# Formulation of Decimation N Algorithm Expressions for Digital Signal Processor 

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#### Abstract

In this article, expression of algorithm of Decimation 人) or Multi-rate unnuing convolution algorithms operating filter output fast by processing Ranming Convolution filter with filter bank is reviewed according to each value of $n(n>=3)$ gnd then the part which is changed in regularly with value of $n$ is expressed as an algorithm.


Keywords: Decimation, Multi-rate running convolutinnalgorithm, FFT (Fast Fourier Transform), DSP (Digital Signal Processar)

## 1. Introduction

For fast algorithms reducing complexity of FIR filter operation, there are two methods: indirect method of operation frequencies inçuding TTF (Fast Fourier Transformation) and direct method of reducing operation in tupe [1]. However, fast algorithms are not widely used because its hardware design requires pipeline structure which demands large area for the design and it is not effective in software design with DSP (Digital Signal Processor). That is, reduction in operations of these fast algorithms is the result of inefficient structure with low effectiveness [2, 3]. Nevertheless, if it can be designed effectively with process using MAC (Multiplication and Accumulation), reduction of operations will be significant. Accordingly, direct methods of fast agorithms reducing operations of MAC have been developed [4, 5]. And fast algorithm asing pseudocirculant matrix with indirect method FFT was also presented.

In this article, expression of algorithm of Decimation N of Multi-rate running convolution algorithms operating filter output fast by processing Running Convolution filter with filter'bank is reviewed according to each value of $n(n>=3)$ and then the part which is changed regularly with value of n is expressed as an algorithm.

## 2. Deémation Algorithm

Generally, filter is expressed in frequencies as follows:

$$
\begin{equation*}
Y(\mathrm{z})=H(\mathrm{z}) X(\mathrm{z}) \tag{1}
\end{equation*}
$$

In this formula, $Y(\mathrm{z}), H(\mathrm{z}), X(\mathrm{z})$ are output signal $y[n]$, impulse response $h[n]$ and z transform of input signal $x[n]$, respectively[6-9].

### 2.1. The Algorithm of Decimation 3

Here, in case of $n=3$, it is decomposed with polyphase and then the subsequent formulas are reviewed.

$$
\begin{align*}
& X(z)=X_{0}\left(z^{3}\right)+z^{-1} X_{1}\left(z^{3}\right)+z^{-2} X_{2}\left(z^{3}\right) \\
& Y(z)=Y_{0}\left(z^{3}\right)+z^{-1} Y_{1}\left(z^{3}\right)+z^{-2} Y_{2}\left(z^{3}\right) \\
& H(z)=H_{0}\left(z^{3}\right)+z^{-1} H_{1}\left(z^{3}\right)+z^{-2} H_{2}\left(z^{3}\right) \tag{2}
\end{align*}
$$

When the polyphase-decomposed formula (2) is substitute into the formula (1), the following relational expression in the $z$ is shown. Then, if expressed with omitting $\left(z^{3}\right)$ of the entire polynomial expressions, it is the formula (3) below.

$$
\begin{aligned}
Y_{0}+z^{-1} Y_{1}+z^{-2} Y_{2}= & {\left[H_{0} X_{0}+z^{-3} H_{2} X_{1}+z^{-3} H_{1} X_{2}\right] } \\
& +z^{-1}\left[H_{1} X_{0}+H_{0} X_{1}+z^{-3} H_{2} X_{2}\right] \\
& +z^{-2}\left[H_{2} X_{0}+H_{1} X_{1}+H_{0} X_{2}\right]
\end{aligned}
$$

This formula can be expressed with matrix like the formula (4) below.

$$
\left[\begin{array}{l}
Y_{0}  \tag{4}\\
Y_{1} \\
Y_{2}
\end{array}\right]=\left[\begin{array}{rrr}
H_{0} & z^{-3} H_{2} & z^{-3} H_{1} \\
H_{1} & H_{0} & z^{-3} H_{2} \\
H_{2} & H_{1} & H_{0}
\end{array}\right]+\left[\begin{array}{r}
X_{0} \\
X_{1} \\
X_{2}
\end{array}\right]
$$

The right side of this formula is called the pseudocirculant matrix and it can be expressed with 9 sub-filters.

In order to reduce number of sub-filters used, $Y_{0}, Y_{1}$, and $Y_{2}$ of the formula (4) are processed respectively as f610ws:

$$
\begin{align*}
Y_{0} & =H_{0} X_{0}+z^{-3} H_{2} X_{1}+z^{-3} H_{1} X_{2} \\
& \left.=H_{0} X_{0}-z^{-3} H_{1} X_{1}-z^{-3} H_{2} X_{2}+z^{-3} H_{1}+H_{2}\right)\left(X_{1}+X_{2}\right)  \tag{5a}\\
Y_{1} & =H_{1} X_{0}+H_{0} X_{1}+z^{-3} H_{2} X_{2} \\
& \left.=-H_{0} X_{0}-H_{1} X_{1}+z^{-3} H_{2} X_{2}+{ }^{( } H_{0}+H_{1}\right)\left(X_{0}+X_{1}\right)  \tag{5b}\\
&
\end{align*}
$$

When processing with the formulas (5a), (5b) and (5c), the formula can be calculated by asing 6 filters including $H_{0}, H_{1}, H_{2}, H_{0}+H_{1}, H_{1}+H_{2}, H_{0}+H_{2}$ and so on. If the given filter is a FIR filter with 60 tabs, it needs 60 multiplication operations. Of the formulas (5a), (5b) and (5c) are used, 6 different filters have each 20 tabs and 120 multiplication operations are required in total. However, one third of them are operated in low speed and 40 multiplication operations per sample are required $[10,11]$. Therefore, if the proposed algorithm is used, number of multiplication operations is reduced from 60 to 40. The formulas (5a), (5b) and (5c) can be expressed by using Toom-Cook Algorithm as follows [12]:

$$
\left[\begin{array}{l}
Y_{0}  \tag{6}\\
Y_{1} \\
Y_{2}
\end{array}\right]=C_{3}\left\{A_{3}\left[\begin{array}{l}
H_{0} \\
H_{1} \\
H_{2}
\end{array}\right] * A_{3}\left[\begin{array}{c}
X_{0} \\
X_{1} \\
X_{2}
\end{array}\right]\right\}
$$

In this formula, $*$ stands for inner product and $A_{3}$ and $C_{3}$ are as follows, respectively:

$$
A_{3}=\left[\begin{array}{lll}
1 & 0 & 0  \tag{7}\\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 1
\end{array}\right] \quad C_{3}=\left[\begin{array}{cccccc}
1 & -z^{-3} & -z^{-3} & 0 & 0 & z^{-3} \\
-1 & -1 & z^{-3} & 1 & 0 & 0 \\
-1 & 1 & -1 & 0 & 1 & 0
\end{array}\right]
$$

Number of rows in $A_{3}$ is that of sub-filters used and $C_{3}$ Stands for combination of sub-filters for expressing output.

The structure of filter by the formula (6) and (7) is like Figure 1(a)

(b) Structure with inserting a sampler

(a) Basic structure, (b) Structure with inserting a sampler, (c) Final structure

In Figure 1 (a), $Y_{0}\left(\mathrm{z}^{3}\right), Y_{1}\left(\mathrm{z}^{3}\right)$, and $Y_{2}^{2}\left(\mathrm{z}^{3}\right.$ are signals inserted with zero sample, down sampler 3 and up sampler 3 can be inserted like Figare $1^{\prime \prime}$ (b). And in Figure 1(b), filter of $z^{-3}$ and down sampler 3 can be exehangeable in position by using Noble Identity and it can induce an effective structure shown in the final Figure 1 (c). As seen in Figure 1 (c), inputted samples are reduced by one third of the speed by using down sampler and delay element and it is focused to delay element and up sampler one by one in the output.

The sub-filter $\tilde{A}_{3}$ except for matrix bllaterally symmetrical to the Identity matrix in $A_{3}$ of the formula (7) is as follows:


### 2.2. The Algorithm of Decimation

When the formula (1) is decomposed with polyphase of 4 , the relation of its input and output ean be expressed in pseudocirculant matrix as follows:

$$
\left[\begin{array}{c}
Y_{0}  \tag{9}\\
Y_{1} \\
Y_{2} \\
Y_{3}
\end{array}\right]=\left[\begin{array}{rrrr}
H_{0} & z^{-4} H_{3} & z^{-4} H_{2} z^{-4} H_{1} \\
H_{1} & H_{0} & z^{-4} H_{3} z^{-4} H_{2} \\
H_{2} & H_{1} & H_{0} z^{-4} H_{3} \\
H_{3} & H_{2} & H_{1} & H_{0}
\end{array}\right]\left[\begin{array}{c}
X_{0} \\
X_{1} \\
X_{2} \\
X_{3}
\end{array}\right]
$$

In the formula (9), $Y_{0}, Y_{1}, Y_{2}$ and $Y_{3}$ are processed as follows:

$$
\begin{align*}
& Y_{0}=H_{0} X_{0}+z^{-4} H_{3} X_{1}+z^{-4} H_{2} X_{2}+z^{-4} H_{1} X_{3} \\
& \left.\left.=H_{0} X_{0}-z^{-4} H_{1} X_{1}+z^{-4} H_{2} X_{2}-z^{-4} H_{3} X_{3}+z^{-4} H_{1}+H_{3}\right)^{( } X_{1}+X_{3}\right) \tag{10a}
\end{align*}
$$

$Y_{1}=H_{1} X_{0}+H_{0} X_{1}+z^{-4} H_{3} X_{2}+z^{-4} H_{2} X_{3}$

$$
\left.\left.\left.=-H_{0} X_{0}-H_{1} X_{1}-z^{-4} H_{2} X_{2}-z^{-4} H_{3} X_{3}+{ }^{( } H_{0}+H_{1}\right)^{( } X_{0}+X_{1}\right)+z^{-4}\left(H_{2}+H_{3}\right)^{( } X_{2}+X_{3}\right)(10 \mathrm{~b})
$$

$$
\begin{align*}
Y_{2} & =H_{2} X_{0}+H_{1} X_{1}+H_{0} X_{2}+z^{-4} H_{3} X_{3} \\
& \left.\left.=-H_{0} X_{0}+H_{1} X_{1}-H_{2} X_{2}+z^{-4} H_{3} X_{3}+H_{0}+H_{2}\right)^{( } X_{0}+X_{2}\right) \tag{10c}
\end{align*}
$$

$$
\begin{align*}
Y_{3} & =H_{3} X_{0}+H_{2} X_{1}+H_{1} X_{2}+H_{0} X_{3} \\
& \left.\left.\left.\left.=-H_{0} X_{0}-H_{1} X_{1}-H_{2} X_{2}-H_{3} X_{3}+{ }^{( } H_{0}+H_{3}\right)^{( } X_{0}+X_{3}\right)+{ }^{( } H_{1}+H_{2}\right)^{( } X_{1}+X_{2}\right) \tag{10d}
\end{align*}
$$

The formula (10) can be expressed by using Toom-Cook Algorithm as follows:

$$
\left[\begin{array}{c}
Y_{0} \\
Y_{1} \\
Y_{2} \\
Y_{3}
\end{array}\right]=C_{4}\left\{A_{4}\left[\begin{array}{c}
H_{0} \\
H_{1} \\
H_{2} \\
H_{3}
\end{array}\right] * A_{4}\left[\begin{array}{c}
X_{0} \\
X_{1} \\
X_{2} \\
X_{3}
\end{array}\right]\right\}
$$

In this formula, $A_{4}$ and $C_{4}$ are as follows, respectively.
$A_{4}=\left[\begin{array}{lllll}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1\end{array}\right]$
The sub-filter $\tilde{A} \mathbb{A}_{\text {except }}$ for mantix bilaterally symmetrical to the Identity matrix in $A_{4}$ of the formula (12) is as follows:


### 2.3. The Algorithm of Decimation 5

When the formula (1) is decomposed with polyphase of 5, the relation of its input and output can be expressed in pseudocirculant matrix as follows:

$$
\left[\begin{array}{c}
Y_{0}  \tag{14}\\
Y_{1} \\
Y_{2} \\
Y_{3} \\
Y_{4}
\end{array}\right]=\left[\begin{array}{rrrrr}
H_{0} & z^{-5} H_{4} & z^{-5} H_{3} & z^{-5} H_{2} & z^{-5} H_{1} \\
H_{1} & H_{0} & z^{-5} H_{4} & z^{-5} H_{3} & z^{-5} H_{2} \\
H_{2} & H_{1} & H_{0} & z^{-5} H_{4} & z^{-5} H_{3} \\
H_{3} & H_{2} & H_{1} & H_{0} & z^{-5} H_{4} \\
H_{3} & H_{2} & H_{1} & H_{0}
\end{array}\right]\left[\begin{array}{c}
X_{0} \\
X_{1} \\
X_{2} \\
X_{3} \\
X_{4}
\end{array}\right]
$$

In the formula (9), $Y_{0}, Y_{1}, Y_{2}, Y_{3}$ and $Y_{4}$ are processed as follows:

$$
\begin{align*}
Y_{0}= & H_{0} X_{0}+z^{-5} H_{4} X_{1}+z^{-5} H_{3} X_{2}+z^{-5} H_{2} X_{3}+z^{-5} H_{1} X_{4} \\
= & H_{0} X_{0}-z^{-5} H_{1} X_{1}-z^{-5} H_{2} X_{2}-z^{-5} H_{3} X_{3}-z^{-5} H_{4} X_{4} \\
& \left.+z^{-5}\left(H_{1}+H_{4}\right)\left(X_{1}+X_{4}\right)+z^{-5}\left(H_{2}+H_{3}\right)^{( } X_{2}+X_{3}\right) \tag{15a}
\end{align*}
$$

$$
\begin{align*}
Y_{2}= & H_{2} X_{0}+H_{1} X_{1}+H_{0} X_{2}+z^{-5} H_{4} X_{3}+z^{-5} H_{3} X  \tag{15b}\\
= & -H_{0} X_{0}+H_{1} X_{1}-H_{2} X_{2}-z^{-5} H_{3} X_{3}-z^{-5} H_{4} X_{4} \\
& \left.\left.+\left(H_{0}+H_{2}\right)^{( } X_{0}+X_{2}\right)+z^{-5}\left(H_{3}+H_{4}\right) X_{3}+X_{4}\right)
\end{align*}
$$

$$
Y_{3}=H_{3} X_{0}+H_{2} X_{1}+H_{1} X_{2}+H_{0} X_{3}+z^{-5} H_{4} X_{4}
$$

$$
=-H_{0} X_{0}-H_{1} X_{1}-H_{2} X_{2}-H_{3} X_{3}+z^{-5} H_{4} X_{4}
$$

$$
\begin{equation*}
+\left(H_{0}+H_{3}\right)\left(X_{0}+X_{3}\right)+\left(H_{1}+H_{2}\right)\left(X_{1}+X_{2}\right) \tag{15~d}
\end{equation*}
$$



$$
\begin{align*}
Y_{4}= & H_{4} X_{0}+H_{3} X_{1}+H_{2} X_{2}+H_{1} X_{3}+H_{0} X_{4} \\
= & -H_{0} X_{0}-H_{1} X_{1}+H_{2} X_{2}-H_{3} X_{3}-H_{4} X_{4} \\
& \left.\left.\left.+\left(H_{0}+H_{4}\right)^{( } X_{0}+X_{4}\right)+{ }^{( } H_{1}+H_{3}\right)^{( } X^{-}+X_{3}\right) \tag{15e}
\end{align*}
$$

The formula (15) can be expressed by using Toom-Cook Algorithm as follows:


In this formula, $A_{5}$ and $G_{5}$ are as follows, respectively:


The sub-filter $\tilde{A}_{5}$ except for matrix bilaterally symmetrical to the Identity matrix in $A_{5}$ of the formula (12) is as follows:

$$
\tilde{A_{5}}=\left[\begin{array}{llllll}
1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{array}\right]
$$

### 2.4. The Algorithm of Decimation $N$

When the formula (1) is decomposed with polyphase of N , the relation of its input and output can be expressed in pseudocirculant natrix as follows:

In the formula (9), in frder to express $Y_{0}, Y_{1}, Y_{2}, Y_{3} \ldots$ and $Y_{\mathrm{N}-1}$, the following precedent expressions atequired.
$Y_{\mathrm{m}}=\mathrm{e}_{0} H_{0} \mathrm{X}_{0}+\mathrm{e}_{1} H_{1} X_{\mathrm{N}}++\mathrm{e}_{\mathrm{N}-1} H_{\mathrm{N}-1 \mathrm{H}_{\mathrm{N}}}+\mathrm{k}_{1}\left(H_{\mathrm{bl} 1}+H_{\mathrm{c} 2}\right)\left(X_{\mathrm{bl}}+X_{\mathrm{cl}}\right)+\ldots+\mathrm{k}_{\mathrm{j}}\left(H_{\mathrm{bj}}+H_{\mathrm{cj}}\right)\left(X_{\mathrm{bj}}+X_{\mathrm{c} j}\right)$
In the expression 1: $\mathrm{kj} \mathrm{H}(\mathrm{j} \mathrm{j}+\mathrm{Hcj})(\mathrm{Xbj}+\mathrm{Xcj})$, the range of value of $j$ is as follows:
If N is an odd number.

- $j=(N-1) / 2$

If N is anteren number,

- If ris án even number or $0, \mathrm{j}=\mathrm{N} / 2-1$
-firm is an odd number, $\mathrm{j}=\mathrm{N} / 2$
The expression 2; Of items in the formula of Ym (here, m is $0 \sim \mathrm{~N}-1$ ), value of $e a$ of eaHaXa(here, a is $0 \sim \mathrm{~N}-1$ ) can predict whether it is a positive number or a negative number. (But, in $\mathrm{kj}(\mathrm{Hbj}+\mathrm{Hcj})(\mathrm{Xbj}+\mathrm{Xcj})$, value of $k j$ is always plus)

In case N is an odd number,

- If $m$ is an even number or 0 , value of $a$ is plus in case of $m / 2$ only and the rest is minus.
- If m is an odd number, value of a is a is plus in case of $(\mathrm{N}-1) / 2+(\mathrm{m}+1) / 2$ and the rest is minus.

In case N is an even number,

- If $m$ is an even number or 0 , value of $a$ is plus in two cases of $m / 2$ and $N / 2+m / 2$ and the rest is minus.
- If m is an odd number, value of a is always minus.

The expression 3: Of items in the formula $Y_{\mathrm{m}}($ here, m is $0 \sim \mathrm{~N}-1$ ), combination of b and c in the formula $\left(H_{\mathrm{b}}+H_{\mathrm{c}}\right)\left(X_{\mathrm{b}}+X_{\mathrm{c}}\right)$ (here, $\left.\mathrm{b}<\mathrm{c}\right)$ is as follows:

Step 1: In case of $Y_{0}$,

- First, If ea is plus in the expression 1 , and then it will be excluded.
- For b , starting from the smallest and for c , starting from the largest, the combination is made.
- The formula $\mathrm{b}+\mathrm{c}=\mathrm{N}$ is made.

Step 2: In case of $Y_{\mathrm{m}}(0<\mathrm{m}<\mathrm{n})$

- First, if ea is plus in the expression 2, and then it will be expluded.
- There is always the combination with $b=0$ and $c=m . S$
- Among the rest, starting from thesmallest of $b$ seek the combination of $b+c=m$.
- If there are remaining yalues, among tithen, the smallest value is b. And seek combinations of $\mathrm{b}+\mathrm{c}=\mathrm{m}+\mathrm{N}$ and then every combination can be found.
The expression 4: Of items in the expression $Y_{\mathrm{m}}$ (here, m is $0 \sim \mathrm{~N}-1$ ), $z^{-\mathrm{N}}$ on values of $\mathrm{e}_{\mathrm{a}}$ and $\mathrm{k}_{\mathrm{j}}$ in the formulas e $\left(H_{3} X_{\mathrm{a}}(\right.$ here $, \mathrm{is}, 0 \sim \mathrm{~N}-1)$ and $\mathrm{k}_{\mathrm{j}}\left(H_{\mathrm{b}}+H_{\mathrm{c}}\right)\left(X_{\mathrm{b}}+X_{\mathrm{c}}\right)($ here, $\mathrm{b}<\mathrm{c})$ respectively are as follotes:
- $\mathrm{e}_{\mathrm{a}} H_{\mathrm{a}} X_{\mathrm{a}}:$ If $a<\mathrm{m}, z^{-\mathrm{N}}$ exists ime $\mathrm{a}_{\mathrm{a}}$.
- $\mathrm{k}_{\mathrm{j}}\left(H_{\mathrm{b}}+H_{\mathrm{c}}\left(X_{\mathrm{b}}+X_{\mathrm{c}}\right) \cdot\right.$ If $\mathrm{b} 1>\mathrm{m}, z^{-\mathrm{N}}$ exists in $\mathrm{k}_{\mathrm{j}}$

Example: Calculate a fogmula of $Y_{4}$ if $\mathrm{N}=7$.
For basic expressions, there are three expressions of $\left(H_{\mathrm{b}}+H_{\mathrm{c}}\right)\left(X_{\mathrm{b}}+X_{\mathrm{c}}\right)$ since $\mathrm{j}=(7-$ 1) $/ 2=3$ according to the expression 1 mentioned above.

Therefore, the expression of $Y_{4}$ can be expressed as follows:

$$
\begin{align*}
Y_{4} & =\mathrm{e}_{0} H_{1} \mathrm{X}_{9}+\mathrm{e}_{1} H_{1} X_{1}+\mathrm{e}_{2} H_{2} X_{2}+\mathrm{e}_{3} H_{3} X_{3}+\mathrm{e}_{4} H_{4} X_{4}+\mathrm{e}_{5} H_{5} X_{5}+\mathrm{e}_{6} H_{6} X_{6} \\
& 4 \mathrm{k}\left(\mathrm{H}\left(H_{\mathrm{b} 1}+H_{\mathrm{c} 2}\right)\left(X_{\mathrm{b} 1}+X_{\mathrm{c} 1}\right)+\mathrm{k}_{2}\left(H_{\mathrm{b} 2}+H_{\mathrm{c} 2}\right)\left(X_{\mathrm{b} 2}+X_{\mathrm{c} 2}\right)+\mathrm{k}_{3}\left(H_{\mathrm{b} 3}+H_{\mathrm{c} 3}\right)\left(X_{\mathrm{b} 3}+X_{\mathrm{c} 3}\right)\right. \tag{a1}
\end{align*}
$$

By applying the expression 2 to the formula a1, value of plus or minus signs of $e_{0}, e_{1}$, $e_{2}, e_{3}, e_{4}, e_{5}$, and $e_{6}$, are calculated as follows:

$$
\begin{align*}
Y_{4}= & -\mathrm{e}_{0} H_{0} \mathrm{X}_{0}-\mathrm{e}_{1} H_{1} X_{1}+\mathrm{e}_{2} H_{2} X_{2}-\mathrm{e}_{3} H_{3} X_{3}-\mathrm{e}_{4} H_{4} X_{4}-\mathrm{e}_{5} H_{5} X_{5}-\mathrm{e}_{6} H_{6} X_{6} \\
& +\mathrm{k}_{1}\left(H_{\mathrm{b} 1}+H_{\mathrm{c} 2}\right)\left(X_{\mathrm{b} 1}+X_{\mathrm{c} 1}\right)+\mathrm{k}_{2}\left(H_{\mathrm{b} 2}+H_{\mathrm{c} 2}\right)\left(X_{\mathrm{b} 2}+X_{\mathrm{c} 2}\right)+\mathrm{k}_{3}\left(H_{\mathrm{b} 3}+H_{\mathrm{c} 3}\right)\left(X_{\mathrm{b} 3}+X_{\mathrm{c} 3}\right) \tag{a2}
\end{align*}
$$

By applying the expression 3 to the formula a2, combinations of $\mathrm{bj}+\mathrm{cj}$ in $\left(H_{\mathrm{bj}}+\right.$ $\left.H_{\mathrm{cj}}\right)\left(X_{\mathrm{bj}}+X_{\mathrm{cj}}\right)$ are calculated as follows:

$$
\begin{align*}
Y_{4}= & -\mathrm{e}_{0} H_{0} \mathrm{X}_{0}-\mathrm{e}_{1} H_{1} X_{1}+\mathrm{e}_{2} H_{2} X_{2}-\mathrm{e}_{3} H_{3} X_{3}-\mathrm{e}_{4} H_{4} X_{4}-\mathrm{e}_{5} H_{5} X_{5}-\mathrm{e}_{6} H_{6} X_{6} \\
& +\mathrm{k}_{1}\left(H_{0}+H_{4}\right)\left(X_{0}+X_{4}\right)+\mathrm{k}_{2}\left(H_{1}+H_{3}\right)\left(X_{1}+X_{3}\right)+\mathrm{k}_{3}\left(H_{5}+H_{6}\right)\left(X_{5}+X_{6}\right) \tag{a3}
\end{align*}
$$

By applying the expression 4 to the formula 3, if values of $k_{1} \sim k_{3}$ are calculated, the final expression of $Y_{4}$ is as follows:

$$
\begin{align*}
Y_{4}= & -H_{0} X_{0}-H_{1} X_{1}+H_{2} X_{2}-H_{3} X_{3}-H_{4} X_{4}-z^{-7} H_{5} X_{5}-z^{-7} H_{6} X_{6} \\
& +\left(H_{0}+H_{4}\right)\left(X_{0}+X_{4}\right)+\left(H_{1}+H_{3}\right)\left(X_{1}+X_{3}\right)+z^{-7}\left(H_{5}+H_{6}\right)\left(X_{5}+X_{6}\right) \tag{a4}
\end{align*}
$$

$\tilde{A}_{\mathrm{N}}$ except for matrix bilaterally symmetrical to the Identity matrix in $A_{\mathrm{N}}$ can be calculated as follows:

First, number of Nx m. the matrix $\mathrm{a}_{\mathrm{ik}}(\mathrm{i}=1,2,3, \ldots, \mathrm{~m}$ and $\mathrm{k}=1,2,3, \ldots, \mathrm{~N})$ is as follows

$$
\begin{aligned}
& a_{11}: a_{12}: a_{13}: \ldots: a_{1 N} \\
& a_{21}: a_{22}: a_{23}: \ldots: a_{2 N} \\
& \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
& a_{m 1}: a_{m 2}: a_{m 3}: \ldots: a_{m N}
\end{aligned}
$$



Here, $\mathrm{m}={ }_{\mathrm{N}} C_{2}-\llcorner\mathrm{N} / 2\lrcorner$
The condition is one row must contain two and the rest of them have value of 0 .
Every value of the matrix is initiatized to $0, b$
$\mathrm{i}=1, \mathrm{k}=1$
$j=k$
if $(\mathrm{k}+\mathrm{j})=\mathrm{N}$ then goto 1 N
$\mathrm{a}_{\mathrm{ik}}=1$
$j=j+1$
$\mathrm{a}_{\mathrm{ij}}=1$
if $j=N$ thengote 11
$i=i+$

$k=k+1$
$j=k$
if $k=N$ then $e n$
goto 4
And in values of $P_{\mathrm{N}}$ and $Q_{\mathrm{N}}$ can be divided. For value of row of the first 1 from eachrou in the $\tilde{A}_{\mathrm{N}}$,

- in ease N is an even number, if it is same or less than $\mathrm{N} / 2$, the row is PN and other case is QN .
- in case N is an odd number, if it is same or less than $(\mathrm{N}+1) / 2$, the row is $P_{\mathrm{N}}$ and other case is $Q_{\mathrm{N}}$.


## 3. Conclusion

The expression of decimation N algorithm can be fixed as the expression mentioned above. Just there is difference in number of sub-filters in number of rows of $\tilde{A}_{\mathrm{N}}$ but there is no difference in number of rows of $\tilde{A}_{\mathrm{N}}$. It is difference resulted from processing the expression in $\tilde{A}_{\mathrm{N}}$. And there is difference in values of matrix in $\tilde{A}_{5}$ but it can change
according to order of items in the expressions and there is no difference in the result. In case of using $\tilde{A}_{5}$ suggested here rather than that of from $\tilde{A}_{5}$, there is a constant regulation and values of $\tilde{A}_{\mathrm{N}}$ can be easily calculated through the algorithms suggested here without necessity of calculation of each expression.

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