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Study on an Improved PSO Algorithm and its Application for Solving Function Problem

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Abstract

Particle swarm optimization(PSO) algorithm has the advantages of simplicity and easy implementation, but it exits the weaknesses of the being easy to fall into local minimum and premature convergence. In order to overcome these weaknesses of PSO algorithm, the inertia weight and learning factor are improved and the PSO algorithm is initialized by using chaotic optimization in order to propose an improved PSO(WLFCPSO) algorithm. In the proposed WLFCPSO algorithm, chaotic optimization strategy is used to initialize the parameters of PSO algorithm in order to obtain the more reasonable initialization parameters. The adaptive inertia weight adjustment strategy is used to control the adjustment ability of inertia weight in order to keep the diversity of the inertia weight. The dynamic linear adjustment strategy for learning factors is used to gradually reduce the cognitive ability of the individual and improve the global search ability of particles. In order to prove the effectiveness of the proposed WLFCPSO algorithm, several benchmark functions are selected. The experiment results show that the proposed WLFCPSO algorithm has the rapider convergence speed and higher solution precision for solving high-dimension function optimization problems.

Keywords: *Particle swarm optimization; chaotic optimization; adaptive adjustment; diversity; function optimization*

1. Introduction

Particle swarm optimization(PSO) algorithm was proposed by Dr. James Kennedy and Dr. R.C. Eberhar in 1995[1]. This algorithm is derived from the simulation foraging behavior of the bird and fish. It is easy to be described and implemented, adjusts the less parameters, uses relatively small size of population, needs the less number of evaluated functions, takes on fast convergence speed, parallel processing and good robustness. It can find the global optimal solution of solving problem with the larger probability, and has higher computational efficiency than the traditional random method. This algorithm has deep intelligence background and is suitable for scientific research and engineering application. So the PSO algorithm has caused the extensive attention of researchers in the field of evolutionary computation, and has obtained a lot of research results in a few years. It is widely applied in the field of function optimization, neural network training, fuzzy system control, classification, pattern recognition, signal processing, robot technology and so on[2-5].

For complex optimization problems, the basic PSO algorithm exits some defects of premature convergence, slow convergence speed and so on. Many scholars put forward the corresponding improvement strategies in order to improve the performance of PSO algorithm. Fan *et al*. [6] proposed the hybrid Nelder-Mead (NM)-Particle Swarm Optimization (PSO) algorithm based on the NM simplex search method and PSO for the optimization of multimodal functions. The hybrid NM-PSO algorithm is very easy to implement, in practice, since it does not require gradient computation. Liang *et al.* [7] proposed a variant of particle swarm optimizers (PSOs) that we call the comprehensive

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learning particle swarm optimizer (CLPSO), which uses a novel learning strategy whereby all other particles' historical best information is used to update a particle's velocity. Kelley *et al*., [8] proposed the use of a weighted sum of Debye functions to approximate more general complex permittivity functions. Chen *et al*., [9] proposed an innovative hybrid recursive particle swarm optimization (HRPSO) learning algorithm with normalized fuzzy c-mean (NFCM) clustering, particle swarm optimization (PSO) and recursive least-squares (RLS) to generate radial basis function networks (RBFNs) modeling system with small numbers of descriptive radial basis functions (RBFs) for fast approximating two complex and nonlinear functions. Lin *et al*., [10] proposed an improved particle swarm optimization (IPSO) algorithm to adapt the learning rates in the backpropagation process of the RBFNs to improve the learning capability. Kao and Zahara[11] proposed a hybrid method combining two heuristic optimization techniques, genetic algorithms (GA) and particle swarm optimization (PSO), for the global optimization of multimodal functions. Hou *et al.,* [12] proposed an improved hybrid particle swarm algorithm, named the multigrouped mutation particle swarm optimization (MMPSO) to enhance the performance of the simple particle swarm optimization method. Seo *et al*., [13] proposed a auto-tuning multigrouped particle swarm optimization (AT-MGPSO) algorithm for multimodal function optimization. Han *et al*., [14] proposed a new approach coupling adaptive particle swarm optimization (APSO) and a priori information for function approximation problem to obtain better generalization performance and faster convergence rate. Lin *et al*., [15] proposed an improved particle swarm optimization (IPSO) to adapt the learning rates of the recurrent FL-based FNN online. The control performance of the proposed recurrent FL-based FNN controller with IPSO is verified by some simulated and experimental results. Liu *et al*.[16] proposed a radial basis function neural network coupling quantum-behaved particle swarm optimization algorithm (RBF-QPSO). In RBF-QPSO approach, QPSO algorithm is used to find the optimal culture conditions with the established RBF estimator as the objective function. Lin and Wu [17] proposed an efficient learning algorithm, called symbiotic particle swarm optimization (SPSO), combined symbiotic evolution and modified particle swarm optimization for tuning parameters of the recurrent functional neural fuzzy network (RFNFN) . Fan and Chang[18] proposed a novel parallel multi-swarm optimization (PMSO) algorithm with the aim of enhancing the search ability of standard single-swarm PSOs for global optimization of very large-scale multimodal functions. Neyestani *et al.,* [19] proposed a modified PSO (MPSO) mechanism to deal with the equality and inequality constraints in the economic dispatch (ED) problems. Qasem and Shamsuddin[20] proposed a time variant multi-objective particle swarm optimization (TVMOPSO) of radial basis function (RBF) network for improving classification accuracy and diagnosing the medical diseases. Tian *et al*., [21] proposed quantum-behaved particle swarm optimization (QPSO) to estimate the heat source without a priori information on its functional form, which is classified as the function estimation by inverse calculation. Juang *et al*., [22] proposed an adaptive fuzzy PSO (AFPSO) algorithm, based on the standard particle swarm optimization (SPSO) algorithm. Vaisakh *et al*., [23] proposed an evolving ant direction particle swarm optimization algorithm for solving the optimal power flow problem with non-smooth and non-convex generator cost characteristics. Ganeshkumar *et al*., [24] proposed an enhanced particle swarm optimization (EPSO) based on BLX-α crossover and Non-uniform mutation from Genetic Algorithm (GA) for extracting optimal rule set and tuning membership function for fuzzy logic based classifier model. Tatsumi *et al*., [25] proposed a PSO with a new chaotic system derived from the steepest descent method for a virtual quartic objective function with perturbations having its global minima at the personal and global bests, where elements of each particle's position are updated by the proposed chaotic system or the standard update formula. Qasem *et al*., [26] proposed a new memetic multiobjective evolutionary algorithm applied to a radial basis function (RBF) network design based on multi-objective particle swarm optimization augmented with local search features. Jiang *et al*., [27] proposed particle swarm optimization with age-group topology (PSOAG), a novel age-based particle swarm optimization (PSO) for nonlinear function optimization and data clustering problems for performance evaluation. Li *et al*.[28] proposed a novel model (CSPSO-C RBFNN) based on a radial basis function neural network (RBFNN), chaos theory, self-adaptive particle swarm optimization (PSO). The modified PSO algorithm was used to optimize the RBFNN connection weights. Tan *et al*.[29] proposed a composite particle swarm optimization (CPSO) based on combining one particle's historical best information and the global best information into one learning exemplar to guide the particle movement for solving low solution quality and slow convergence speed on multimodal function optimization. Rabanal *et al.*, [30] proposed a parallelizing particle swarm optimization algorithm for functional programming environment. Lu *et al*., [31] proposed an improved dynamic particle swarm optimization algorithm, which uses a new and effective exponential decreasing inertia weight (EDIW) strategy. Yu *et al*., [32] proposed a hybrid particle swarm optimization and genetic algorithm to optimize Radial Basis Function (PSO-GA-RBF) neural network for prediction of annual electricity demand. Li *et al*., [33] proposed a novel model(CSPSO-KHMRBFANN) based on combining chaos theory, self-adaptive particle swarm optimization (PSO) algorithm, K-harmonic means (KHM) clustering and radial basis function artificial neural network (RBF ANN). Abuhasel *et al.*, [34] proposed a hybrid particle swarm optimization (PSO) integrating neural network with fuzzy membership function (NEWFM) technique for epileptic seizure classification tasks based on brain electroencephalography (EEG) signals. Chang [35] proposed a modified particle swarm optimization(PSO) algorithm for solving multimodal function optimization problems.

For the premature convergence, the poor search effect and low convergence precision of basic PSO algorithm for high dimensional function optimization problem, an improved PSO(WLFCPSO) algorithm based on improving the inertia weight and learning factor and chaotic optimization strategy is proposed to improve the optimization performance of the proposed WLFCPSO algorithm. Several benchmark functions are selected to prove the effectiveness of the proposed WLFCPSO algorithm.

The rest of this paper is organized as follows. Section 2 briefly introduces particle swarm optimization. Section 3 proposed an improved ACO algorithm based on improving the inertia weight and learning factor and chaotic optimization strategy. Section 4 introduces the principle and flow of the proposed WLFCPSO algorithm. In this section, the principle and steps of the WLFCPSO algorithm are introduced in detail. Section 5 applies the WLFCPSO algorithm for solving the complex optimization problems. Finally, the conclusions are discussed in Section 6.

2. Particle Swarm Optimization

The PSO algorithm is a population-based search algorithm based on simulating the social behavior of birds[1]. In basic PSO algorithm, individuals, referred to as particles. The positions of particles within the search space are changed based on the social-psychological tendency of individuals in order to delete other individuals. Namely, the velocity(v) and position(x) of each particle will be changed by the particle best value (pB) and global best value (gB). The velocity and position updating of the particle is shown by the followed expression:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_1 \left(pB_{ij}(t) - x_{ij}(t)\right) + c_2 r_2 \left(gB_{ij}(t) - x_{ij}(t)\right) \tag{1}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{2}$$

where $v_{ij}(t+1)$, velocities of particle *i* at iterations *j*, $x_{ij}(t+1)$, positions of particle i^{th} at iterations j^{th}. w is inertia weight to be employed to control the impact of the previous history of velocities. t denotes the iteration number, c_1 is the cognition learning factor, c_2 is the social learning factor, r_1 and r_2 are random numbers uniformly distributed in [0, 1]. Generally, the value of each component in V can be clamped to the range $[-V_{\max}, V_{\max}]$ to control excessive roaming of particles outside the search space.

3. Improvement of Basic PSO Algorithm

Although the PSO algorithm has the advantages of simple structure, less parameters and fast convergence, it is easy to fall into local extreme value, and it can not obtain globally optimal solution. There are two main reasons for this phenomenon: (1) the character of optimization function. (2) due to the inappropriate parameter design and population size of algorithm in the operation process. The two reasons will rapidly disappear the diversity of particles in the process of calculation, and cause the premature phenomenon. The inertia weight and learning factor are improved and the parameters of PSO algorithm is initialized by using chaotic characteristic in order to the global optimization ability in this paper.

3.1. Adaptive Inertia Weight Adjustment

The following index is used to evaluate the premature convergence of population:

$$\Delta = | f_g - f_p | \tag{3}$$

where f_g is the fitness value of global best particle, f_a is the fitness value of all particles, f_p is the average fitness value of fitness value with better than the f_a. Δ is used to evaluate the premature convergence of population. If Δ is smaller, the population tends to be premature convergence.

For the fitness value f_i of the i^{th} particle, adaptive inertia weight adjustment strategy is described as follows:

(1) $f_i > f_p$

At this time, the particle is close to the global optimum value, it is the better particle in the population. So the inertia weight should take the smaller value in order to avoid escaping global optimum value. The inertia weight adjustment of the particle is given:

$$w = w_{med} - \left| \frac{f_i - f_p}{f_g - f_p} \right| \times (w_{med} - w_{\min}) \tag{4}$$

where $w_{\min}$ is the minimum value of w, w_{med} is the intermediate value of the range of w. If the fitness value of the particle is better, the inertia weight of the particle is smaller to strengthen the local optimization.

(2) $f_a < f_i < f_p$

At this time, the particle is the general particle in the population, it has better global optimization ability and local optimization ability. The inertia weight of the particle is adjusted by using the nonlinear decreasing strategy in the following expression.

$$w = w_{max} - (w_{max} - w_{min}) \times \frac{t_{Iter}}{T_{max}} \quad (5)$$

where t_{Iter} is current iteration, T_{max} is the maximum number of iteration.

(3) $f_i < f_a$

At this time, the particle is poor particle, which should give a large inertia weight. The inertia weight is adjusted by using the control parameters k_1 and k_2.

$$w = 1.5 - \frac{1}{1 + k_1 \times \exp(-k_2 \times |f_g - f_p|)} \quad (6)$$

where if the k_1 is more than one, it is used to control the upper bound of w. If the k_1 is larger, the upper bound of w is larger. If the k_2 is more than zero, it is used to control the adjustment ability of w.

3.2. Chaotic Optimization for Initializing Parameters

Chaotic optimization is a novel optimization method, which utilizes the unique ergodicity of chaotic system to optimize the search. It does not require the objective function that has continuous and differentiable character. Its basic idea is to generate a set of chaotic variables with the same number of optimization variables, then the chaos is introduced into the optimization variable to take on chaotic state by using carrier mode. At the same time, the ergodicity range of chaotic motion is amplified to the range of optimization variable, then the chaotic variable is directly used to search.

Logistic mapping is a typical chaotic system, and it is used to generate chaotic variable. The mapping expression is given:

$$z_{k+1} = \mu z_k (1 - z_k) \qquad k = 1,2,3,4,\cdots, \mu \in (2,4] \quad (7)$$

where μ is control parameter, when μ =4.0 and $0 \le z_k \le 1$, Logistic is complete in a state of chaos. The chaotic characteristics of μ =4.0 is used In this paper.

3.3. Improvement of Learning Factors:

In the PSO algorithm, the values of learning factors reflect the intensity of information exchange between particles. The values and proportional relationship of learning factors (c_1 and c_2) determine the motion direction of particles and the convergence results of the algorithm. It can be seen that larger or smaller values of learning factors are not conducive to the optimization of particles. The influences of learning factors are considered in here, so a dynamic linear adjustment strategy for learning factors is proposed in this paper. The expression is given:

$$\begin{cases} c_1 = 2.4 - \dfrac{1.4 t_{Iter}}{T_{max}} \\ c_2 = 0.9 + \dfrac{1.6 t_{Iter}}{T_{max}} \end{cases} \quad (8)$$

In here, learning factor c_1 can be linearly reduced with the number of iterations and learning factor c_2 can be linearly increased with the number of iterations. The improvement of learning factors gradually reduces the cognitive ability of the individual and improve the global search ability of particles when the number of iterations is increased. In the early time of the algorithm, this strategy can improve the global search ability of particles in the whole search space. In the end of the algorithm, this strategy can converge to the global optimum to the particles.

4. Principle and Flow of the WLFCPSO Algorithm

In the basic PSO algorithm, when the particle find a current best position, other particles in the neighborhood filed will quickly move this particle. If the best position is a local optimum value, the particles in the population will be unable to research in the solution space. So it is easy to fall into local extremum value, and does not provide a mechanism for escaping from local optimal solution. In order to make the large search space for finding the possible solution space of the optimal solution, and fully and effectively use the historical knowledge, the search is focused on the possible space of the optimal solution in order to converge to the global optimum with greater probability. So the adaptive inertia weight adjustment strategy, chaotic optimization strategy and dynamic linear learning factor adjustment strategy are introduced into the basic PSO algorithm in order to propose an improved PSO(WLFCPSO) algorithm. In the proposed WLFCPSO algorithm, chaotic optimization strategy is used to initialize the parameters of PSO algorithm in order to obtain the more reasonable initialization parameters. The adaptive inertia weight adjustment strategy is used to control the adjustment ability of inertia weight in order to keep the diversity of the inertia weight. The dynamic linear adjustment strategy for learning factors is used to gradually reduce the cognitive ability of the individual and improve the global search ability of particles.

The flows of the proposed WLFCPSO algorithm are described:

Step1. Initialize parameters

Randomly generate population size(m),the number of maximum iteration($T_{\max}$), fitness error(ε), and other relevant parameters.

Step2. Dynamically initialize the position and velocity of each particle

Randomly generate n-dimensional vectors $Z_1 = (Z_{11}, Z_{12}, \cdots, Z_{1n})$, the value of each component is between 0 and 1, n is the number of variables in the objective function. According to equation(7), n vectors(z_1 , z_2 , z_3 ,, $\cdots$, z_n) are obtained. Then each component of z_i is carrier to the corresponding variable interval.

Step3. Calculate the fitness value of each particle

According to the objective function, the fitness value of each particle is calculated. The M solutions of the better performance are selected from the initial population (N) as the initial solution.

Step4. The best position of particle is $Pbest$ and global best of population is $Gbest$.

Step5. If the fitness of particle is better than $Pbest$, the current fitness is set as $Pbest$. The optimal individual extreme value is selected as $Gbest$.

Step6. The velocity and position of the particle is updated according to the equation(1) and equation(2).

Step7. The optimal position $P_g = (P_{g1}, P_{g2}, P_{g3}, \cdots, P_{gD})$ is dynamically optimized.

Step8. The corresponding adaptive strategies are used according to the different fitness value of the particle. The inertia weight is adjusted according to the equation(3) to equation(6). The learning factors are adjusted according to the equation(8). Then return to Step3.

Step9. Determine the end condition meets the termination. If it meets the termination, next to Step.10. Otherwise return to Step 3.

Step10. Output the global optimal position.

5. Experimental Results and Comparative Analysis

In order to prove the performance of the proposed WLFCPSO algorithm, several benchmark functions are selected in this paper. At the same time, the basic PSO algorithm, chaotic PSO(CPSO) algorithm and quantum PSO(QPSO) algorithm are selected to compare with the proposed WLFCPSO algorithm. The initial parameters of the basic PSO, CPSO and QPSO algorithms are selected after testing. The selected ones are those that gave the best computational results. The initial parameters are obtained as follows: population size $m = 70$, the number of maximum iteration $T_{\max} = 500$, fitness error $\varepsilon = 0.001$, max velocity $v_{\max} = 100$. The experiment environment is: Matlab 2010b, hardware configuration: the Pentium IV, 2.0GB RAM with Windows 7 operation system. Each algorithm is independently run 30. Several benchmark function expressions from the famous benchmarks are shown in Table 1.

Table 1. Benchmark Function Expression (30D)

Function	Function Expression	Optimum	Range
f_1	$f(x) = \sum_{i=1}^{30} x_i^2$	$f_{\min} = 0$	**[-100,100]**
f_2	$f(x) = \sum_{i=1}^{D-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2$	$f_{\min} = 0$	**[-2.048,2.048]**
f_3	$f(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1)$	$f_{\min} = 0$	**[-2.56,2.56]**
f_4	$f(x) = \sum_{i=1}^{30} (x_i^2 - 10\cos(2\pi x_i) + 10)$	$f_{\min} = 0$	**[-50, 50]**
f_5	$f(x) = -20\exp(-0.2\sqrt{\sum_{i=1}^{n} \frac{x_i^2}{n}}) - \exp(\sum_{i=1}^{n} \cos(2\pi x_i)/n) + 20 + e$	$f_{\min} = 0$	**[-32,32]**
f_6	$f(x) = \sum_{i=1}^{30} (y_i^2 - 10\cos(2\pi y_i) + 10)$ $y_i = \begin{cases} x_i, if \ \lvert x_i \rvert \geq 0.5 \\ round(2x_i)/2, if \ \lvert x_i \rvert < 0.5 \end{cases}$	$f_{\min} = 0$	**[-50,50]**

In order to describe the performance of these algorithms, the best optimal value and average value are selected to illuminate the performance of proposed WLFCPSO algorithm. The experiment results are shown in Table 2.

Table 2. The Experimental Results (30D)

Function	Method	Optimal value	Min value	Average value
f_1	PSO	0	6.256 271e-005	3.203 604e-004
	CPSO		1.357 462e-030	2.447 328e-018
	QPSO		4.503 065e-032	3.814 395e-020
	WLFCPSO		7.540 732e-042	5.960 463e-024
f_2	PSO	0	1.863 064e-000	5.243 864e+001
	CPSO		8.264 074e-004	3.074 851e-003
	QPSO		3.064 861e-005	1.645 917e-003
	WLFCPSO		3.064 972e-007	3.940 562e-004
f_3	PSO	0	2.046 682e-012	4.742 803e-010
	CPSO		4.647 073e-062	7.906 532e-048
	QPSO		2.943 682e-065	1.734 832e-050
	WLFCPSO		1.634 801e-077	5.946 830e-061
f_4	PSO	0	5.745 074e-010	4.046 034e-007
	CPSO		7.463 880e-011	3.754 608e-008
	QPSO		1.463 964e-014	9.432 869e-012
	WLFCPSO		3.534 8015-019	2.463 765e-016
f_5	PSO	0	2.745 886e-007	6.953 068e-006
	CPSO		6.643 856e-011	4.708 327e-009
	QPSO		3.846 582e-013	6.074 681e-011
	WLFCPSO		6.457 092e-017	8.192 326e-015
f_6	PSO	0	3.836 005e-003	3.843 845e-001
	CPSO		4.128 640e-007	2.463 892e-005
	QPSO		1.871 473e-010	3.224 083e-007
	WLFCPSO		3.843 065e-017	8.137 404e-013

As can be seen from Table 2, for the given six benchmark functions, the proposed WLFCPSO algorithm can obtain the best solution for all six benchmark functions by observing and analyzing the experiment results. And the proposed WLFCPSO algorithm has better optimization performance than the basic PSO algorithm, CPSO algorithm and QPSO algorithm. So the experiment results prove that the proposed WLFCPSO algorithm takes on the better searching ability. The comparison results show that the proposed WLFCPSO algorithm is more capable to obtain the global optimization solution and overcome the premature phenomenon for the high-dimensional function problems. So the proposed WLFCPSO algorithm has higher solving accuracy for complex function.

6. Conclusion

The PSO algorithm is based on social adaptation of knowledge for working, and all individuals are considered to be of the same generation. It has many advantages, such as the simple algorithm, the fewer control parameters and good convergence performance, and so on. But it is easy to fall into local extremum value, and does not provide a mechanism for escaping from local optimal solution. So the adaptive inertia weight adjustment strategy, chaotic optimization strategy and dynamic linear learning factor adjustment strategy are introduced into the basic PSO algorithm to make the large search space for finding the possible solution space of the optimal solution, and fully and effectively use the historical knowledge. An improved PSO(WLFCPSO) algorithm is proposed in this paper. In order to prove the performance of the proposed WLFCPSO algorithm, six benchmark functions are selected. And the basic PSO algorithm, chaotic PSO(CPSO) algorithm and quantum PSO(QPSO) algorithm are selected in order to prove the optimization performance of the proposed WLFCPSO algorithm. The comparison results show that the proposed WLFCPSO algorithm is more capable to obtain the global optimization solution and overcome the premature phenomenon for the high-dimensional function problems.

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