huang, "Singularity detection and processing with wavelet", *IEEE Transaction on information theory*, vol. 38, (1992), pp. 617-643.

- [24] Z. R. Struzik, "Removing divergences in the negative moments of the multi-fractal partition function with the wavelet transformation", CWI Report, InformationSystems (INS-R9803), (1998) January 31.

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