

# A Study on Denoising Method for Image of Mixed Noise

Zhao Xiaofeng and Xie Hongsheng

*Shandong Women's University*  
*zhxf0929@sina.com*

## **Abstract**

*In view of the images polluted by mixed noise in actual image processing, in this paper, the hybrid filter algorithm combined by median filtering and wavelet transform is generalized to two-dimensional fractional wavelet domain, verified the effectiveness and advantages of hybrid filtering algorithm combined by median filtering and two-dimensional fractional wavelet transform. And put forward a method to calculate the optimal fractional order in the case of unknown noise, eventually get better image denoising effect and has certain actual application value.*

**Key words:** *Mixed noise; Median filtering. Two-dimensional fractional wavelet transforms; Image denoising*

## **1. Introduction**

Digital image molding and processing has become the current field of research and use of objects, the scientific research and is widely used in social life each domain, for work and life has brought great convenience and benefits. However, in the process of image collection and transmission, by outside noise pollution. Surrounding conditions, such as precision outside factors can affect the image of the real degree, the existence of various noise in the image, thus acquired a lot of images is derived from the original image and noise signal overlap, the noise will cover part of the image information, bring adverse effect for image processing of the follow-up work. To get high quality original image, image denoising is an important link in the process of image processing. The purpose of image denoising is to get the original image from the noise in the image restoration with removing the noise at the same time, and keep the important information of the original image as much as possible.

Image signal in the process of formation, transmission and terminal treatment etc, often by different kinds of noise pollution. And the complexity of the actual application environment, noise characteristic is very complex, most of the image noise is not a single, and the output image has been noise pollution, noise size is unknown, therefore, need to be mixed for unknown noise filter method to research. Discussed in this paper, the mixed noise, it is to point to by a zero mean Gauss white noise and impulse noise (positive and negative pulses, the salt and pepper noise) of two types of noise. They are widely exist in reality in various typical noise.

In recent years, the time-frequency analysis as a considerable vitality gradually developed signal processing method, and a great deal of attention, has been widely used in many fields and become a hot research topic in the field of signal processing. In time and frequency domain analysis of signal, can reflect the signal spectrum change over time, both the two aspects of time domain and frequency domain information of signals. Typical time-frequency analysis theory is: based on the Fourier transform of short-time Fourier transform, multiresolution analysis of wavelet transform is suitable for the fractional Fourier transform of chirp signal etc. The traditional Fourier transform, the window size is fixed does not change with frequency, and there is no discrete orthogonal basis. Wavelet transform is window Fourier transform is the succession and development

of the localization idea, and overcomes the drawback of traditional Fourier transform, can be in different positions have different on the surface of the time-frequency resolution, is the signal time domain, frequency domain (Fourier domain) of multi-resolution analysis, in the Fourier domain or can also be thought of as a scalable band pass filter. Thus has good time-frequency localization performance, can be caused by the rough to the subtle observation signal, step by step is a effective tool in the field of signal processing.

Now, the existing image denoising methods are mostly based on the characteristics of the original image, the noise probability function *etc.*, to analyze noise and spectrum characteristics, *etc.* In both the noise spectrum and frequency domain signal without overlapping, low-frequency filtering method is very effective. When white noise signal containing noise, the frequency spectrum of the signal and noise can have overlap, the low frequency filter method denoising effect is not ideal, namely it in removing noise at the same time, also filter out the part of the edge information of original image, the image edge is relatively fuzzy, but if can effectively keep the edge details of the high frequency filter leads to expand outside noise consequences, denoising effect is not good. Researchers have proposed a variety of ways to filter out mixed noise in image, this paper USES the method is the method of wavelet image denoising and median filtering and the combination of improved. Median filter to remove impulse noise effect is more ideal, and wavelet transform the Gauss noise filtering effect is good, but if the role separately mixed filter out noise, the effect is not satisfactory. By a large number of analysis and simulation experiments show that combining the median filter and wavelet transform of hybrid filter algorithm is effective and feasible.

## 2. Related Works

According to the image and noise superimposition relationship, noise can be divided into multiplicative noise and additive noise and additive noise has nothing to do with the input image signal, multiplicative noise and image signal, in this article generally considered the additive noise. Common additive white Gaussian noise and salt and pepper noise (or called the impulse noise), among them, the amplitude and power spectral density is uniformly distributed random distribution on the noise is Gaussian white noise; Have essentially the same amplitude, but distribution random noise is salt and pepper noise.

### 2.1. Median filtering algorithm based on impulse noise detection

Median filtering is a field of pixel operation window pixels in order of size in the middle of the value as the output value to the center pixel. This article uses the method is to detect all possible first impulse noise points, then use filter window is not impulse noise pollution in the pixel gray value, as the center pixel gray value estimate. Due to the impulse noise identification process in advance, even if the pulse noise pollution is heavier, the median filtering method based on impulse noise detection, also can get better filtering results.

Given the definition of correlation windows [1, 2]:  $C$  is window for noise detection;  $B$  is the filter window;  $[x_{mn}]$  said the image of the corresponding matrix;  $C[x_{mn}]$  is point  $x_{mn}$  operation center of the window. By the literature [3], the measuring point window contains a pixel point  $x_{mn}$  centered  $(2n+1) \times (2n+1)$  pixels, then the window all the points in the mean and standard deviation are:

$$x'_{mn} = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N x_{ij} \quad (1)$$

$$a_{mn} = \sqrt{\frac{\sum_{i=1}^N \sum_{j=1}^N (x_{ij} - x'_{ij})^2}{(N-1)^2}} \quad (2)$$

Image pixel gray value is always a part with the surrounding pixels continuous gradient, when the noise appeared on the original pixels, which is due to its pixel, on a convex concave under as impulse noise point on the minimum or maximum of gray value of image, and then its neighborhood gray scale of pixels within the gap is bigger. Thus, according to the continuity of image grayscale and impulse noise jump degeneration, the original pixels and impulse noise points are available to distinguish and identify. If again according to probability theory,  $\rho$  probability density for the impulse noise and the noise of gray value of positive and negative probability are  $\rho / 2$ . Therefore, in order to detect the noise more efficiently, can inspect point  $x_{ij}$  specific size of the grey value. If you point  $x_{ij}$  grey value of more than one parameter  $t$ , could be the noise points. That is,  $x_{ij} \in [0, t]$  or  $[255-t, 255]$ . According to the experience of the simulation, it shows that the noise pixel grayscale range between  $[0, 10]$  and  $[245, 255]$ , so the parameter  $t$  value in the range of  $(5, 10]$ .

For impulse noise detection and identification, combined with the literature [10], using a simple and rapid and practical effective impulse noise detection method, which satisfies the conditions (3), the point is  $x_{mn}$  impulse noise point, the opposite is not:

$$\begin{cases} |x_{mn} - x'_{mn}| > k \times a_{mn} \\ 0 \leq x_{mn} \leq t \\ 255 \geq x_{mn} \geq 255 - t \end{cases} \quad (3)$$

among them,  $x'_{mn}$  and  $a_{mn}$  respectively are the pixel values of the mean and standard deviation within  $C$ ,  $k$  is constant threshold, and  $k > 0$ ,  $t$  as gray parameters, and  $t \in [5, 10]$  [4-5].

For  $[x_{mn}]$  impulse noise point distribution, define a binary identity matrix  $B_{mn}$  whose dimension is the same as the original image [6]. If the noise in the identity matrix  $B_{mn} = 0$ , it corresponds to the pixel is impulse noise point in the original image; If the noise identification matrix  $B_{mn} = 1$ , prove this point is not polluted in the original image pixels. Initialize all the elements in the matrix ( $B_{mn}$ ) and is set to 1, impulse noise detection, on the basis of the result to its elements assignment, the income of a binary image ( $B_{mn}$ ), reflects the distribution of impulse noise.

Specific impulse noise filtering method is: first, choose the appropriate  $D$  filtering window, select the dimension of  $4 \times 4$ . In the impulse noise identification matrix ( $B_{mn}$ ), to identify 1 point does not make processing, identification of 0 point processing by the following formula [7]:

$$y_{mn} = \begin{cases} M(C[x_{mn}]), & B_{mn} = 0 \text{ and } s \text{ is odd number} \\ \{M_1(C[x_{mn}]) + M_2(C[x_{mn}])\} / 2, & B_{mn} = 0 \text{ and } s \text{ is even number} \\ x_{mn}, & B_{mn} = 1 \text{ or } s = 0 \end{cases} \quad (4)$$

among them,  $s$  is number of signal points in the filter window is  $D$ , you have an odd number of signal points in  $M(C[x_{mn}])$  to  $D$  pixel values in order of size when the median,  $M_1(C[x_{mn}])$  and  $M_2(C[x_{mn}])$  to  $D$  have an even number of signal points in

pixel value according to the size of the order in the middle of the two values,  $y_{mn}$  is  $x_{mn}$  pixel values of the processed.

For the processed the identity of the matrix  $B'_{mn}$  calculated by the following formula [8]:

$$B'_{mn} = \begin{cases} 1, & y_{mn} \neq x_{mn} \\ B_{mn}, & y_{mn} = x_{mn} \end{cases} \quad (5)$$

After median filtering can be obtained by the algorithm of matrix  $[y_{mn}]$ , also need to determine whether the binary matrix  $B'_{mn}$ , after surviving the noise of the logo is 0 points. If there is the matrix  $[y_{mn}]$ , then repeat the above steps, until the binary pulse noise identifier with no longer exist in matrix  $B'_{mn}$  zero point. At this point, the image  $[y_{mn}]$  is whole the output of median filtering algorithm.

## 2.2. Based on noise estimation of two-dimensional fractional wavelet transform filter algorithm

Fractional wavelet transform multi-resolution analysis of wavelet transform theory to promote the fractional Fourier domain, its associated with the evolution of the wavelet transform and the theory of fractional order, comprehensive characteristics of the two came into being at the same time, is a new kind of time and frequency domain analysis method. Compared with the wavelet transform, the biggest advantages of fractional wavelet transform added a variable  $p$  order of time, can be more flexible to adjust the wavelet coefficients. Good application prospects of fractional wavelet transform and the signal processing method based on fractional wavelet transform the potential applications of self-evident.

Secondary fractional order wavelet transform are defined as shown below:

$$W(a_{mn}, b') = \iiint B_{\rho_1, \rho_2}(x, y, x', y') f(x, y) \times h_{a_{mn}, b'}(x', y') dx dy dx' dy' \quad (6)$$

Fractional domain is expressed as:

$$W(a_{mn}, b') = (a_m a_n)^{0.5} \iint H(a_m \mu, a_n \nu) \times \exp(j2\pi\mu b_{x'}, j2\pi\nu b_{y'}) \times F\{F^{\phi_1, \phi_2}[F(x, y)](x', y')\}(\mu, \nu) d\mu d\nu \quad (7)$$

In it,  $h_{a_{mn}, b'}(x', y')$  is the scale of the mother wavelet and translation function, as shown below:

$$h_{a_{mn}, b'}(x', y') = (a_m a_n)^{-0.5} h\left(\frac{x' - b_{x'}}{a_m}, \frac{y' - b_{y'}}{a_n}\right) \quad (8)$$

$(a_m a_n) = (a_m, a_n)$  is discrete scale,  $b' = (b_{x'}, b_{y'})$  is displacement scale.

Secondary fractional order wavelet reconstruction formula is:

$$f(x, y) = \frac{1}{C} \iint F \left\{ \sum \sum \iint \frac{1}{a_m a_n} W(a_{mn}, b') \times H(a_m \mu, a_n \nu) \exp(-j2\pi\mu b_{x'}, -j2\pi\nu b_{y'}) \times db_{x'} db_{y'} \right\} (x', y') B_{-\rho_1, -\rho_2}(x, y, x', y') dx' dy' \quad (9)$$

In image denoising, the pros and cons of various algorithms are ultimately comes down to the evaluation on the quality of the images, so you need a reasonable image quality assessment standards. Reference literature both at home and abroad and the evaluation standard are relatively uniform. After a lot of research and verification, the current widely used in scientific research personnel peak signal-to-noise ratio as evaluation indexes. Defined as: including noise image peak signal-to-noise ratio  $p0$  and denoising image peak signal-to-noise ratio  $p1$ , as shown below:

$$p0 = -10 \lg \left( \frac{\sum_{i=1}^m \sum_{j=1}^n N^2(i, j)}{m \times n \times 255^2} \right) \quad (10)$$

$$p1 = -10 \lg \left( \frac{\sum_{i=1}^m \sum_{j=1}^n (X(i, j) - X'(i, j))^2}{m \times n \times 255^2} \right) \quad (11)$$

$X(i, j)$  is Primitive noise image,  $X'(i, j)$  is for image denoising is estimated,  $N(i, j)$  is the estimated noise. After denoising images of  $p1$  value, the greater the denoising processing after the image more close to the original image, the image denoising effect is better.

Objective image quality index used peak signal to noise ratio of  $p$  value, the optimal  $p$  values for images, need to accurately estimate the residual noise, here is mainly to have nothing to do with the image of the zero mean Gauss white noise estimation. The input image is often has been polluted by the noise of the image, contains  $x_{mn}$  and noise of the image noise standard deviation  $a_{mn}$  is unknown, estimate of the image  $P$  values equivalent to the estimate of the standard deviation of  $a_{mn}$ .

Image noise estimation methods for the standard deviation of  $a_{mn}$  [9]: subtract with noise image denoising image difference to estimate the value of the noise signal, this method needs to use effectively filter out noise and well keep the edge details of both filter, will contain noise image into several parts, respectively estimates that each part  $a_{mn}$ , choose the more appropriate as  $a_{mn}$  actual estimated values; To estimate the noise of the image by image statistical features of the standard deviation  $a_{mn}$ .

Based on two-dimensional fractional wavelet transform in image denoising, the optimal  $p$  value selection is a very important link. This paper puts forward a optimal  $p$  value selection method based on noise estimation, now define the input noise unknown root mean square error of the  $M$  and unknown input noise peak signal to noise ratio of  $Pu$  objective evaluation criteria for the new image denoising, such as formula (12) and (13) [11.12]:

$$M = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n (f(i, j) - N(i, j) - X'(i, j))^2 \quad (12)$$

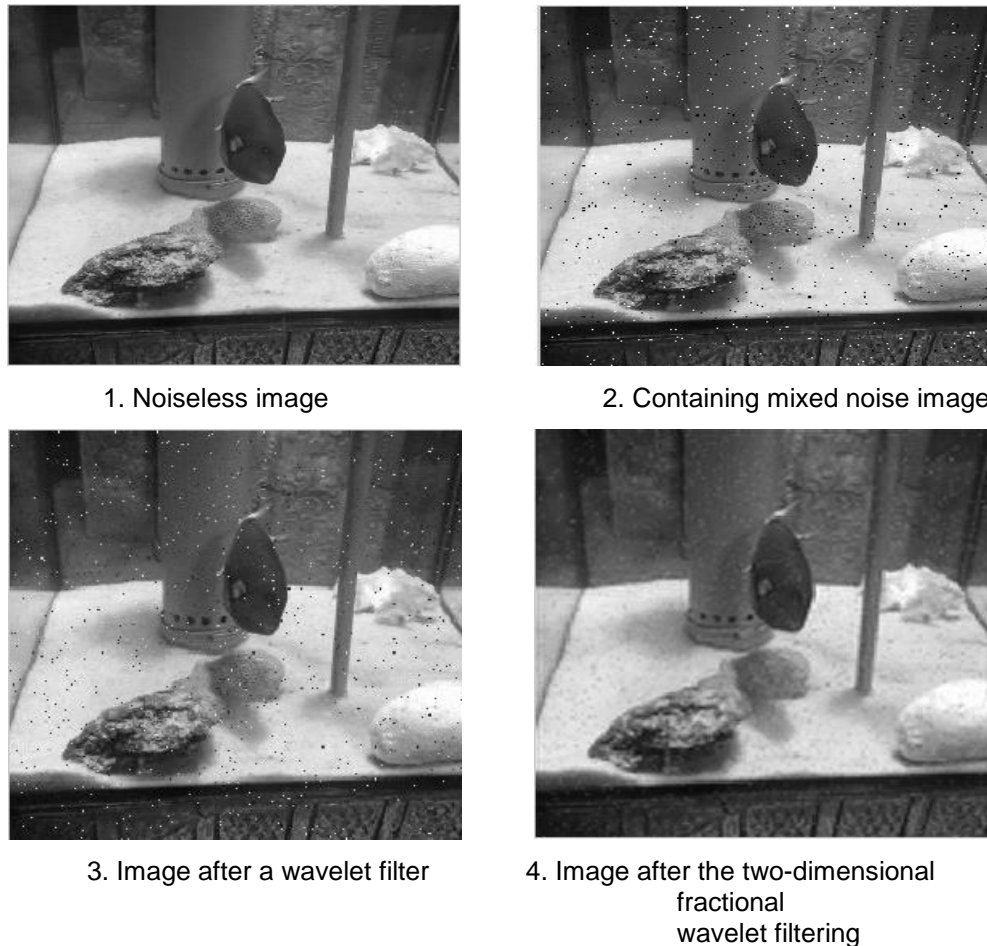
$$Pu = -10 \lg \frac{M}{255^2} \quad (13)$$

Among them,  $f(i, j)$  is the original noise image,  $X'(i, j)$  is estimated after denoising, image noise  $N(i, j)$  is estimated.

$N(i, j)$  approximates to the real value of noise,  $f(i, j) - N(i, j)$  more close to the original image without noise, can be thought of as  $f(i, j) - N(i, j)$  on the estimate of  $X'(i, j)$ , and the fractional wavelet transform denoising after  $X'(i, j)$  can also be

considered for  $X(i,j)$  kind of estimate, by comparing two kinds of estimates have been the  $P_u$  can be used as  $X'(i,j)$  standard to evaluate the gap between two estimates,  $P_u$ , the greater the degree of approximation, the higher of the two, this time can be thought of  $X'(i,j)$ ,  $p$  value is the most optimal, when  $X'(i,j) = X(i,j)$ ,  $P_u$  is equal to  $p$ . And the  $P$  value by the same token, the  $M$  value is smaller, the  $P_u$  value, the greater the  $X'(i,j)$  and  $X(i,j)$  higher degree of approximation, show the better denoising effect. The optimal corresponding fractional time calculated  $P_u$  is the optimal estimates of the  $p$  values [13.14].

To illustrate the proposed by Gaussian noise background, based on second order fractional wavelet transform denoising method is effective, here to unwater commonly used standard image as an example, the noise standard deviation of Gaussian noise pollution for 30 cases, the wavelet transform and secondary fractional order wavelet transform denoising method to do a comparison, to verify the superiority of this method with high efficiency.



**Figure 1. Denoising of Unwater Image**

By above knowable, the residual noise wavelet transform denoising is larger, and the secondary fractional order wavelet transform denoising based on better retain the useful detail information, in addition to the more noise, after denoising result in accordance with the original image better.

### 3. Hybrid Filter

Image gray scale is always continuous with the surrounding pixels and the gradient, and impulse noise point of gray level by which the original image grey value with the grey value of noise, with the surrounding pixels not continuous, so the high frequency part of the frequency domain, the wavelet domain and fractional wavelet transform domain coefficient is larger. If the threshold processing directly in the time-frequency domain, must choose larger threshold. And the image edge details information is in the high frequency need to select a smaller threshold. If too much of a threshold choice, can filter in addition to the details of wavelet coefficients at the same time, the image can get blurred.

To solve the problems existing in the hybrid filter, based on the two-dimensional fractional wavelet threshold denoising and median filtering, this paper proposes a hybrid unknown image noise filtering method. To filter the impulse noise accurately, do not destroy the image pixels are not impulse noise pollution status, and then in the two-dimensional fractional wavelet time-frequency domain on threshold shrinkage filter Gauss noise, the specific steps are as follows:

- (1) By impulse noise detection, identification pulse noise pixel, to distinguish the impulse noise and Gauss noise:
- (2) Using median filter method, impulse noise in image signal with noise:
- (3) For the rest of the standard deviation of  $a_{mn}$  zero means Gauss white noise:
- (4) One of the best fractional order  $p$  value to calculate the filtering results:
- (5) Transform the images, median filter to the two-dimensional fractional wavelet time-frequency domain:
- (6) In order  $p$  two-dimensional fractional wavelet time-frequency domain, shrinking threshold denoising method is used for the secondary filter:
- (7) Solving the  $-p$  order two-dimensional fractional wavelet transform, reconstruction after denoising image

### 4. Experiment and analysis

To illustrate the proposed hybrid unknown noise background, based on hybrid filter image denoising method is effective, it is commonly used in image processing of standard Lena image as an example, the choice of several groups of different mixed noise of impulse noise and Gauss white noise, with the median filter and wavelet transform denoising filtering methods such as comparison, evaluate the effectiveness of the proposed denoising method.

Specific denoising process is: the impulse noise detection, according to the type (1) according to (2), (3) the impulse noise filtering, and then estimate the standard deviation of the residual noise.  $a_{mn}$ , then estimate the rest of the zero mean Gauss white noise, and then use the Pu optimal method to calculate the optimal order number  $p$ , the optimal fractional wavelet domain filtered Gauss white noise. In this method, impulse noise detection adopts  $7 \times 7$  windows, according to the simulation experience value, took the four parameters  $k$ , obtains the best filtering effect, take 5 parameter  $t$ . The simulation experiment results of various methods for filtering are shown in Table 1. Figure 2 is  $a_{mn}$  variety of filter method for  $a_{mn}=40$  pulse noise and  $a_{mn}=40$  white noise filtering of the output image after mixed noise of Gauss.

**Table 1. Lena Image Denoising P value Comparison (/db)**

Noise	{20,20}	{30,20}	{40,30}	{50,30}	{40,30}	{50,30}	{60,30}
Noise of image	20.03	18.67	18.05	17.37	16.85	16.15	15.35
Median filtering	28.04	27.58	26.30	25.19	24.48	24.19	23.78
Wavelet transform	27.63	27.19	26.39	26.03	25.40	25.01	24.29
Method in the paper	28.24	28.04	27.49	27.10	26.30	26.08	25.58

The Table 1 shows that different proportion and the size of the mixed noise background, the proposed hybrid filtering algorithm is used to remove the mixed noise in the image of the proceeds of the P value is the highest.

Mixed noise consists of impulse noise and Gauss white noise, as the change of noise ratio, the effect of various denoising algorithm strength is different. When impulse noise ratio is greater than the Gauss white noise, median filtering alone effect is obviously enhanced, when the ratio is greater than the impulse noise and Gauss white noise, enhance the effect of wavelet transform, separate the effect of median filtering. Visible, median filter for noise suppression ability is used to filter out the narrow pulse signal is very effective, but for short tail distribution noise such as Gauss noise, median filtering showed poor filtering performance.



1. Noiseless image



2. Containing mixed noise image



3. Image after a median filter



4. Image after the wavelet denoising





5. Image after the hybrid filter

### Figure 2. Denoising of Lena image

By above knowable, this article uses multiple filter method is superior to single average filter or wavelet denoising method. Separate images become very smooth, after median filtering and separate the wavelet denoising method is not able to completely filter to remove the impulse noise point, this paper adopts the hybrid algorithm of filtering effect is better than both.

To sum up, this paper is on the basis of the existing image denoising algorithms, performance improvement, and puts forward a new hybrid filtering algorithm, this algorithm can more reasonably selecting two-dimensional fractional wavelet transform, the optimal order estimates of the number of  $p$ , finally get good denoising effect, and it has certain actual application value.

## 5. Conclusion

In practical application was hybrid image of unknown noise pollution, this paper combined with two-dimensional fractional wavelet threshold denoising and median filtering, presents a new hybrid unknown image noise filtering method, the two-dimensional fractional wavelet transform theory applied to image denoising. Verified by the experiment, the optimal  $p$  values based on wavelet transform method is accurate and practical, and 2 d wavelet transform and median filter, such as method, based on the mixture of two-dimensional fractional wavelet transform image denoising method can make the image and noise in fractional wavelet domain not overlap, as much as possible in order to better filter out noise, the method combining the median filter can effectively remove impulse noise excellent properties and two-dimensional fractional wavelet threshold denoising method good ability of filtered Gauss white noise.

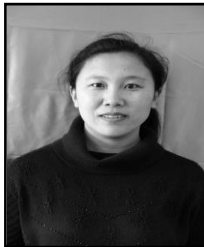
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## Authors



**Zhao Xiaofeng**, obtained Bachelor's Degree of Science from Shandong Normal University in 2004, and obtained Master's Degree of Engineering from Shandong University in 2011. She is currently researching on Digital Image Processing Theory and Application.



**Xie Hongsheng**, obtained Ph.D in Computer Science from CUMT in 2007. His research interests are focused on pattern recognition and machine learning.