# Projector Calibration Method using Orthogonal Gray Code Combined with Trapezoid Phase Shift Code 

Yu Xiaoyang, Meng Xiaoliang, Wu Haibin*, Zang Tianjian and Xiao Zhenyu<br>The higher educational key laboratory for Measuring and Control Technology and<br>Instrumentations of Heilongjiang province, Harbin University of Science and<br>Technology, Harbin 150080, China<br>1263886567@qq.com<br>Abstract


#### Abstract

In coded-structured light three dimensional measurementsystem, system calibration plays an important role for the measurement accuracy. Projector calibration is an important part of the system calibration. Therefore, this paper proooses a projector calbration method with simple calibration process and high accuracy. Thi method ebmbines Zhang's plane model calibration method with orthogonal Gray code + trapezoid phase shift code. During the calibration process, this paper uses Gray code pattern and wapezoid phase shift code pattern to establish the relation of projector intage and camert point coordinates, the corner extraction accuracy of this paper's method is higher than the traditional orthogonal Gray code. According to the image coordinates in the projector's perspective, we program and calculate the projector's internal and external parameters matrix based on the Zhang's plane model calibration toolbox. The results show that this paper's method is simple and flexible, the maximum relative error of the culibration parameters is $0.027 \%$, and it meets the requirements of system calibration in ehgmeering or industrial fields.


Keywords: Coded-strictured light, System calibration; Projector calibration; Orthogonal Gray code + trapezord phase shifteode

## 1. Introduction

Coded-structured rivhe three dimensional measurement system has the advantage of noncontact, high accufacy, high speed. Therefore, it has been widely used in the area of industrial inspection, nấhine vision, medicine, face recognition and so on [1-3]. In coded-structured light system, projector calibration is one of the important parts of the system. The representaive paper abroad is presented by Zhang [4] who used the camera to give the projector visualization, the calibration process needs to project the phase shift pattern, but when ye calculate the corner point coordinates, the program needs to calculate the arctangent function, the speed is slower. Currently the main domestic research achievements of the projector calibration method are as follows:

In 2008, Xiaolin Dai [5] put forward a projector calibration method based on phase matching technology. Although the accuracy of this calibration method is high, the algorithm is complex and the calibration process is slow. In 2009, Zhongwei Li [6] proposed a projector calibration method using the circular landmark point. This method used two different sets of coding image to establish the relation of projector and camera, but the error of the extracted center point is big. In 2011, Dong Zhang [7] put forward a kind of projector calibration method base on color circular landmark pattern, this method used the common calibration checkerboard to project the color circular landmark pattern, according to the center
coordinates of the color circular landmark pattern, it calculated the projector's parameters. But this method is complex and the accuracy is not very high. In 2013, Zhao Wagng [8] proposed a kind of projector calibration method based on beam method, this method improves the calibration accuracy, but the calibration process is complex.

In order to improve the calibration accuracy and speed of the projector, it has great significance to study how to improve the accuracy of corner point extraction. Therefore, this paper will put forward a new projector calibration method which combines the Zhang's plane model calibration method and orthogonal Gray code + trapezoid phase shift code, then verify the projector calibration results after calibration.

## 2. Calibration Principle

### 2.1. Projector Imaging Model

Because the projector can be regarded as a reverse camera, its imaging model can be regarded as a reverse camera pinhole model, the imasing model 1 shown in Figure 1. A specific point in the space $Q$ intersected with the point $q$ of the igaging plane. In order to get the two dimensional coordinate of $q$, this paper proposed a mod by projecting the orthogonal phase shift coding pattern to the sufface of the plane model, then calculated the two dimensional coordinate through the camera images finally we can use the image coordinate $(u, v)$ and the world coordiate $\left(X_{w A} Y_{v}, \mathcal{Z}_{w}\right)$ to calculate the internal and external parameters matrix of the projecter.


Figure 1. Projector Imaging Model

### 2.2. Calibration Procedure

This paper uses the checkerboard plane model to set up the relation between the projector images and camera images. In the calibration process, this paper used the Gray code + trapezoid phase shift code pattern of horizontal and vertical direction to project the images [9]. The main steps are as follows:
(1) Take a checkerboard plane model image using the camera, and project the horizontal and vertical Gray code and trapezoid phase shift code images to the surface of the plane model, and use the camera to take these images, the calibration plane model images are shown in Figure 2.


Figure 2. Checkerboard Calibration Plane Model
(2) Use the images taking from (11, Use) the calibration toolbox to extract the corner sub-pixel coordinates of the checkerboard.
(3) The Gray code and trapezoid phase shift code combines the low error rate of Gray code, simple and easy realization high accuracy and high sampling density of trapezoid phase shift code. So this paper uses the Gray code + trapezoid phase shift code to accomplish the calibration of projector. Put the white paper on the surface of the checkerboard plane model, then project horizontal and vertical Gray code and trapezoid phase shift code images. In order to improve the accuracy of comner point extraction, this paper used five Gray code images and three trapezoid phase shiftimages to project. First, project the five Gray code in turn then use the camera take the images, then project the three trapezoid phase shift images and use the camera take the images the code image is shown in Figure 3.


Figure 3. Gray Code + Trapezoid Phase Shift Code Pattern
The intensity function of trapezoid phase shift pattern can be denoted by the following equations:

$$
\begin{align*}
& I_{1}(x, y)=\left\{\begin{array}{lr}
I^{\prime}(x, y)+I^{\prime \prime}(x, y) \quad x \in[0, T / 6) \text { or }[5 T / 6, T] \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y)(2-6 x / T) & x \in[T / 6, T / 3) \\
I^{\prime}(x, y) & x \in[T / 3,2 T / 3) \\
I^{\prime}(x, y)+I^{\prime}(x, y)(6 x / T-4) & x \in[2 T / 5,2 T / 6)
\end{array}\right.  \tag{1}\\
& I_{2}(x, y)=\left\{\begin{array}{lr}
I^{\prime}(x, y)+I^{\prime \prime}(x, y)(6 x / T) & x \in[0, T / 6) \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y) & x \in[T / 6, T / 2) \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y)(4-6 x / T) & x \in[T / 2,2 T / 3)
\end{array}\right.  \tag{2}\\
& I^{\prime}(x, y) \quad x \in[2 T / 3, T) \\
& I_{3}(x, y)=\left\{\begin{array}{l}
I^{\prime}(x, y) \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y)(6 x / T-2) \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y) \\
I^{\prime}(x, y)+I^{\prime \prime}(x, y)(6-6 x / T)
\end{array}\right. \tag{3}
\end{align*}
$$

where $I_{l}(\mathrm{x}, \mathrm{y}), I_{2}(\mathrm{x}, \mathrm{y}), I_{3}(\mathrm{x}, \mathrm{y})$ are the intensity function of the three patterns respectively, $I^{\prime}(\mathrm{x}, \mathrm{y})$ and $I^{\prime \prime}(\mathrm{x}, \mathrm{y})$ are the minimum intensity and intensity modulation of the pixel point( $\mathrm{x}, \mathrm{y}$ ), T is the distance of one trapezoid phase shift code period. The decode principle of the trapezoid phase shift code $1 s$ similan with the sin phase shift code, we can get the intensity function by projecting the three images inturen, then calculate the relative phase value.

The trapezoid phase shift code phase value can be denoted by the following equation:

$$
\begin{equation*}
\sum_{r(x, y)}=\frac{I_{m}(x, y)-I_{L}(x, y)}{I_{H}(x, y)-I_{L}(x, y)} \tag{4}
\end{equation*}
$$

Where $I_{L}(x, y) I_{M}\left(x, y\right.$ and $I_{H}(x, y)$ are the minimum value, medium value and maximum value of the pixel point $(\mathrm{x}, \mathrm{y})$ intensity respectively. $r(x, y)$ is the relative phase value, the minimum value of the pixel point ( $\mathrm{x}, \mathrm{y}$ ) is zero, and the maximum value is one. Therefore, the range of $(x, y)$ is from 0 to 1 .

In order to get the linear value of the phase value in one period, this paper uses the following equation to get a linear curve:

$$
\begin{equation*}
r_{\psi}=2 \times \text { round }\left(\frac{N-1}{2}\right)+(-1)^{N+1} \frac{I_{M}(x, y)-I_{L}(x, y)}{I_{H}(x, y)-I_{L}(x, y)} \tag{5}
\end{equation*}
$$

where ${ }_{4}$ is a relative phase value, round is the upward integer function, use the equation (5) to get rid of the triangular, and the $r(x, y)$ range is from 1 to 6 , then the images can be projected to the object linearly, and get the depth information of the three dimensional coordinate.

The decode major problem of the Gray code + trapezoid phase shift code is to calculate the absolute phase, firstly calculate the period value of the Gray code and relative phase value, then combines the two values to calculate the absolute phase value, the equation is as follows:

$$
\begin{equation*}
R(x, y)=6 k+2 \times \text { round }\left(\frac{N-1}{2}\right)+(-1)^{N+1} r(x, y) \tag{6}
\end{equation*}
$$

Where $R(\mathrm{x}, \mathrm{y})$ is the absolute phase value, k is the period value of Gray code.
(4) Use the absolute phase values and the corner coordinate extracted in step (2) to calculate the image coordinate in the projector imaging plane.
(5) Repeat the above steps until we get at least three different perspective image coordinates of the projector to accomplish the internal parameters calibration of the projector, then choose another images to calculate the external parameters matrix of the projector.

The projector calibration flow chart is shown in Figure 4.


Figure4. Projector Calibration Flow Chart

## 3. Verification Experment

This paper uses the Autodesk 3ds Max 2010 software to verify the calibration experiment. The device schematic diagram can be shown in Figure 5.


Figure 5. The Device Schematic Diagram
The parameters in the verification experiment are shown in Table 1.

Table 1. The Parameters of the Projector

| Device parameters |  | Coordinate rotation |  | Coordinate translation |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Projector | $N^{p} \times M^{p}$ | $1024_{\times} 768$ | $\omega^{p}$ | $21.8^{\circ}$ | $t_{\mathrm{x}}$ | 0 |
|  | $\beta_{1}^{p}$ | $20^{\circ}$ | $\varphi^{p}$ | $-180^{\circ}$ | $t_{\mathrm{y}}$ | 0 |
|  | $\beta_{2}^{p}$ | $15^{\circ}$ | $\rho^{p}$ | $90^{\circ}$ | $t_{\mathrm{z}}$ | 539.516 |

In the experiment, there's a distance between the camera and the calibration model plane, in order to ensure the camera can take all the corners of the calibration model plane, the checkerboard plane model should consider the range of the camera. In this paper, when we calculate the internal parameters, the size of the model plane is $210 \times 270$. When we calculate the external parameters, the size of the model plane is $140 \times 180$.

After calibration, the parameters matrix can be denoted as $y^{\text {. }}$

$$
\left.a_{-} c=\left[\begin{array}{cccc}
0.1589 & 1306.1 & 52 & .0593 \\
1051.8 & -0.0989 & -0.0736 & 293.6144 .108 \\
0 & -0.3705
\end{array}\right)=0.9288 \quad 0539.6104\right] \text {, }
$$

The projector calibration parameterseareshown in Pable 2. Note that the projector in the simulation environment is off-axis projection.

The data in the Table 2 shows the correctness of the projector calibration method, the projector calibration error is caused by théquantization error when locating the corner coordinates using the orthogonal Gray oode + trapezoid phase shift code.

Table 2. The Projector Calibration Parameters using the Method of this Paper


In Table 2, the relative error between the calibration value and standard value is $0.027 \%$, the error is very small, it can even be ignored, it verified the correctness of this paper's calibration method.

Because this paper adopted the whole image feature points rather than the feature points near the image center, this ensures the integrity of the calibration parameters and avoids the complex and unstable of the nonlinear optimization, and it improves the accuracy of the calibration.

This paper also uses the orthogonal Gray code to calibrate the projector, and the calibration parameters results are shown in Table 3.

Table 3. The Parameters using the Orthogonal Gray Code

| System <br> parameters | Calibration <br> value | Real <br> value | Relative <br> error | System <br> parameters | Calibration <br> value | Real <br> value | Relative <br> error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r_{11}$ | 0 | 0 | 0 | $r_{32}$ | 0.3714 | 0.3704 | $0.27 \%$ |
| $r_{12}$ | 0.9290 | 0.9286 | $0.043 \%$ | $r_{33}$ | -0.9281 | -0.9286 | $0.054 \%$ |
| $r_{13}$ | 0.3714 | 0.3704 | $0.27 \%$ | $t_{\mathrm{x}}$ | 0 | 0 | 0 |
| $r_{21}$ | 1 | 1 | 0 | $t_{\mathrm{y}}$ | 0 | 0 | 0 |
| $r_{22}$ | 0 | 0 | 0 | $t_{\mathrm{z}}$ | 545.3828 | 539.516 | $1.09 \%$ |
| $r_{23}$ | 0 | 0 | 0 | $\beta_{1}$ | $20.0124^{\circ}$ | $20^{\circ}$ | $0.062 \%$ |
| $r_{31}$ | 0 | 0 | 0 | $\beta_{2}$ | $15.0322^{\circ}$ | $15^{\circ}$ | $0.21 \%$ |

In the actual experiment, we also use the orthogonal Gray code and this paper's methbd to measure the plane model, the measurement error is shown in Table 4.

Table 4. The Measured Data Comparjson

|  | Standard value/mm | Orthogonal Gray code |  | Orthogonal Gray code and trapezoid phase shift code |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Measured value/mm | Error/mm | Measured value/mm) | Error/mm |
| 1 | 50 | 49.440 | -0.560 | 49.950 | -0.050 |
| 2 | 50 | 50.116 | 0.116 | 50.006 | 0.006 |
| 3 | 50 | 50.656 | 0.656 | 50.042 | 0.042 |
| 4 | 50 | 49.106 | -0.894 | 49.913 | -0.087 |
| 5 | 50 | 50.445 | 0.445 | 50.020 | 0.020 |
| 6 | 50 | 50.210 | 0.220 | 50.004 | 0.004 |

According to Table 4 the accuracyof ysing the onthogonal Gray code is significantly lower than the method of this paper, bedause the trapezaid phase shift code is continuous coding, the corner point extraction accuracy is higher(Therefore, the accuracy of this paper's method is higher than the accuracy of asing the orthogonal Gray code.

## 4. Conclusion

In this paper, we conbine the Zhang's plane model calibration method with the orthogonal Gray code and trapezoid phase shift code, use the coding pattern to calculate the image coordinates on the imaging plane of the projector. The results show that the maximum relative error is $0.027 \%$ some parameters error is very small, it can even be ignored. This paper also used the Othogonal Gray code to compare with the method of this paper, the results show that the aecuracy of this paper's method is high.

## Acknowledgements

The article studies by the Education Department of Heilongjiang province science and technology research projects (12521069) funding.

## References

[1] R. J. Valkenburg and A. M. Melvor, "Accurate 3d measurement using a structured light system," Image and Vision Computing, vol. 16, no. 2, (1998), pp. 99-110.
[2] S. Zhang and P. S. Huang, "Novel method for structured light system calibration," Opt. Eng., vol. 45, no. 8, (2006), pp. 83601-83608.
[3] R. Q. Yang, "Robust and accurate surface measurement using structured light," IEEE Transactions on Instrumentation and Measurement, vol. 57, no. 6, (2008), pp. 1275-1280.
[4] S. Zhang, "High-resolution, real-time 3-D shape measurement," Thesis for the Doctor's Degree in Engingeering. Stony Brook University, (2005).
[5] X. Dai, Y. Zhong, C. Yuan and Y. Ma, "Research on projector calibration in one-camera 3-D measurement systems", Machinery Design \& Manufacture, (2008).
[6] Z. Li, Y. Shi, K. Zhong and C. Wang, "Projector calibration algorithm for the structured light measurement technique," OPTA OPTICA SINICA, vol. 29, no. 11, (2009), pp. 3061-3065.
[7] D. Zhang and L. Tang, "A method of calibrating projector based on projecting color circular marks," Computer \& Digital Engineering, vol. 39, no. 4, (2011), pp. 123-127.
[8] Z. Wang, J. Huang, J. Gao and Q. Xue, "Calibration of the structured light measurement system with bundle adjustment," Jounral of Mechanical Engineering, vol. 49, no. 8, (2013), pp. 32-40.
[9] L. Shan, "3D measurement method and technology based on combined time encoding structured light," Harbin University of Science and Technology, (2010).


