MR Image Segmentation based on Local Information Entropy Geodesic Active Contour Model

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Abstract

In this paper, an improved Geometric Active Contour model is presented. This model introduced a window function to research the mean information of each pixel's neighbor region, and constructed a novel signed pressure force function based on the entropy to drive the contour towers the boundary. In order to improve the efficiency and stability of the algorithm, this paper adopts two valued level set method to realize the segmentation process. The experimental results show that this method can obtain satisfied result even when the images have intensity inhomogeneity and weak edges.

Keywords: GAC model, signed pressure force function, Local information entropy, Image segmentation

1. Introduction

At present, active contour models (ACM) [1] are widely used in image segmentation. This model was initially put forth by Kass *et al.* Generally speaking, the method based on ACM is usually classified into two types: edges-based models [1, 2] and region-based methods [3]. The edges-based models can well segment the target boundary with strong edges, which utilize the gradient information of images, but fail to segment the image with weak boundaries and noise. The region-based models utilize the grey mean or variance can well overcome the influence of weak boundaries or noise. However, this models are sensitive to intensity inhomogeneous, thus it can not well segment images with bias field.

Li *et al.* [4] proposed the local binary fitting (LBF) model, which utilizes the local image information to segment objects with intensity inhomogeneity and achieve the better segmentation results. Although the LBF model utilizes local mean information can better overcome the intensity inhomogeneous of images, but it can not well segment the images with strong noise and weak boundaries. Zhang *et al.* proposed a novel region-based GAC model in paper [5] can well segment the images with noise and weak boundaries. This model constructs a signed pressure force function based on global information to replace the edge-stopping function of the GAC model. This model assumed the grey information of mage is homogeneous, so it can not well segment the images with bias field.

From the research, most medical images local grey information presented gauss distribution and local statistical information can well describe the grey information distribution of the image. This paper can segment the medical images with bias field or weak boundaries and noise better by constructs a novel signed pressure force function based on

local entropy information which could utilized the local statistical to describe grey information. This method can get better segment result for this model can make full use of the grey information of images.

2. Background

Geodesic active contour (GAC) model is put forward by V. Caselles et al. [2] in 1997, and the energy functional of this model is defined as follows:

$$E^{GAC} = \int_0^{L(C)} g(|\nabla I[C(s)]|)ds \tag{1}$$

Where $L(C)$ is the length of contour C, ∇I is the gradient of image I, $g(\cdot)$ is the edge stopping function and s is the arc length unit of image. The level set evolution equation corresponding to formula (1) as follows:

$$\frac{\partial \phi}{\partial t} = g(|\nabla I|)\left(div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) + \alpha\right) + \nabla g \cdot \nabla \phi \tag{2}$$

Where $div(\cdot)$ is divergence operator, α is a constant. $g(\nabla I)$ is the edge-stopping function that decreasing with the image gradient values $|\nabla I|$ increase.

Generally, medical images often exists fuzzy boundaries or weak edges and noise. Because of the GAC model only utilize the gradient information but can not utilize comprehensive features of images to drive the evolution curves to segment the target boundary, which lead to the evolution contours over the weak boundaries or fall into the local optimal.

3. Image Information Entropy

For the image, $I : \Omega \in \mathbf{R}^2 \to \mathbf{R}$, the information of the pixel $x \in \Omega$ is:

$$E(x) = -\sum_{w(x-y)} p(I(y)) \log p(I(y)) \tag{3}$$

Where $p(I(y))$ is a Gaussian distribution function in the local region, $w(x-y)$ is a window function which centered at the pixel point x. The information entropy can achieve the maximum when the grey distribution is uniform.

Image information entropy [6] also has a widely application in image processing, especially in image segmentation. Such as the ACM based on regional fitting with local entropy proposed by He Chuanjiang [7] *et al.* and Amitava Chatterjee *et al.* utilize maximum fuzzy entropy to segment brain CT images [8].

4. Signed Pressure Force function based on Local Information Entropy

Zhang *et al.* proposed a region-based GAC (RGAC) [5] model, which constructs a signed pressure force (spf) function replace g of the GAC model. The definition of spf as follows:

$$spf = \frac{I - \frac{c_1 + c_2}{2}}{\max\left(\left|I - \frac{c_1 + c_2}{2}\right|\right)} \tag{4}$$

Where $\max\left(|\cdot|\right)$ indicates the absolute maximum of grey difference, grey mean c_1 and c_2 respectively defined as follows:

$$c_1 = \frac{\iint_\Omega I \cdot H(\phi)dxdy}{\iint_\Omega H(\phi)dxdy} \qquad c_2 = \frac{\iint_\Omega I \cdot \left(-H(\phi)\right)dxdy}{\iint_\Omega \left(-H(\phi)\right)dxdy} \tag{5}$$

Because of the RGAC model still rely on intensity homogeneity, and medical image usually include the bias field, so this model cannot to segment the object boundary well.

The grey information of most medical images in local region presents Gaussian distribution according to literature [9]. This paper introduced window function w to research the grey distribution of local image and introduced the image information entropy into spf to construct a novel drive force function which defined as follows:

$$lespf = \frac{\int_\Omega w(x-y)\left(E_m(x) + E_p(x)\right)dy}{\max\left(\left|\int_\Omega w(x-y)\left(E_m(x) + E_p(x)\right)dy\right|\right)} \tag{6}$$

Where $E_p(x) = -p_b\left(I(y)\right)\cdot\log\left(p_b\left(I(y)\right)\right)$ and $E_m(x) = -p_m\left(I(y)\right)\cdot\log\left(p_m\left(I(y)\right)\right)$ respectively present image information of background region and target region, Ω is the whole image region, $w(x-y)$ is window function, $p_b(I(y))$ and $p_m(I(y))$ respectively present Gaussian function of background and target regions and they respectively computed the posterior probability of background and target regions. According to literature [9], we can know:

$$p_b(I(y)) = \frac{1}{2\pi\sigma_b^2}\exp\left(-\frac{(I(y)-f_b)^2}{2\sigma_b^2}\right) \qquad p_m(I(y)) = \frac{1}{2\pi\sigma_m^2}\exp\left(-\frac{(I(y)-f_m)^2}{2\sigma_m^2}\right) \tag{7}$$

Where f_b and f_m respectively present grey mean of background and target regions:

$$f_b = \frac{\int_\Omega w(x-y)I(y)H(\phi(y))dy}{\int_\Omega w(x-y)H(\phi(y))dy} \qquad f_m = \frac{\int_\Omega w(x-y)I(y)\left(1-H(\phi(y))\right)dy}{\int_\Omega w(x-y)\left(1-H(\phi(y))\right)dy} \tag{8}$$

Where σ_b and σ_m respectively present variance of background and target regions:

$$\sigma_b^2 = \frac{\int_\Omega w(x-y)(I(y)-f_b)^2 H(\phi(y))dy}{\int_\Omega w(x-y)H(\phi(y))dy} \qquad \sigma_m^2 = \frac{\int_\Omega w(x-y)(I(y)-f_m)^2\left(1-H(\phi(y))\right)dy}{\int_\Omega w(x-y)\left(1-H(\phi(y))\right)dy} \tag{9}$$

In local region, grey changes present Gaussian distribution according literature [9]. Take a point x in contour, suppose Ω_y is a local region which is controlled by window function w and centered at point x. Ω_b and Ω_m respectively present background and object regions that the image region Ω_y divided into inside and external by the contour line. If $x\in\Omega_b$, $E_p(x)$ get a smaller value and $E_m(x)$ get a bigger value, then $lespf > 0$. This situation can drive evolution curves shrinkage to object boundary. In contrast, if $x\in\Omega_m$, $lespf < 0$, it can drive evolution curves expansion to object boundary. As we can know, the

grey information in local region satisfies Gaussian distribution, thus the maximum posterior probability of inside and external regions is 1 when the contours approach to object boundary. Moreover, we can know that $-p_b(I(y))\log p_b(I(y)) \approx 0$ and $-p_m(I(y))\log p_m(I(y)) \approx 0$, therefore $lespf \approx 0$ and the contour line will stop evolution at the object boundary. From the formula (6), the range of variation of function $lespf$ is [-1, 1] satisfies definition of spf function based on the above discussion.

Take $lespf$ instead of spf in the RGAC model; get the corresponding gradient descent flow equations as follows:

$$\frac{\partial \phi}{\partial t} = lespf \cdot |\nabla \phi| \cdot \left[div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) + \alpha \right] + \nabla lespf \cdot \nabla \phi \tag{10}$$

Because of grey values in local region of medical images changes slowly, the derivative of $lespf$ approach to 0, writing as $\nabla lespf \approx 0$. Simultaneously, $div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \cdot |\nabla \phi| = \Delta \phi$, and $G_\sigma * \phi$

can be used to approach $\Delta \phi$ in this paper. This paper through utilize Gauss filter instead of curvature regularization. From the above discussion, formula (10) can be written brief as:

$$\frac{\partial \phi}{\partial t} = \alpha \cdot lespf \cdot |\nabla \phi| \tag{11}$$

5. Experimental Results and Analysis

Figure 1 is the segmentation of left ventricle MR image. Figure 1(a)-(b) is the original image with initial curve and the true segmentation result. Figure 1(c) is the segmentation result of C-V model. This model can not well segment the images with bias field or weak boundaries. Figure 1(d) is the result of GAC model which iterate 600 times. This model can only precede single direction evolution. Figure 1(e) is the result of RGAC model which iterate 100 times. This model can easy occur over-segmentation. Figure 1(f)-(g) respectively presents the result of LBF and LGD model which iterate 100 and 200 times. The parameter of them is $\lambda_1 = 1.0, \lambda_2 = 1.0$ and kernel parameter sigma=15. Both of them can well segment images with bias field, but fail to segment noise image. Figure 1(h) is segmentation result of this paper method iterate 100 times with parameter sigma=15 and $\alpha = 100$. This method can well segment the images with bias field and weak boundaries.

In order to better respond or compare segmentation accuracy and segmentation efficiency of each model, we utilize 20 groups left ventricle MR images for experiment segmentation test. The size of image is 110×110, and utilize the Jaccard Similarity（JS）index [10] to quantitative analysis the segmentation accuracy.

(a)　　　　　　　(b)　　　　　　　(c)　　　　　　　(d)

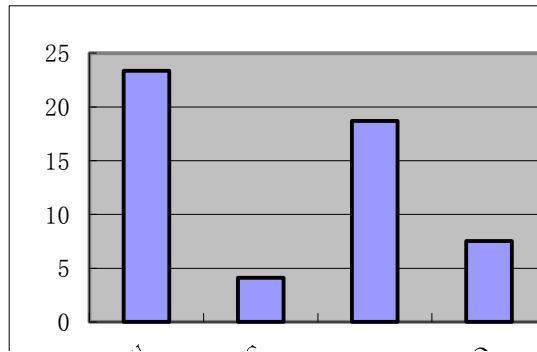|			
(e)	(f)	(g)	(h)
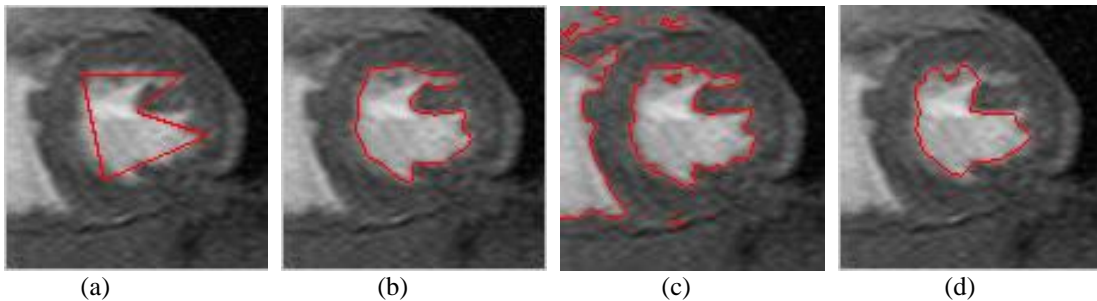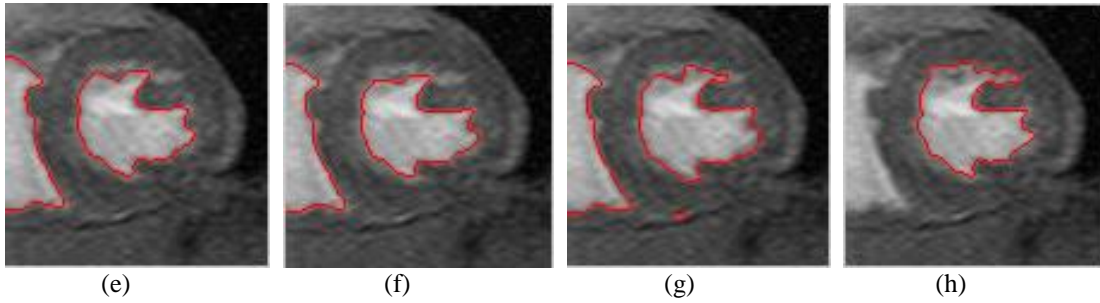
Figure 1. The segmentation result of left ventricle magnetic resonance (MR) image. Figure 1(a)-(b) respectively present original image with initial curve and accurate segmentation result. Figure 1(d)-(h) respectively present segmentation result of C-V model, GAC model, RGAC model, LBF model, LGD model and this paper method

As shown in the figure, the segmentation accuracy of this paper method is more than 90%. The C-V model can not well segment the images with bias field. The GAC model can easily produces boundary leakage. The RGAC model fails to segment the images with bias field. Due to lots of convolution operation, the segmentation efficiency of LBF and LGD model is lower. This paper method can well segment the images with bias field. Due to this method adopt window function to computer, the average iteration time no more than 3 seconds and iteration times less than 200 times. This situation is still in reasonable range.
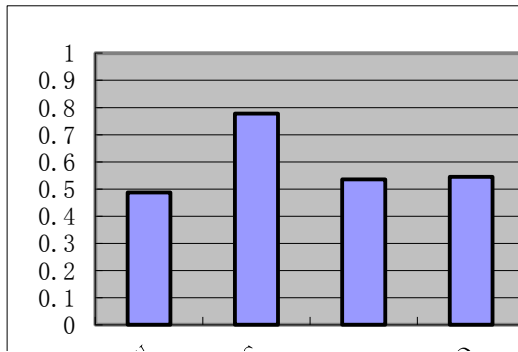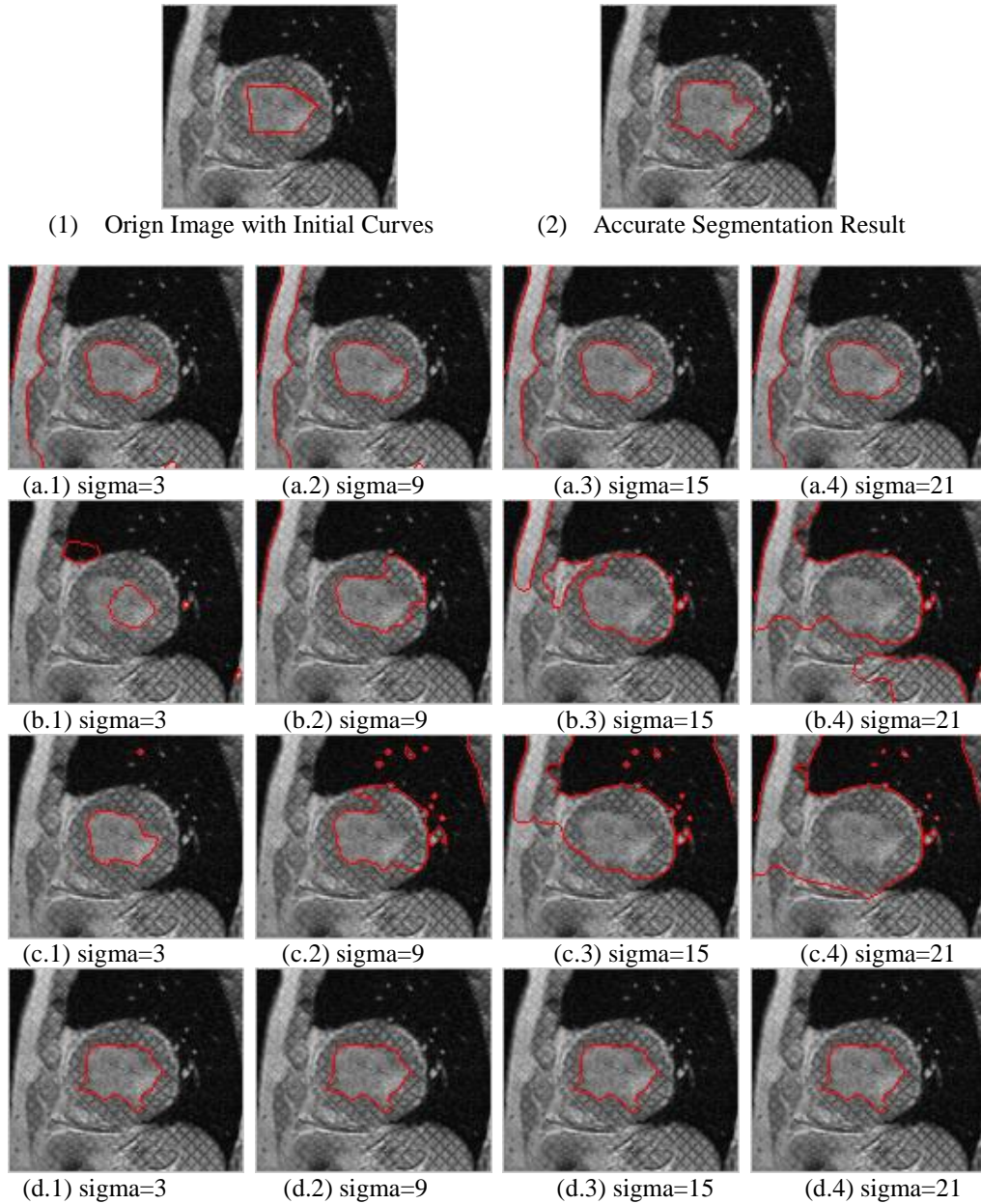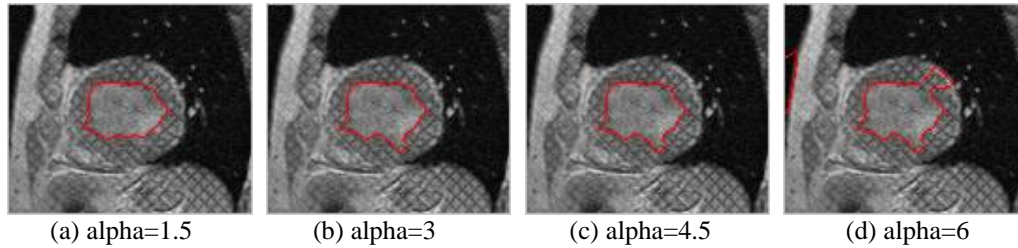

Figure 2. Segmentation Accuracy   **Figure 3. Segmentation Efficiency**

Figure 4 is the segmentation result of left ventricle MRI with sigma for [3, 9, 15, 21]. Initial image exits of noise, bias field, weak boundaries and labeled line. Figure 4(1)-(2) respectively present the origin image with initial curve and the accurate segmentation result. Figure 4(3) is the result of RGAC model iterates 100 times with the different sigma. This model can well overcome noise and weak boundaries, but easy cause to segment homogenous images wrong. Figure 4(4)-(5) respectively present the result of LBF and LGD model iterates 100 and 200 times with the parameters $\lambda_1 = 1.0$, $\lambda_2 = 1.0$. The LBF model and LGD model can not well segment the images included bias field with the sigma increase. Figure 4(6) is the result of this paper method iterates 150 times with parameter $\alpha = 3$. This method can well segment the images with the intensity inhomogeneous. Figure 5 is the segmentation result of this paper method iterates 150 times with sigma=15 and $\alpha = [1.5, 3, 4.5, 6]$.

Figure 6 and Figure 7 respectively present the segmentation accuracy and efficiency with the sigma. From the figure, this paper method is obviously better than RGAC, LBF and LGD model. With the increase of sigma, the segmentation accuracy of LBF and LGD model decrease.

(1)   Orign Image with Initial Curves   (2)   Accurate Segmentation Result

(a.1) sigma=3   (a.2) sigma=9   (a.3) sigma=15   (a.4) sigma=21

(b.1) sigma=3   (b.2) sigma=9   (b.3) sigma=15   (b.4) sigma=21

(c.1) sigma=3   (c.2) sigma=9   (c.3) sigma=15   (c.4) sigma=21

(d.1) sigma=3   (d.2) sigma=9   (d.3) sigma=15   (d.4) sigma=21

Figure 4. The first row is the segmentation result of origin image with initial curves. The second to fifth row respectively present the segmentation result of RGAC model, LBF model, LGD model and this method based on the size of sigma is [3, 9, 15, 21]

| (a) alpha=1.5 | (b) alpha=3 | (c) alpha=4.5 | (d) alpha=6 |

Figure 5. The segmentation result with the influence of the parameter α in this paper
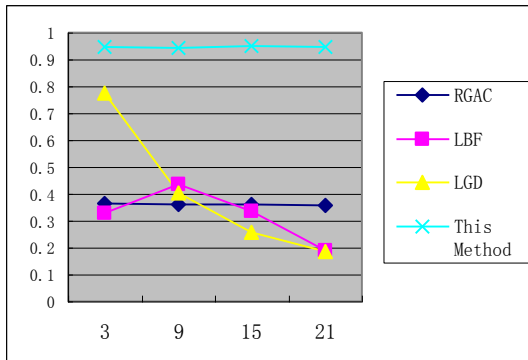
Figure 6. The influence of segmentation accutare with sigma
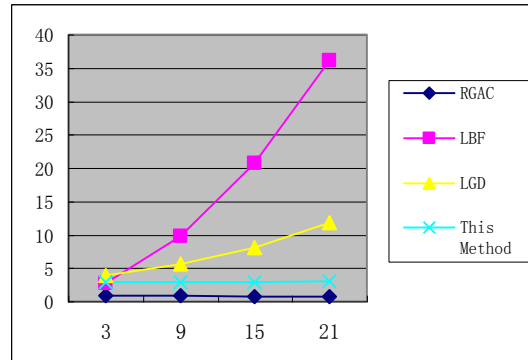
Figure 7. The influence of segmentation efficiency with sigma

6. Conclusions

This paper proposed an improved GAC model for the images with noise, bias field and weak boundaries. This paper model utilizes the window function to investigate the grey information of local region, while introduce the local information entropy to describe the distribution of the grey information. Compared with GAC model, this method can achieve the high segmentation accuracy. Compared with C-V and RGAC model, this method can well segment the images with bias field. And compared with LBF and LGD model, this method has more choice of kernel parameters.

Acknowledgements

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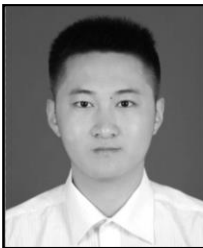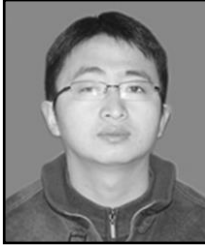
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