

## A New Modeling and Analysis Approach Based on State-Space Averaging for Converters

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### Abstract

*4-quadrant converters (4QCs) have been widely applied in power active VAR and harmonic compensators, FACTS, electric drive systems, and a variety of other applications. The state-space averaging model is usually quite facile and suitable for converter topologies with only two switch states, like half-bridge 4QC units. When directly used in topologies with multiple switch states, however, the modeling method becomes overly complicated. Due to this, plus the fact that multi-phase PWM 4QCs and even other varieties of converters are mostly multi-unit-symmetrical, a novel method is necessary. By researching the inner link between multi-phase topologies and half-bridge units, including inter-phase symmetry configurations, and control relations, this paper proposes a general unified modeling approach for application to multi-phase 4QCs. A series of conclusions are derived, alongside theoretical analysis of the model. The validity of the model and its theory is confirmed using single-phase and three-phase 4QCs and their numerical simulating waveforms as special cases. The proposed method may also extend to the modeling, analysis, and general research of other varieties of converter topologies.*

**Keywords:** *4-quadrant converters, state-space averaging, multi-unit-symmetrical, general unified modeling, theoretical analysis*

### 1. Introduction

Mathematical models of power converters are usually developed specifically for application to their system design, simulation, and analysis of stability. A simple and effective research method, state-space averaging (SSA) has been used by researchers and developers to successfully model many different types of power converters. SSA was first proposed by Middlebrook and CuK to study DC/DC converters [1].

At present, 4-quadrant converters (4QCs) are commonly applied in power active VAR and harmonic compensators of power systems, active power filters (APFs), FACTS, electric drive systems, and more. Effective modeling and analysis of 4QCs has attracted the widespread interest of experts and scholars. The state-space averaging model is convenient and appropriate for converter topologies with only two switch states, such as half-bridge 4QC units [2]. A linear model including state space equations can be used to describe each switch state of a circuit; but when directly used in topologies with multiple switch states, the model becomes overly complicated and unsuitable. Because of this, plus the fact that multi-phase PWM 4QCs and even other varieties of converters are mostly characterized as multi-unit/symmetrical, a novel modeling method becomes necessary. After considering the SSA model of a half-bridge 4QC unit with ready-made model and analysis results, which is quite simple, this study researches the inner link between multi-phase topologies and half-bridge units, inter-phase symmetry configuration, and control

relations in order to propose a general unified modeling approach for multi-phase 4QCs. A series of conclusions regarding the general unified analysis of our multi-unit/symmetrical 4QC model is derived, along with a thorough theoretical analysis based on the model. This paper likewise provides a series of valuable formulas and conclusions based on quantitative analysis of results, using single-phase and three-phase 4QCs as specific examples. Considering the majority of converter topologies in the field of power electronics are characterized as multi-unit/symmetrical, this study provides a modeling and analysis approach with broad potential application to the many kinds of converter topologies combined with multi-unit/symmetrical association.

## 2. SSA Model and Analysis Results of Half-Bridge 4QC Unit

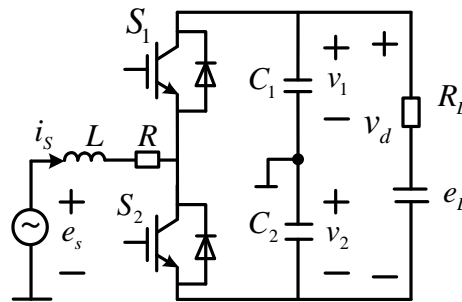


Figure 1. Topology of a 4QC Half-Bridge Unit

The mathematical model using SSA for the topology of a 4QC half-bridge unit is established according to Figure 1 [3]:

$$\begin{bmatrix} L\dot{i}_s \\ C\dot{v}_1 \\ C\dot{v}_2 \end{bmatrix} = \begin{bmatrix} -R & -d & d' \\ d & -1/R_L & -1/R_L \\ -d' & -1/R_L & -1/R_L \end{bmatrix} \begin{bmatrix} i_s \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1/R_L \\ 0 & 1/R_L \end{bmatrix} \begin{bmatrix} e_s \\ e_L \end{bmatrix} \quad (1a)$$

$$v_d = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_s \\ v_1 \\ v_2 \end{bmatrix} \quad (1b)$$

where  $R$  represents the sum of the equivalent resistances of the AC-side input filter inductor and power switch in the formula, parameter  $d$  is the duty ratio of the power switch  $S_1$ , and  $d'$ , the duty ratio of  $S_2$ , satisfies:

$$d' = 1 - d \quad (2)$$

Using the same model as Formula (1), the results of the steady-state analysis based on the 4QC half-bridge unit is as follows.

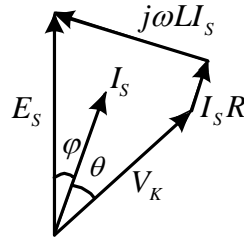
The duty ratio  $d$  can be expressed as:

$$d = \frac{1}{2} + \frac{m}{2} \sin(\omega t - \varphi - \theta) \quad (0 \leq m \leq 1) \quad (3)$$

where  $m$  represents the modulation ratio of the PWM wave-form in the formula:

$$m = 2V_K / V_d \quad (4)$$

where  $V_d$  is the average value of the output voltage  $v_d$ ,  $V_K$  is the peak value of the fundamental voltage of the PWM of the  $K$  point as shown in Figure 1. The relationship between it and  $\varphi$ ,  $\theta$  is as shown in the phasor diagram in Figure 2.



**Figure 2. Phasor Diagram of 4QC AC Circuit**

The DC-side output voltage contains the DC component and the second harmonic component, expressed as:

$$v_d = V_d + v_N \tag{5}$$

where the DC component can be expressed as:

$$V_d = \frac{mI_s R_L}{4} \cos \theta + e_L \tag{6}$$

Through further analysis it can be re-written:

$$V_d = \frac{1}{2} (e_L + \sqrt{e_L^2 + 2I_s R_L E_s \cos \varphi - 2I_s^2 R_L R}) \tag{7}$$

where  $e_L$  is the DC-side counter electromotive force, the second harmonic component can be expressed as:

$$v_N = \frac{mI_s R_L}{4\sqrt{1 + (\omega R_L C)^2}} \cos[2(\omega t - \varphi) - \theta + \beta] \tag{8}$$

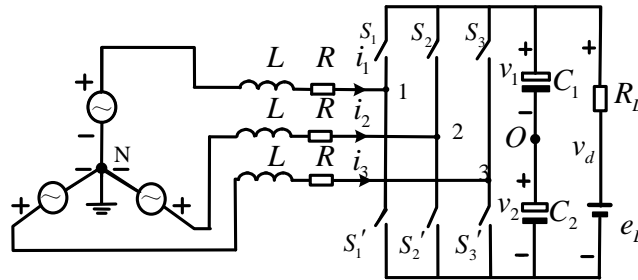
where  $\beta = \pi - \arctg(\omega C R_L)$ .

### 3. General Unified Model based on Multi-Unit Xharacterization of 4QC

All voltage type topologies of bridge 4QCs can be considered that their topologies are combined with many half-bridge 4QC units, as shown in Figure 1. In order to obtain a general mathematical model of a multi-phase 4QC without loss of generality, one assumes that the number of phase of the 4QC is M (M is an integer, M>1), and that R represents the sum of the equivalent resistances of the AC-side input filter inductor and the power switch. M phases of AC-side sine electric potential are symmetrical. The main circuit topology of a 3-phase 4QC is shown in Figure 3, in which M=3. General unified modeling, based on multi-unit/symmetry, is applied to an M-phase 4QC referring to the structure of the topology of a three-phase 4QC. On the condition that the midline exists between the AC neutral N and the DC neutral O, we form the unit model as follows. Note that each phase works independently according to the SSA model of a half-bridge unit as shown in Formula (1):

$$\begin{bmatrix} L\dot{i}_k \\ C\dot{v}_1 \\ C\dot{v}_2 \end{bmatrix} = \begin{bmatrix} -R & -d_k & d'_k \\ d_k & -1/R_L & -1/R_L \\ -d'_k & -1/R_L & -1/R_L \end{bmatrix} \begin{bmatrix} i_k \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1/R_L \\ 0 & 1/R_L \end{bmatrix} \begin{bmatrix} e_k \\ e_L \end{bmatrix} \tag{9}$$

where subscripts of variables k ranging from 1 to M correspond to M unit models, in turn. For example, k subscripts correspond to three phases A, B, and C when M=3, shown as midpoints of each half-bridge between high-sides and low-sides in Figure 3.



**Figure 3. The Topology of 3-Phase Bridge 4QC**

If there is no neutral line between O and N, the first line of Formula (9) can be written as:

$$L\dot{i}_k = e_k - i_k R - (v_{ko} + v_{oN}) = e_k - i_k R - (d_k v_1 - d'_k v_2 + v_{oN}) \quad (k=1,2,\dots,M) \quad (10)$$

where  $v_{ko}$  indicates the voltage between the Kth phase midpoint of the half-bridge unit and DC-side midpoint, while  $v_{oN}$  indicates the voltage between the AC-side midpoint O and the DC-side midpoint N. Using formulas  $d'_k = 1 - d_k$  and  $v_1 = v_2 = v_d / 2$  we obtain:

$$L\dot{i}_k = e_k - i_k R - [(d_k - \frac{1}{2})v_d + v_{oN}] \quad (k=1,2,\dots,M) \quad (11)$$

As for the M-phase 4QC, the common voltage  $v_d$  of M phases must satisfy M formulas, as shown in Formula (11). Considering that there is no neutral line in the AC side, the sum of currents of M phases is zero; also, the M phases of AC-side sine electric potential are symmetrical. M equations are expressed as:

$$v_{oN} = \left( \frac{1}{2} - \frac{1}{M} \sum_{j=1}^M d_j \right) v_d \quad (12)$$

By plugging Formula (12) into Formula (11), we obtain:

$$L\dot{i}_k = e_k - i_k R - (d_k - d_m)v_d \quad (k=1,2,\dots,M) \quad (13)$$

where

$$d_m = \frac{1}{M} \sum_{j=1}^M d_j \quad (14)$$

$d_m$  is the average value of duty ratios of M phases.

Considering that the DC currents of M phases of 4QCs must satisfy the superposition principle, the second line and the third line of Formula (9) are written as:

$$\frac{C}{2}\dot{v}_d = \sum_{k=1}^M (d_k - \frac{1}{2})i_k - \frac{v_d - e_L}{R_L} = \sum_{k=1}^M d_k i_k - \frac{v_d - e_L}{R_L} \quad (15)$$

In fact, this formula is a form of the injection current equation of the DC-side bus.

At this point, Formulas (13) to (15) can be synthesized as a unified SSA model of an M-phase 4QC:

$$Z\dot{X} = AX + BE \quad (16a)$$

where

$$\begin{aligned}
 X &= [i_1 \ i_2 \ \dots \ i_M \ v_d]^T, \quad E = [e_1 \ e_2 \ \dots \ e_M \ e_L]^T \\
 Z &= \text{diag}[L \ L \ \dots \ L \ C/2], \quad B = \text{diag}[1 \ 1 \ \dots \ 1 \ 1/R_L] \\
 A &= \begin{bmatrix} -R & 0 & \dots & 0 & d_m - d_1 \\ 0 & -R & & & d_m - d_2 \\ \vdots & & \ddots & & \vdots \\ 0 & & & -R & d_m - d_M \\ d_1 & d_2 & \dots & d_M & -1/R_L \end{bmatrix}
 \end{aligned} \tag{16b}$$

According to the unified model based on multi-unit association, considering two most widely useful topologies of 4QC are where M is equal to two and three, respectively, we further investigate and analyze the mathematical model of SSA as follows.

(1) Three-phase model

A three-phase 4QC is made up of three half-bridge units as shown in Figure 3. If  $M = 3$  in Formula (16), the SSA model of a three-phase 4QC can be written:

$$\dot{X} = AX + BE \tag{17a}$$

where

$$\begin{aligned}
 X &= [i_1 \ i_2 \ i_3 \ v_d]^T, \quad E = [e_1 \ e_2 \ e_3 \ e_L]^T, \\
 Z &= \text{diag}[L \ L \ L \ C/2], \quad B = \text{diag}[1 \ 1 \ 1 \ 1/R_L], \\
 A &= \begin{bmatrix} -R & 0 & 0 & d_m - d_1 \\ 0 & -R & 0 & d_m - d_2 \\ 0 & 0 & -R & d_m - d_3 \\ d_1 & d_2 & d_3 & -1/R_L \end{bmatrix},
 \end{aligned} \tag{17b}$$

and where  $d_m$  is the average value of duty ratios of the three phases. Using three-phase symmetrical control and Formula(14), we obtain:

$$d_m = \frac{1}{3} \sum_{k=1}^3 d_k = \frac{1}{2} \tag{17c}$$

The three-phase 4QC model as shown in Formula (17) is in accordance with results of previous research [4]-[6].

(2) Single-phase model

Usually, so-called single-phase 4QCs are actually comprised of two half-bridge units. For intuitive analysis, we establish the equivalent topology of a single-phase 4QC as shown in Figure 4. The two half-bridge units have the same structure, while the AC-side input voltage satisfies:

$$e_2 = -e_1 \tag{18}$$

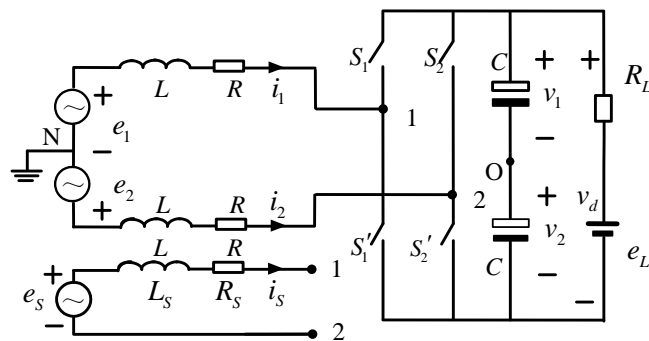


Figure 4. Topology of Single Phase Bridge 4QC

Due to the symmetrical control of two half-bridge units and the symmetry of the AC-side voltage and current, when  $M=2$  in Formula (16), we obtain the SSA model of a single-phase 4QC:

$$Z\dot{X} = AX + BE \quad (19a)$$

where

$$X = [i_1 \quad i_2 \quad v_d]^T, \quad E = [e_1 \quad e_2 \quad e_L]^T$$

$$Z = \text{diag}[L \quad L \quad C/2], \quad B = \text{diag}[1 \quad 1 \quad 1/R_L]$$

$$A = \begin{bmatrix} -R & 0 & d_m - d_1 \\ 0 & -R & d_m - d_2 \\ d_1 & d_2 & -1/R_L \end{bmatrix} \quad (19b)$$

$$d_m = \frac{1}{2}(d_1 + d_2) = \frac{1}{2} \quad (19c)$$

The symmetry voltages of the AC-side input circuit are combined one voltage with filter inductance, so the practical AC-side input circuit of the single-phase 4QC is derived as shown in Figure 4. If the SSA model above satisfies Formula (18), provided that anti-symmetric PWM control is inflicted on the two half-bridge units, we obtain:

$$i_2 = -i_1 \quad (20)$$

The order of single-phase model as shown in Formula (19) can then be reduced. Using the former two lines of Formula (19b), we obtain:

$$L\dot{i}_k = e_k - i_k R + (d_m - d_k)v_d \quad (k=1, 2) \quad (21)$$

Using Formulas (18), (20), and (21) we obtain:

$$e_1 = e_s / 2, \quad e_2 = -e_s / 2, \quad i_1 = i_s, \quad i_2 = -i_s, \quad R_s = 2R, \quad L_s = 2L$$

and complete the simplified form of the SSA model of a single-phase 4QC:

$$\begin{bmatrix} L_s \dot{i}_s \\ \frac{C}{2} \dot{v}_d \end{bmatrix} = \begin{bmatrix} -R_s & -(d_1 - d_2) \\ d_1 - d_2 & -\frac{1}{R_L} \end{bmatrix} \begin{bmatrix} i_s \\ v_d \end{bmatrix} + \begin{bmatrix} e_s \\ \frac{e_L}{R_L} \end{bmatrix} \quad (22)$$

The SSA model of the single-phase 4QC as shown in Formula (22) is a reduced-order model of Formula (19).

#### 4. Static Analysis of 4QC based on Multiunit-Association General Unified Model

Static expressions of the M-phase symmetrical power voltage, input current, and the control law of duty ratio by proper closed-loop control are written as:

$$\begin{cases} e_k = E \sin \alpha_k \\ i_k = I \sin(\alpha_k - \varphi) \\ d_k = \frac{1}{2} + \frac{m}{2} \sin(\alpha_k - \varphi - \theta) \end{cases} \quad (23)$$

where  $\alpha_k = \omega t - (k-1)2\pi/M$  ( $k=1,2, \dots, M$ ), and  $\varphi$  is the AC-side power factor angle.

The former M lines of the M-phase 4QC based on the multi-unit general unified model, as shown in Formula (16), are:

$$L\dot{i}_k = e_k - R i_k - (d_k - d_m)v_d \quad (k=1,2, \dots, M) \quad (24)$$

Plugging Formulas (23) and (14) into the formula above form two equations by deducing and comparing. These two equations then multiply  $\cos\varphi$  and  $\sin\varphi$ , respectively, to obtain and the difference between them. Using Formula (4) we obtain:

$$V_k \cos\theta = E \cos\varphi - IR \quad (25a)$$

Adding the squares of the two equations, we obtain:

$$V_k^2 = E^2 + (IR)^2 + (\omega LI)^2 - 2EI(R \cos\varphi + \omega L \sin\varphi) \quad (25b)$$

Formula (25) indicates the quantitative relationship between both  $\cos\theta$  and  $V_k$ , and the parameters of the AC-side input circuit.

Using the last line of Formula (16), we obtain:

$$\frac{C}{2} \dot{v}_d = \sum_{k=1}^M d_k i_k - \frac{v_d - e_L}{R_L} \quad (26)$$

With the sum term of Formula (26), we obtain:

$$d_k i_k = \frac{1}{2} I \sin(\alpha_k - \varphi) + \frac{m}{4} I \cos\theta - \frac{m}{4} I \cos[2(\alpha_k - \varphi) - \theta] \quad (27)$$

$$\begin{aligned} \sum_{k=1}^M d_k i_k &= \frac{M}{4} m I \cos\theta - \frac{1}{4} m I \sum_{k=1}^M \cos[2(\alpha_k - \varphi) - \theta] \\ &= \frac{1}{4} M m I \cos\theta - \frac{1}{4} m I \sum_{k=1}^M \cos\left[2(\omega t - \varphi) - (k-1)\frac{4\pi}{M} - \theta\right] \end{aligned} \quad (28)$$

Plugging the formula above into Formula (26), we obtain:

$$\begin{aligned} \frac{C}{2} \dot{v}_d + \frac{v_d}{R_L} &= \frac{M}{4} m I \cos\theta + \frac{e_L}{R_L} \\ &- \frac{1}{4} m I \sum_{k=1}^M \cos\left[2(\omega t - \varphi) - (k-1)\frac{4\pi}{M} - \theta\right] \end{aligned} \quad (29)$$

In order to conveniently analyze this, we consider DC-side output voltage a sum of the DC component and pulsating component, *i.e.*  $v_d = V_d + \hat{v}_d$ . Substituting this into the formula above and comparing the two sides of the equation, we obtain:

$$V_d = \frac{M}{4} m I R_L \cos\theta + e_L \quad (30)$$

$$\frac{C}{2} \dot{\hat{v}}_d + \frac{\hat{v}_d}{R_L} = -\frac{1}{4} m I \sum_{k=1}^M \cos\left[2(\omega t - \varphi) - (k-1)\frac{4\pi}{M} - \theta\right] \quad (31)$$

Typically, the counter EMF  $e_L$  is constant. Formula (30) clearly indicates the magnitude of the DC component of the output voltage. By substituting the expressions, including the  $m$  of Formula (4) in calculation of Formula (30), and Formulas (25a) and (30) as quadratic equations, we obtain:

$$V_d = \frac{1}{2} (e_L + \sqrt{e_L^2 + 2MIR_L E \cos\varphi - 2MI^2 R_L R}) \quad (32)$$

The following is an analysis of the differential equation on the AC pulsating component in Formula (31). We deduce that a stable solution must contain the second harmonic. The stable solution of  $\hat{v}_d$  is considered as follows:

$$\hat{v}_d = \sum_{k=1}^M V_{d2} \cos\left[2(\omega t - \varphi) - (k-1)\frac{4\pi}{M} - \theta + \beta_k\right] \quad (33)$$

Substituting the formula above into the Formula (31), and using the transform formulas of trigonometric functions, we obtain:

$$\begin{aligned}
 V_{d2} \sqrt{(1/R_L)^2 + (\omega C)^2} \sum_{k=1}^M \cos[2(\omega t - \varphi) - (k-1) \frac{4\pi}{M} - \theta + \beta_k + \alpha] \\
 = \frac{1}{4} mI \sum_{k=1}^M \cos[2(\omega t - \varphi) - (k-1) \frac{4\pi}{M} - \theta + \pi]
 \end{aligned} \tag{34}$$

where  $\alpha = \arctan(\omega CR_L)$ . By comparing the two sides of the formula above, we obtain:

$$\beta_k = \beta = \pi - \alpha = \pi - \arctan(\omega CR_L) \tag{35}$$

$$V_{d2} = mIR_L / 4 \sqrt{1 + (\omega CR_L)^2} \tag{36}$$

The unified stable expression of the pulsating component of the DC output voltage of the M-phase 4QC is derived by substituting Formulas (35) and (36) into Formula (33).

According to an analysis of conclusions based on the multi-unit general unified model, we create a specific analytic expression of steady-state response for two cases: a single-phase 4QC, and a three-phase 4QC, where M is equal to 2 and 3, respectively.

(1) Three-phase analysis

Using Formula (32), we obtain a stable expression of the DC component of the DC-side output voltage of the three-phase 4QC when M=3.

$$V_d = \frac{1}{2} (e_L + \sqrt{e_L^2 + 6IR_L E \cos \varphi - 6I^2 R_L R}) \tag{37}$$

Looking at Formulas (35), (36), and (33), we see that the pulsating component  $\hat{v}_d$  of the DC-side output voltage of the three-phase 4QC equals zero. This proves that the output ripple components of each half-bridge unit counteract each other, due to three-phase symmetry. In ideal conditions, the ripple of a low frequency is not contained in the stable voltage  $v_d$  with the exception of the ripple of the switch frequency.

(2) Single-phase analysis

In Formula (32) we obtain a stable expression of the DC component of the DC-side output voltage of the single-phase 4QC when M=2.

$$V_d = \frac{1}{2} (e_L + \sqrt{e_L^2 + 4IR_L E \cos \varphi - 4I^2 R_L R}) \tag{38}$$

The formula above takes the following form, where  $E_s = 2E$  and  $R_s = 2R$ , considering the equivalent structure of a single-phase 4QC as shown in Figure 4:

$$V_d = \frac{1}{2} (e_L + \sqrt{e_L^2 + 2IR_L E_s \cos \varphi - 2I^2 R_L R_s}) \tag{39}$$

By analyzing Formulas (35), (36), and (33) we know that the pulsating component of the DC-side output voltage of the single-phase 4QC is expressed as:

$$\hat{v}_d = 2V_{d2} \cos[2(\omega t - \varphi) - \theta + \beta] \tag{40}$$

where the  $\beta$  and  $V_{d2}$  are as shown in Formulas (35) and (36). The expressions of the DC-side output voltage  $V_d$  are the same as in static analysis results found in Formulas (6)-(8) for the half-bridge 4QC unit, but the pulsating magnitude  $V_{d2}$  of the voltage  $\hat{v}_d$  doubles. The influence of the DC-side output voltage and the magnitude of the second harmonic are described quantitatively in Formulas (39), (40), (35), and (36).



## 5. Simulation Results

In order to further verify the above theoretical analysis results of the general model and dynamic behavior of arbitrary multi-phase 4QCs, we create simulations for both single-phase and three-phase 4QCs. Using direct current control [7] -[9], a double-loop control system with DC-side voltage and AC-side current of 4QC is established. Using the parameters of the single-phase 4QC:  $L = 6\text{mH}$ ,  $R = 0.3\Omega$ ,  $C = 1000\mu\text{F}$ ,  $R_L = 26\Omega$ ,  $E_{sm} = 311\text{V}$ ,  $e_L = 300\text{V}$ , on the condition that the given DC-side voltage  $V_d$  equals  $476\text{V}$  and the AC-side power factor angle  $\varphi$  is  $0^\circ$ , two Matlab simulation results for the two single-phase models using either Formula (19) or (22) are identical. The wave shape of the single-phase 4QC with a sudden step load is shown in Figure 5. The parameters of the three-phase 4QC are established:  $L = 6\text{mH}$ ,  $R = 0.3\Omega$ ,  $C = 1000\mu\text{F}$ ,  $R_L = 20\Omega$ ,  $E_m = 311\text{V}$ ,  $e_L = 500\text{V}$ , on the condition that the given DC-side voltage  $V_d$  equals  $700\text{V}$  and the AC-side power factor angle  $\varphi$  is  $0^\circ$ , the transient wave from rectification to active inversion of the three-phase 4QC, using Matlab's three-phase SSA model and results of Formula (17), is shown in Figure 6.

Thus, we prove that both simulation results are wholly consistent with the known SSA model and the theoretical analysis results. The DC-side stable output voltage of single-phase 4QC satisfies Formulas (38)-(40). The second harmonic is contained in the simulations, in addition to the DC component, and the AC-side power factor always equals 1. The DC-side stable output voltage of three-phase 4QCs satisfies Formula (33), in which the pulsating component with low frequency is equal to zero. The phase difference between the AC-side current and the AC-side voltage converts from in-phase to negative-phase automatically through its closed-control system, so that the energy flow reverses accompanied by a sudden increment of  $800\text{V}$  in the counter EMF  $e_L$  of the DC-side load. Notably, this wave does lack certain relevant characteristics such as the ripple of the switching frequency, due to the "average" characteristic of the SSA model. The correctness and feasibility of the SSA model and its theoretical analysis results are verified by the simulation waves.

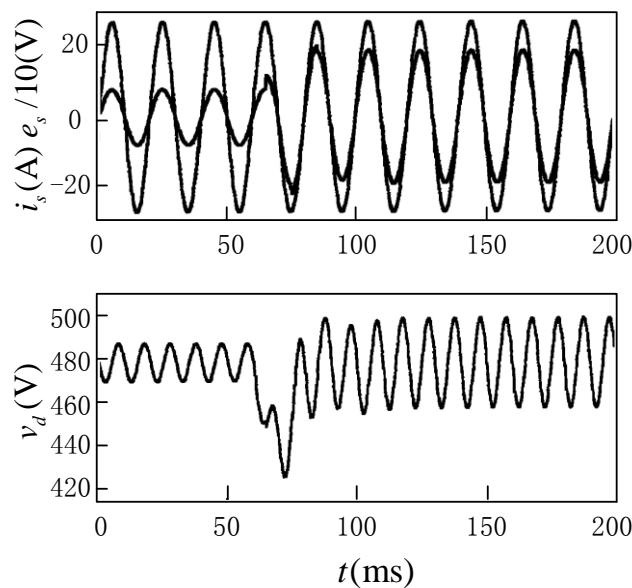
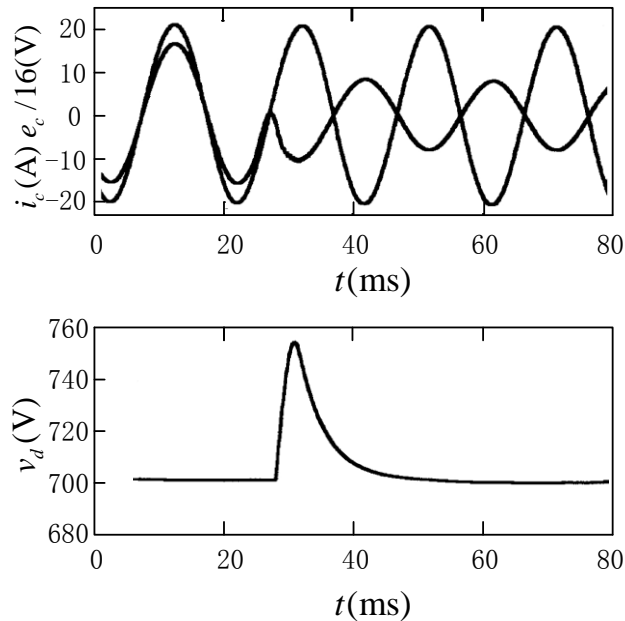


Figure 5. Dynamic Response of Single-Phase 4QC with Step Load



**Figure 6. Dynamic Process of 3-Phase 4QC from Rectification to Active Inversion**

## 6. Conclusions

This study first models a 4QC half-bridge unit composed of the topology of a multi-phase 4QC with the pre-existing SSA model and static analysis results.

As previously discussed, the state-space averaging model is usually facile and well-suited to converter topologies with only two switch states, like half-bridge 4QC units. As directly used in topologies with multiple switch states, however, this modeling method becomes overly complicated. Due to this, plus the fact that multi-phase PWM 4QCs and even other varieties of converters are mostly multi-unit/symmetrical, a novel modeling method is necessary. By researching the inner link between multi-phase topologies and half-bridge units, interphase symmetry configuration, and control relations, this paper proposes a general unified modeling approach for application to multi-phase 4QCs. A series of conclusions regarding the general unified analysis and multi-unit association of 4QCs is derived, along with theoretical analysis of the model. To fully explore the general unified model and form a quantitative analysis of results, we simulate the most common 4QCs (single-phase and three-phase) and form several valuable formulas and conclusions based on these models. This work provides a novel modeling and analysis approach for multi-unit/symmetrical cases, not only for 4QCs but for other types of multi-unit-symmetrical converter topologies.

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