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A Multicast Routing Algorithm Based on Prior Objective Nodes

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Abstract

The minimum Steiner tree problem is an NP-complete problem in multicast routing algorithms. In this paper, an improved algorithm called the prior nodes minimum cost path heuristic (PNMPH) algorithm is presented according to the shortage of the minimum cost path heuristic (MPH) algorithm, in which some paths that pass through prior destination nodes are selected first. It partly shares links in the network and decreases the cost of the multicast routing tree. It is also closer to the optimal solution with the time complexity $O(n^3)$. The simulation results on the existing networks show that the cost of the PNMPH algorithm is lower than that of the MPH algorithm in the case of more than 90%.

***Keywords**: Multicast routing; MPH algorithm; prior nodes; time complexity;*

1. Introduction

In the past few decades, multicast routing trees (or Steiner trees), which are used for effective point-to-multipoint communication (such as webpage broadcasting, video on demand, video conferencing, and VOIP), have seen an extremely fast development over the internet [1, 2]. In order to meet different QoS requirements (such as time delay, cost, band breadth, and packet loss rate), one effective solution is to find multicast QoS routing trees at a minimal cost.

This is often referred to as the Steiner tree problem. In 1983, K. Bharath-Kumar and others proved that the minimal Steiner tree problem is an NP-complete problem [3]. More recently, many researchers are doing research on optimal deterministic algorithms and heuristic approximation algorithms of Steiner trees and have achieved great breakthroughs [4-8]. On the one hand, deterministic algorithms, such as the genetic algorithm and the ant colony algorithm, can finally find optimal solutions without considering the time required. However, the time complexity grows exponentially with the increase of network nodes, thus requiring an extremely fast computing and processing rate, which is not very suitable for network multicast use. On the other hand, although they cannot necessarily find optimal Steiner trees, heuristic algorithms can find multicast trees close to optimal solutions in shorter and more reasonable amounts of time [9-18], thus making them more valuable and meaningful in network application.[1]

In terms of research on approximation algorithms, the performance of the paths found through the earliest heuristic algorithms, in the worst situations, were found to have ratios of less than 2 when compared to optimal solutions. Later on, researchers proposed the KMB algorithm, the ADH algorithm, and the MPH algorithm, the approximate solutions of which have seen improved performance, with a ratio of 2-2/q. Based on findings of the research of X. Yuan in 2002 [4], G. Xue and others put forward the approximation algorithm for the MCOP problem [7, 8], which guarantees that the approximate rate of paths found remains at $(1+\varepsilon)$ by rounding and zooming. As recent as 2013, Hwa-Chun

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Lin and others put forward a multicast tree approximation algorithm dependent on node weight [11]. In 2014, Guanhong Pei did research on the maximal handling capacity of delay-constrained wireless networks and proposed a new type of approximation algorithm [12].

Referring to the latest literature, this article puts forward a novel heuristic approximation algorithm based on optimal multicast project nodes to help solve the Steiner tree problem. The second part of this article defines Steiner trees. The third part provides an overview of existing multicast routing algorithms. The fourth part describes the improved algorithm proposed in this article and analyzes the quality and time complexity of its solutions. Finally, in the fifth and sixth parts, the data from the simulation experiments is provided, and conclusions are drawn.

2. Definitions of Multicast Minimal Generating Trees (Steiner Trees)

Definition 1. G(V,E) is an undirected network graph, with V as its node set, |V|=n as its node number, E as its edge set, and |E|=m as its edge number or link number. Supposing edge cost is $c(e_j)$, and the set of multicast destination nodes is $D=\{d_j\}$,|D|=q, the algorithm of a multicast Steiner generating tree is: In figure G, looking for the generating trees of all nodes in the set D, marked as $T(V_T,E_T)$, with $D \subseteq V_T \subseteq V$, $E_T \subseteq E$, and minimizing the cost C(T) of the whole tree T, which is calculated as follows:

$$C(\mathrm{T}) = \min \sum_{e_j \in E_T} c(\mathrm{e}_j) \tag{1}$$

Definition 2. In the Steiner tree T_q, the node in the set $\bar{D}$ ($\bar{D} \subseteq V_T$ and $\bar{D} \cap D = \varnothing$) is called a non-multicast node, or a Steiner node. The cost of any edges from multicast destination node d_i($i \leqslant q$) to generating tree T_{i-1} is as follows:

$$c(d_i, T_{i-1}) = \min\{c(d_i, \mathrm{v}_k) \mid d_i \in \overline{D \cap \mathrm{V}_{T_{i-1}}}, \mathrm{v}_k \in \mathrm{V}_{T_{i-1}}\} \tag{2}$$

The corresponding PATH(d_i,T_{i-1}) of (2) is the shortest.

(1) When D=V, the problem of the Steiner generating tree is finding the minimum generating tree of graph G. The MRT algorithm can get the optimal solution with time complexity $O(n^2)$.

(2) When q=2, the problem is to find the shortest path between two points. An algorithm like Dijkstra can provide the optimal solution in polynomial time.

Apart from the above two cases, the Steiner tree ($D \neq V, q \neq 2$) problem has been proved to be an NP-complete problem. Aimed at this problem, this article further discusses and puts forward the prior nodes based minimum cost path heuristic algorithm.

3. Multicast Routing Algorithms

3.1. Heuristic Algorithms

In terms of the Steiner tree problem, there are intelligent algorithms, genetic algorithms, and heuristic algorithms, each kind having its own advantages and disadvantages. Intelligent algorithms and genetic algorithms can find optimal solutions more easily but require more time to do so, while heuristic algorithms offer better time complexity and practicability. Three typical heuristic algorithms are often discussed in the existing literature. They are as follows:

(1) The KMB or DNH algorithms, with a time complexity of $O(qn^2)$

(2) The ADH algorithm, with a time complexity of $O(n^3)$

(3) The MPH algorithm, with a time complexity of $O(qn^2)$

The above three algorithms all proved that the ratio between the cost of solving the generation tree and that of an optimal multicast tree is less than 2-2/q. Because $q \leq n$, the

KMB and MPH algorithms are predominant in time complexity. Furthermore, it has been found in many network simulation experiments that the MPH algorithm offers better average performance in most cases. Therefore, the MPH algorithm is a comparatively more excellent heuristic algorithm with which to address the Steiner generation tree problem in terms of time complexity and performance.

3.2. Description of the MPH Algorithm

In 1980, Takahashi and Matsuyama put forward the MPH algorithm to help solve the Steiner tree problem [15]. The following are its steps:

Step 1: Choose any node d_1 from the set D of multicast destination nodes, let i=1, and initialize the generation of tree $T_i=\{d_1\}, V_{T1}=\{d_1\}$;

Step 2: The cost $c(d_k,T_{i-1})$ from $d_k(k\leq q)$ in $\overline{D\cap T_{i-1}}$ to T_{i-1} can reach the minimum value by comparing formula (3):

$$c(d_i,\mathrm{T}_{i-1})=\min\{c(d_k,\mathrm{T}_{i-1})\mid d_k\in\overline{D\cap T_{i-1}}\} \quad (3)$$

By connecting d_i to T_{i-1} through the shortest $PATH(d_i,T_{i-1})$, the updated tree can get the result of $T_i=PATH(d_k,T_{i-1})\cup T_{i-1}$;

Step 3: Unil i>q, find the Steiner generating tree; otherwise, let i=i+1. Then repeat Step 2.

4. Prior Nodes Minimum Cost Path Heuristic Algorithm (PNMPH)

In the topology structure of network G, the minimal cost (or distance) path matrix between any two nodes can be obtained via the Floyd algorithm. It was found through analysis that nodes of some projects pass more paths in those of multicast project, while the nodes of others pass fewer paths. The differences between these projects are defined as follows.

Definition 3. In G(V,E), given the priority of node d_i is a_i, with its initial value 0, every time the shortest path between any two nodes in G passes d_i, $a_i=a_i+1$. Thus, the corresponding priority set $\{a_i\}$ of the multicast node set D is obtained.

According to the MPH algorithm and the ideas of network path sharing, in the same conditions, the path through node d_i with larger values a_i is a priority, the subsequently destination nodes added to the multicast tree can share the path, reducing the cost of the whole Steiner generating tree. On these grounds, the improved PNMPH algorithm is proposed in this paper.

4.1. Description of PNMPH Algorithm

The steps of the PNMPH algorithm are as follows:

Step 1: Choose any node $d_1=v_1$ in D, set k=1, and initialize $T_i=\{d_i\}$ and $V_{T1}=\{d_i\}$;

Step 2: Run the shortest-path algorithm. If one of the shortest paths goes through the multicast destination node d_i, then $a_i=a_i+1$(a_i is the priority value of d_i), from which the priority $\{a_i\}$ of each multicast destination node is obtained and sorted from large to small.

Step 3: Calculate the cost $c(d_k,T_{i-1})$ from $d_k(k\leq q)$ in $\overline{D\cap T_{i-1}}$ to T_{i-1}, according to sorted$\{a_i\}$. Connect di to T_{i-1} through the shortest $PATH(d_k,T_{i-1})$, so the updated tree can be obtained from $T_i=PATH(d_k,T_{i-1})\cup T_{i-1}$.

Step 4: Until i>q, find the Steiner generating tree; otherwise, let i=i+1, and repeat Step 3.

4.2. An Example of the PNMPH Algorithm

An example the PNMPH algorithm is shown in Figure 1. Figure 1(a) shows a simple topology map G of an undirected network, in which nodes in red are node D of multicast destination nodes and others are non-multicast destination nodes. Numbers marked between two nodes are the link costs or distances. Destination nodes are nodes originally added to the generating tree, T_1={A}. When determining the Steiner generating tree with the MPH algorithm, the order of the nodes added to the tree is A-D-E-F, resulting in the total cost of the generating tree being 11. According to the PNMPH algorithm proposed in this paper, the generating graph is found. In the second step of the algorithm, the multicast destination nodes are sorted as E-D-F. The node first added to multicast generating tree is node E, as shown in Figure 1(b). Destination nodes D and F were added to the tree, and the final multicast Steiner tree obtained is shown in Figure 1(c). Its total cost is 9, which is better than the cost of 11 generated by the MPH algorithm.

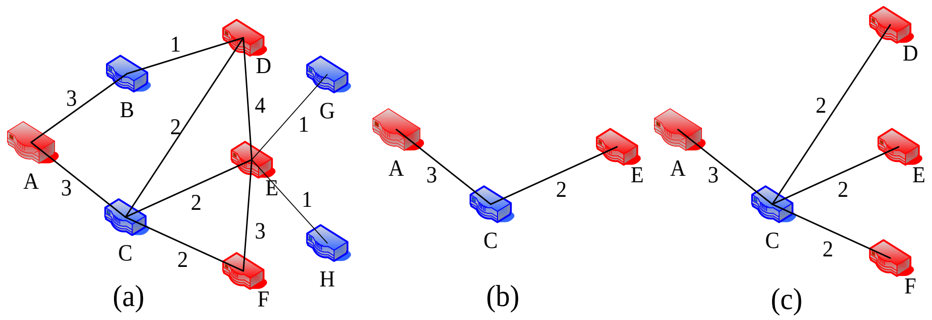


Figure 1. An Example of the PNMPH Algorithm

4.3. Analysis of PNMPH Algorithm

Theorem 1. Supposing that the general cost of multicast tree obtained through the PNMPH algorithm is C_{PNMPH}, that the general cost of optimal multicast tree is C_{OPT}, and that the node number of multicast project is q, then $C_{PNMPH} / C_{OPT} \leq 2(1-1/q)$.

Proof. In the related literature, H. Takahashi and others proved that there is corresponding relationship between each i and node's pair (t_{j-1},t_j), in which i,j=2,3,…q, and $v_1,v_2,\ldots v_k$ is from 1 to k [7]. Let i and number pair $[t_{q(i)-1}, t_{q(i)}]$ have a one to one relationship.

$$\min\{t_{q(i)-1}, t_{q(i)}\} < i \leq \max\{t_{q(i)-1}, t_{q(i)}\} \tag{4}$$

$$c(v_i, V_{i-1}) \leq c(v_{t_{q(i)-1}}, v_{t_{q(i)}}) \tag{5}$$

$$c(v_{t_q}, v_{t_1}) \geq (2/q)\cdot C_{OPT} \tag{6}$$

By inequality (4), (5), and (6), then we have the following:

$$\begin{aligned} C_{PNMPH} &= \sum_{i=2}^{q} c(v_i, V_{i-1}) \leq \sum_{i=2}^{q} c(v_{t_{q(i)-1}}, v_{t_{q(i)}}) \\ &= \sum_{j=2}^{q} c(v_{t_{j-1}}, v_{t_j}) \leq 2(1-1/q)\cdot C_{OPT} \end{aligned} \tag{7}$$

Thus, Theorem 1 is proven.

Theorem 2. The time complexity of the PHMPH algorithm is $O(n^3)$

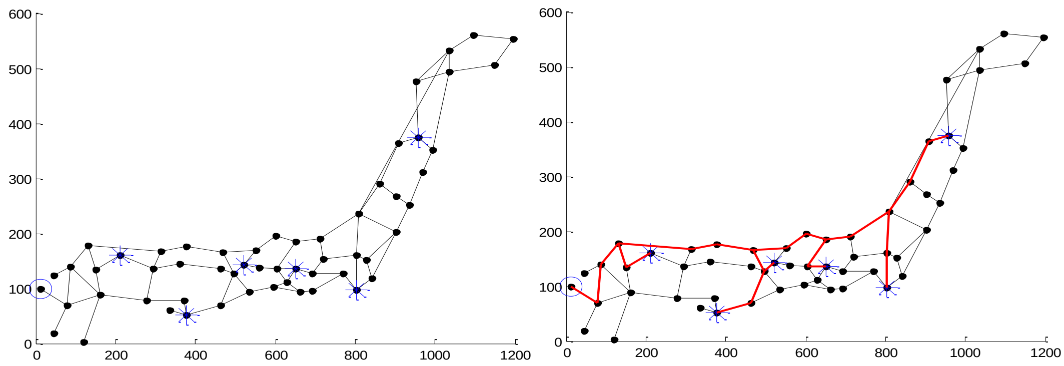
Proof. The time complexity in step 1 of the PNMPH algorithm is constant. Step 2 searches for the shortest path between any destination nodes, and calculates the priority $\{a_i\}$ of each destination node.

With the Floyd algorithm, the time complexity is $O(n^3)$. Steps 3 and 4 respectively find the shortest paths to multicast tree T for q nodes. The time complexity of the path of a node to multicast tree is O(n), and the total time complexity is O(q*n).

According to the above four steps, the time complexity of PNMPH is $O(n^3+ qn)=O(n^3)$, and Theorem 2 is proven.

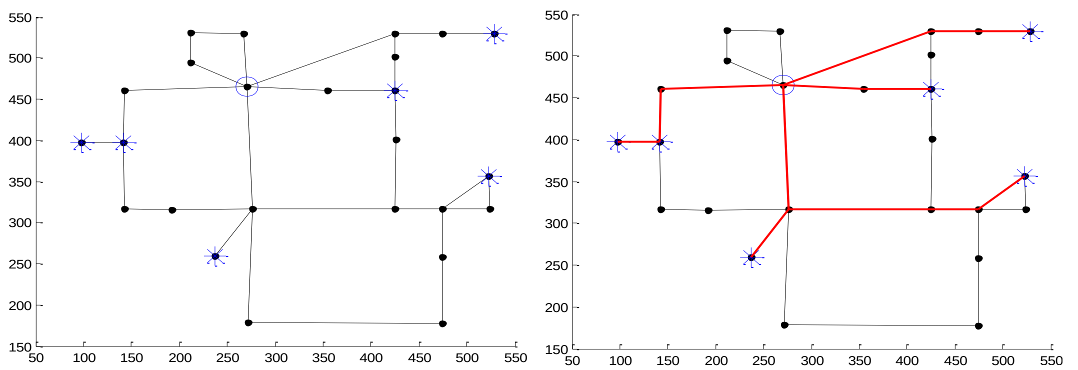
5. Simulation Experiment

In order to verify the performance and efficiency of the PHMPH algorithm, this study employed the NTT network and CERNET networks mentioned in literature [16] for the execution of simulation calculation and to analyze the parameters accordingly. The experiment was done on a computer equipped with 2GB of RAM and an Intel Core Duo CPU that runs at 1.66GHz. Figure 2(a) and Figure 3(a) show the network structure diagrams of the simulation experiments. There are 57 nodes and 81 edges in the NTT network, as well as 25 nodes and 30 edges in the CERNET network. The cost with each edge is delay. The network's parameters can be found at the following URL: http://code.google.com/p/efptas/downloads/list. In Figure 2(b) and Figure 3(b), the blue star nodes are sets of multicast destination nodes, and the red lines are the Steiner tree. As shown in Figure 2(a) and Figure 3(a), six nodes in blue star are chosen for the multicast destination nodes randomly, and then the final multicast routing tree in red lines could be found by the PNMPH algorithm as shown in Figure 2(b) and Figure 3(b).



(a) (b)

Figure 2. The NTT Network Topology and Its Multicast Routing Tree



(a) (b)

Figure 3. The CERNET Network Topology and Its Multicast Routing Tree

Table 1 and Table 2 show the comparisons of Steiner trees generated via the PMH algorithm and the PNMPH algorithm respectively in the simulation experiment. With the two algorithms, q destination nodes in the two networks were randomly generated. The Steiner generating trees were obtained, and the corresponding total costs were recorded. The simulations experiments ran ten times, and the average values were calculated. The corresponding results are shown in the tables below.

Analyzing the experimental results shows that, when the value of q is smaller or close to n, the costs obtained by the two algorithms are close. The results also show that, in most cases, the total cost from the PNMPH algorithm was better than that from the MPH algorithm.

Table 1. A Comparison of Two Algorithms on the NTT Network

NO.	q=6	q=16	q=26	q=36	q=46
MPH	86.0907	150.3528	180.9843	232.7648	265.7491
PNMPH	86.0907	151.7528	187.0367	239.2719	266.8369

Table 2. A Comparison of Two Algorithms on the CERNET Network

NO.	q=3	q=8	q=13	q=18	q=23
MPH	34.3433	82.3672	95.7649	109.6478	126.9567
PNMPH	34.3433	84.1334	102.5765	111.9733	127.6191

6. Conclusion

It was shown in the simulation experiment that, by comparing the novel PNMPH algorithm suggested in this article to the existing MPH algorithm, both of which have a similar time complexity when q is close to n, in most cases, the Steiner generation tree obtained via the PNMPH algorithm had a lower cost in general, and it is closer to optimal an solution. In conclusion, the algorithm put forward in this article is practically meaningful in heuristic algorithm research on the Steiner tree problem.

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References

[1] Zuqing Zhu, Suoheng Li, Xiaoliang Chen, Design QoS-Aware Muli-Path Provisioning Strategies for Efficient CLOUD-Assisted SVC Video Streaming to Heterogeneous Clients, IEEE Transactions on Multimedia, vol.15, no.4, **(2013)**, pp.758-768.
[2] Szymanski, T.H., Max-Flow Min-Cost Routing in a Future-Internet with Improved QoS Guarantees, IEEE Transactions on Communications, vol.61, no.4, **(2013)**, pp.1485-1497.
[3] Ying Xiao, Thulasiraman K., Xi Fang, Dejun Yang, Guoliang Xue, Computing a Most Probable Delay Constrained Path: NP-Hardness and Approximation Schemes, IEEE Transactions on Computers, vol.61, no.5, **(2012)**, pp.738-744.
[4] Yuan X, Heuristic algorithms for multiconstrained quality-of-service routing, IEEE/ACM Transactions on Networking, vol.10, no.2, **(2002)**, pp.244-256.
[5] Chen S, Song M, Sahni S, Two techniques for fast computation of constrained shortest paths, IEEE/ACM Transactions on Networking, vol.16, no.1, **(2008)**, pp.105-115.
[6] Huang J, Liu Y, MOEAQ: A QoS-aware multicast routing algorithm for MANET, Expert Systems with Applications, vol.37, no.2, **(2010)**, pp.1391–1399.
[7] G. Xue, A. Sen, W. Zhang, J. Tang and K. Thulasiraman, Finding a path subject to many additive QoS constraints, IEEE/ACM Transactions on Networking, vol.15, no.1, **(2007)**, pp.201-211.
[8] G. Xue, W. Zhang, J. Tang and K. Thulasiraman, Polynomial time approximation algorithms for multi-constrained QoS routing, IEEE/ACM Transactions on Networking, vol.16, no.3, **(2008)**, pp.656-669.
[9] Waxman BM, Routing of multipoint connections, IEEE Journal on Selected Areas in Communications, vol.6, no.9, **(1988)**, pp.1617-1622.
[10] Gang Feng, Korkmaz, T., A Fast Hybrid ε-Approximation Algorithm for Computing Constrained Shortest Paths, IEEE Communications Letters, vol.17, no.7, **(2013)**, pp.1471-1474.

[11] Hwa-Chun Lin, Hsiu-Ming Yang, An Approximation Algorithm for Constructing Degree-Dependent Node-Weighted Multicast Trees, IEEE Transactions on Parallel and Distributed Systems, vol.25, no.8, **(2014)**, pp.1976-1985.
[12] Guanhong Pei, Parthasarathy, S., Srinivasan, A., Vullikanti, A.K.S., Approximation Algorithms for Throughput Maximization in Wireless Networks With Delay Constraints, IEEE/ACM Transactions on Networking, vol.21, no.6, **(2013)**, pp.1988-2000.
[13] Ting Lu, Jie Zhu, Genetic Algorithm for Energy-Efficient QoS Multicast Routing, IEEE Communications Letters, vol.17, no.1, **(2013)**, pp.31-34.
[14] Yi-Sheng Su, Szu-Lin Su, Jung-Shian Li, Joint Topology-Transparent Scheduling and QoS Routing in Ad Hoc Networks, IEEE Transactions on Vehicular Technology, vol.63, no.1, **(2014)**, pp.372-389.
[15] TAKAHASHI H, MATSUYAMA A., An approximate solution for the Steiner problem in graphs, Math Japonica, vol.24, **(1980)**, pp.573-577.
[16] Jun Huang, Xiaohong Huang, Yan Ma, Routing with multiple quality of services constraints: An approximation perspective, Journal of Network and Computer Applications, vol.35, no.2, **(2012)**, pp.465-475.
[17] Xi Fang, Dejun Yang, Guoliang Xue, MAP: Multiconstrained Anypath Routing in Wireless Mesh Networks, IEEE Transactions on Mobile Computing, vol.12, no.10, **(2012)**, pp.1893-1906.
[18] Dai Huijun, Qu Hua, Zhao Jihong, QoS routing algorithm with multi-dimensions for overlay networks, China Communications, vol.10, no.10, **(2013)**, pp.167-176.

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