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Degrees of Freedom for Complex MIMO Multiple-Way Relay Channel

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Abstract

The amount of diversity or the number of degrees of freedom (DOF) can be increased by multiple antennas in wireless communication systems. In this paper, we study the DOF of multiple-input multiple-output (MIMO) multiple-way relay channel. There are two groups of source nodes and several one relay node which are equipped with multiple antennas in this kind of channel. Each source node in one group exchanges independent message with the source node in the other group via the relay node. We first show the upper bound on the total DOF for this channel. Then we extend it to the complex MIMO multiple-way relay channel and present the corresponding upped bound.

Keywords: *Degrees of Freedom (DOF); capacity; multiple-way relay channel; MIMO*

1. Introduction

Multiple antennas are important to improve the performance of wireless channel. In general, the spectral efficiency of MIMO channels is much higher than that of the conventional signal antenna channels. Some research on MIMO channel [1-3], including the study of channel capacity and the design of communication schemes, illustrates a great improvement of performance.

There is recent interest in the degrees of freedom (DOF) of MIMO communication system. Time, space, and frequency all offer DOF [4][5], spatial dimensions are especially interesting for how they may be accessed with distributed processing. Earlier work [6-8] by some researchers has determined the DOF of various multiuser MIMO systems. Then [9][10] derived the DOF for single-user point-to-point MIMO channel and two users MIMO multiple-access channel. [11] has shown that in the high signal-to-noise radio (SNR) regime, the capacity of a channel with m transmit, n receive antennas and i.i.d. Rayleigh faded gains between each antenna pair is given by:

$$C\left(SNR\right) = \min\left\{m, n\right\} \log SNR + O\left(1\right)$$

Therefore, the number of DOF is the minimum of m and n.

[12] considered the DOF of a multilayer distributed relay network. Based on [14-15] derived the upper bound on the DOF of the MIMO two-way X relay channel. In this paper, we focus on the MIMO system with relay, which is widely used for many kinds of networks. For example, in a cooperative multicell communication system with several base stations and several users via relays, the relays can help exchange information between them. Also, in ad hoc network and wireless mesh network, some users in one group exchange information with some users in the other group via relay

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nodes. Specially, this paper discusses the complex MIMO multiple-way relay channel. In this kind of channel, there are multiple relay nodes, each node is equipped with different number of antennas, and each group consists of different number of source nodes.

The remainder of this paper is organized as follows. Section 2 describes the MIMO multiple-way relay channel model. Section 3 shows the upper bound on the DOF of the MIMO multiple-way relay channel model. Section 4 extends Section 3 to more complex situation, and derives the corresponding upper bound on the DOF. Section 5 concludes the paper.

Notations: Boldface uppercase letters denote the matrices and boldface lowercase letters denote the vector. $(\cdot)^H$ and $Tr(\cdot)$ are the Hermitian transpose and trace operators, respectively.

2. MIMO Multiple-Way Channel Model

A MIMO multiple-way relay channel is shown in Figure 1 of Section 6. The channel consists of two groups of nodes and one relay node. There are L source nodes in each group. Each source node is equipped with M antennas, and the relay node is equipped with N antennas. Each node $A_i (i = 1, 2, \cdots, L)$ on the left side (group A) needs to send an independent message to each node $B_j (j = 1, 2, \cdots, L)$ on right side (group B) via the relay. So does the each source node on the group B.

The total transmission is implemented in two phases. Firstly, in the multiple–access phase, the signals are transmitted to the relay from all the source nodes. The received signal at the relay are given by

$$\mathbf{y}_r = \sum_{i=1}^{L} \mathbf{H}_{A_i,r}\mathbf{x}_{A_i} + \sum_{j=1}^{L} \mathbf{H}_{B_j,r}\mathbf{x}_{B_j} + \mathbf{n}_r \tag{1}$$

Where $\mathbf{y}_r$ and $\mathbf{n}_r$ denote the $N \times 1$ received signal vector and the additive white Gaussian noise (AWGN) vector at the relay, respectively. $\mathbf{x}_{A_i}$ ($\mathbf{x}_{B_j}$) is the $M \times 1$ transmitted signal vector by source A_i (B_j) with the power constraint $\mathrm{E}\left(Tr\left\{\mathbf{x}_{A_i}\mathbf{x}_{A_i}^H\right\}\right) \le P$ $\left(\mathrm{E}\left(Tr\left\{\mathbf{x}_{B_j}\mathbf{x}_{B_j}^H\right\}\right) \le P\right)$; $\mathbf{H}_{A_i,r}$ ($\mathbf{H}_{B_j,r}$) is the $N \times M$ channel matrix from source A_i (B_j) to the relay. The entries of the channel matrix $\mathbf{H}_{A_i,r}$ ($\mathbf{H}_{B_j,r}$) and those of the noise $\mathbf{n}_r$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables with unit variance.

Secondly, in the broadcast phase, the relay node forms a new signal $\mathbf{x}_r$ and broadcasts it to all the source nodes. The received signals at the source nodes are given by

$$\mathbf{y}_{A_i} = \mathbf{H}_{r,A_i}\mathbf{x}_r + \mathbf{n}_{A_i} \tag{2}$$

$$\mathbf{y}_{B_j} = \mathbf{H}_{r,B_j}\mathbf{x}_r + \mathbf{n}_{B_j} \tag{3}$$

Where $\mathbf{y}_{A_i}$ ($\mathbf{y}_{B_j}$) and $\mathbf{n}_{A_i}$ ($\mathbf{n}_{B_j}$) denote the $M \times 1$ received signal vector and the AWGN vector at the source node, respectively; $\mathbf{x}_r$ is the $N \times 1$ transmitted signal vector by the relay with the power constraint $\mathrm{E}\left(Tr\left\{\mathbf{x}_r\mathbf{x}_r^H\right\}\right)\leq P$; $\mathbf{H}_{r,A_i}$ ($\mathbf{H}_{r,B_j}$) is the $M\times N$ channel matrix from the relay to source node. It is assumed that $\mathbf{H}_{r,A_i}$ ($\mathbf{H}_{r,B_j}$) and $\mathbf{n}_{A_i}$ ($\mathbf{n}_{B_j}$) contain i.i.d. zero-mean complex Gaussian random variables with unit variance.

In this paper, we assume that perfect channel state information (CSI) is available at all source nodes and the relay. In addition, it is assumed that the source nodes and the relay node operate in full-duplex mode.

The total DOF of the above MIMO multiple-channel is defined as follow:

$$d=\sum_{i=1}^{L}\sum_{j=1}^{L}d_{A_i,B_j}+\sum_{j=1}^{L}\sum_{i=1}^{L}d_{B_j,A_i}=\lim_{SNR\to\infty}\frac{R(SNR)}{\log(SNR)} \tag{4}$$

Where d_{A_i,B_j} (d_{B_j,A_i}) is the DOF from source node A_i (B_j) to B_j (A_i), $R(SNR)$ is the sum rate as a function of SNR. Because the noise samples are assumed to have unit variance, SNR is defined as $SNR \triangleq P$

3. An UpperBound on the DOF of the MIMO Multiple-Way Relay Channel

In this section, an upper bound on the DOF of the MIMO multiple-way relay channel is derived.

Theorem 1:

Consider a MIMO multiple-way relay channel with two groups of source nodes and one relay node. Each group consists of L source nodes, where each one is equipped with M antennas. And the relay node is equipped with N antennas. The total number of DOF is upper bound by $2\min\{LM,N\}$, *i.e.*,

$$d\leq 2\min\{LM,N\}. \tag{5}$$

Proof: The network information flow of one direction is first considered, i.e., from A_i to B_j via the relay, as shown in Fig.2 of Section 6.

In the first phase (cut 1), $A_i(i=1,2,\cdots,L)$ simultaneously transmit information to the relay. We assume that $A_i(i=1,2,\cdots,L)$ fully cooperate, then the channel essentially becomes a $LM\times N$ MIMO channel. From [13], we can derive the DOF that is $\min\{LM,N\}$. Similarly, in the second phase (cut 2) we can obtain the same result. Combining the two phase and the cut-set theorem[14], we can obtain the DOF of one direction

$$\begin{aligned}\sum_{i=1}^{L}\sum_{j=1}^{L}d_{A_i,B_j}&\leq\min\left\{\min\{LM,N\},\min\{LM,N\}\right\}\\&=\min\{LM,N\}\end{aligned} \tag{6}$$

For the other direction, the similar result can be derived as follow:

$$\sum_{j=1}^{L}\sum_{i=1}^{L} d_{B_j,A_i} \le \min\{\min\{LM,N\},\min\{LM,N\}\} = \min\{LM,N\} \quad (7)$$

So

$$d = \sum_{i=1}^{L}\sum_{j=1}^{L} d_{A_i,B_j} + \sum_{j=1}^{L}\sum_{i=1}^{L} d_{B_j,A_i} \le 2\min\{LM,N\} \quad (8)$$

Proof completed.

Remark: We can notice that the factor 2 is needed since the assumption of full-duplex mode in this scheme. If the mode turns into half-duplex, the factor of 2 is not needed.

From theorem 1, we can see that when $N \le LM$, the DOF of this channel is upper bounded by the twice the number of the relay node's antennas, which is therefore the bottleneck for the spectrum efficiency of the network.

4. An Upper Bound on the DOF of the Complex MIMO Multiple-Way Relay Channel

In practical application, only one relay node usually cannot meet the demand, so more relay nodes should be needed to be deployed. We can extend the one relay node (Fig.1) to more relay nodes model (Fig.3). For convenience, we call this kind of model complex MIMO multiple-way relay channel. In this section, we derive the upper bound on the DOF of the complex MIMO multiple-way relay channel.

Theorem 2:

Consider a complex MIMO multiple-way relay channel with two groups of source nodes and X relay nodes. Each group consists of L source nodes, where each one is equipped with M antennas. And each relay node is equipped with N antennas. The total number of DOF is upper bound by $2\min\{LM,N\}$, i.e.,

$$d \le 2\min\{LM,N\}. \quad (9)$$

Proof: The process of proof is similar to theorem 1. The network information flow of one direction is also first considered. Then we cut this network into X+1 MIMO channel. Thus we have

$$\sum_{i=1}^{L}\sum_{j=1}^{L} d_{A_i,B_j} \le \min\left\{\begin{array}{l}\min\{LM,N\},\\ \underbrace{\min\{N,N\},\cdots,\min\{N,N\}}_{X-1},\\ \min\{LM,N\}\end{array}\right\} = \min\{LM,N\} \quad (10)$$

So

$$d = \sum_{i=1}^{L}\sum_{j=1}^{L} d_{A_i,B_j} + \sum_{j=1}^{L}\sum_{i=1}^{L} d_{B_j,A_i} \le 2\min\{LM,N\} \quad (11)$$

Proof completed,

Obviously, more relay nodes with the same number of antennas cannot change the upper bound on the DOF.

However, there is another more complex situation. From above situations, we notice that every source node has the same number of antennas M, so does every relay node. In the rest of this section, we will derive the upper bound on the DOF of the complex MIMO multiple-way relay channel with different antennas.

Theorem 3:

Consider a complex MIMO multiple-way relay channel with two groups of source nodes and X relay nodes. Group A consists of L_1 source nodes where A_i $(i = 1, 2, \cdots, L_1)$ is equipped with a_i antennas, and Group B consists of L_2 source nodes where B_i $(j = 1, 2, \cdots, L_2)$ is equipped with b_j antennas. The relay node k is equipped with c_k $(k = 1, 2, \cdots, X)$ antennas. The total number of DOF is upper bound by:

$2\min\left\{\sum_{i=1}^{L_1} a_i, c_1, c_2, \cdots, c_X, \sum_{j=1}^{L_2} b_j\right\}$, i.e.,

$$d \le 2\min\left\{\sum_{i=1}^{L_1} a_i, c_1, c_2, \cdots, c_X, \sum_{j=1}^{L_2} b_j\right\} \tag{12}$$

Proof: The process of proof is similar to theorem 2.

From theorem 3, we can see that the minimum number of antennas among all the groups and relay nodes is the bottleneck for the spectrum efficiency of the network.

5. Conclusions

In this paper, we considered the MIMO multiple-way relay channel and the complex MIMO multiple-way relay channel. Then we obtained their corresponding upper bounds on the DOF, from which we can analyze the networks more effectively. By means of network coding and interference alignment, the upper bounds could be achieved, we will leave it to the future work..

6. Graph

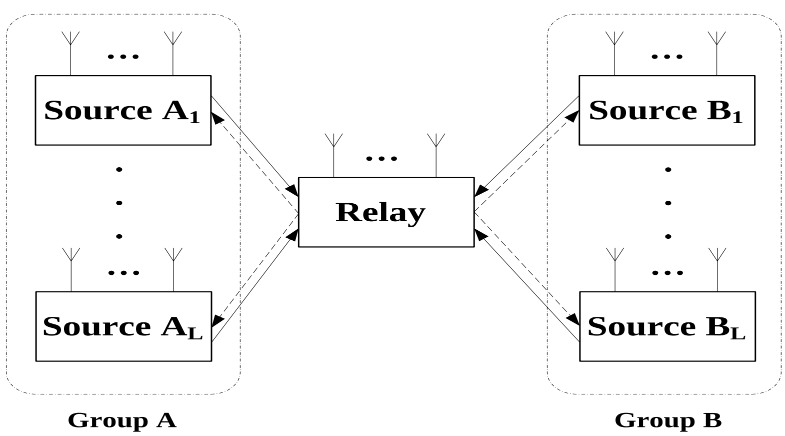

Figure 1. MIMO Multiple-way Relay Channel

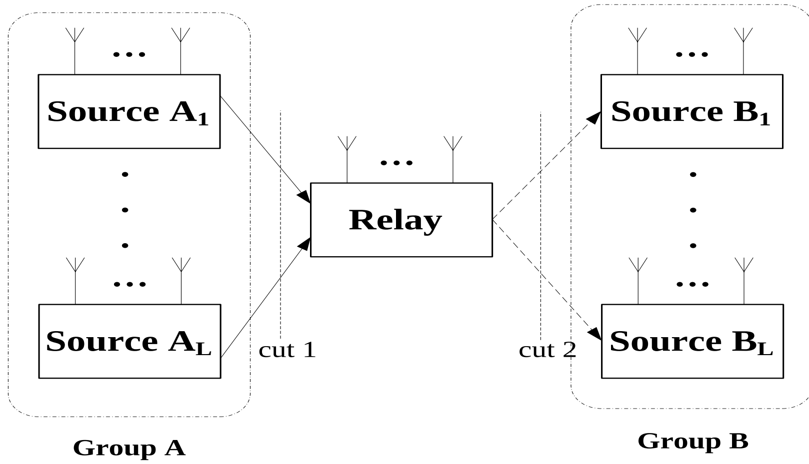

Figure 2. One Direction of Network Information Flow for the MIMO Multiple-way Relay Channel

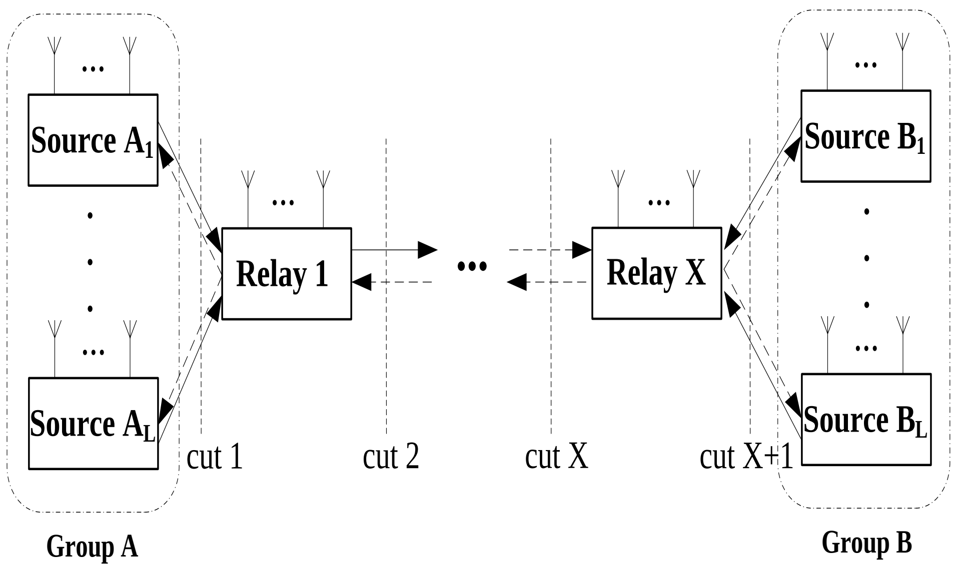

Figure 3. Complex MIMO Multiple-way Relay Channel

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