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Design of Almost Perfect Complementary Sequence Pairs

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Abstract

Sequences with very low out-of-phase auto-correlation and cross-correlation function are widely used for synchronization in mobile communication and multimedia systems where reliable data transmissions are required. For optimum detection, synchronization sequences usually have out-of-phase correlation values that are very low or zero. However sequences with ideal correlation function are very rare. Thus, this paper presents a new algorithm to generate almost perfect complementary sequence pairs with ideal bipolar correlation. Using the auto-correlation properties of these synchronization sequences, we can obtain a correlation circuit whose output contains two peak values equal in magnitude and opposite in polarity at zero and middle shifts. This correlation circuit can be used to double-check the synchronization timing and thus reduce the synchronization search time.

Keywords: *Auto-correlation, cross-correlation, almost perfect complementary sequence pair, bipolar correlation*

1. Introduction

Designing sequences with special correlation properties is a very important issue in mobile and multimedia communication systems where reliable data transmissions are required. Sequences with very low out-of-phase auto-correlation and cross-correlation function are widely used for synchronization in these systems [1-9]. When a synchronization sequence with special correlation properties is used, it is periodically inserted in the bit stream to correctly time-align the transmitter and receiver by using correlations. For optimum detection, synchronization sequences usually have out-of-phase correlation values that are very low or zero. If all the out-of-phase auto-correlation coefficients of a sequence are zero, then the sequence is called a perfect auto-correlation sequence [10]. However, (0,1,1,1) is the only perfect auto-correlation sequence. Chu proposed polyphase codes with perfect auto-correlation [11], but when taking hardware complexity into consideration, binary codes are preferred. Golay proposed pairs of complementary binary sequences that have the sum of their aperiodic auto-correlation functions equal to zero for all time shifts except zero [12]. Pairs of aperiodic binary complementary sequences have been known only for relatively few lengths. More recently, Luke proposed pairs of odd-periodic binary complementary sequences, which can be generated using q-ary m-sequences for many lengths [13]. Now consider another sequence with ideal bipolar correlation properties. Such a sequence has two maximum values equal in magnitude and opposite in polarity at the zero and middle shifts. Furthermore, all the out-of-phase coefficients of the sequence are zero. Thus the sequence is optimal for synchronization since its ideal bipolar

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correlation properties help double-check the synchronization timing and improve the synchronization performance [14 - 15]. However (0,0,1,1) is the only one cyclic distinct sequence with ideal bipolar correlation properties. In this paper, we present an algorithm to generate an almost complementary sequence pair (APCSP) with ideal bipolar correlation properties. By using the proposed iterative algorithm, APCSPs of length $N = 2n$ can be easily generated by using one APCSP with a period n that is a multiple of 4. The sidelobe cancellation effect between the auto-correlation functions of an APCSP is an important factor to consider when designing the sequence. The new proposed algorithm is the extended version of the algorithm introduced in [14], which presented a very limited number of APCSPs with lengths of 8, 16, and 32.

2. Definitions

Let $\mathbf{S} = \{s_i\}$ be a binary sequence of period n and let T denote a cyclic shift left operator such that $T\mathbf{S} = (s_1, s_2, \cdots, s_0)$. For two integers i and j, $T^i\mathbf{S} = T^j\mathbf{S}$ if $i \equiv j(\mathrm{mod}\, n)$. The periodic auto-correlation function is defined by

$$R(\tau) = \sum_{t=0}^{n-1} (-1)^{s_t + s_{(t+\tau)\bmod n}}, \quad 0 \le \tau \le n-1, \tag{1}$$

where $\bmod n$ denotes modulo n and $s_t + s_{(t+\tau)\bmod n}$ is computed modulo 2. We call $R(0)$ the in-phase auto-correlation value and $R(\tau)$ $(\tau \ne 0)$ the out-of-phase auto-correlation values. The out-of-phase auto-correlation values are also called sidelobes. Now, let $\|\mathbf{S}\|$ be the number of 1's in $\mathbf{S}$. The periodic auto-correlation function of $\mathbf{S}$ is $n - 2\|\mathbf{S} + T^\tau\mathbf{S}\|$ and the number of disagreements between $\mathbf{S}$ and $T^\tau\mathbf{S}$ is $\|\mathbf{S} + T^\tau\mathbf{S}\| = 2K$, where the non-negative integer K is given by

$$K = \|\mathbf{S}\| - (\text{number of coincident 1's between } \mathbf{S} \text{ and } T^\tau\mathbf{S}). \tag{2}$$

Then, the auto-correlation function becomes [16]

$$R_s(\tau) = n - 4 \cdot K. \tag{3}$$

Let $\mathbf{S} = \{s_i\}$ be a binary sequence of period n. Periodic repetition but with reversal of the signs of alternate periods gives the odd-periodic sequence $\hat{\mathbf{S}} = \{\hat{s}_i\}$. This means that

$$\hat{s}_i = \begin{cases} s_i, & 0 \le i \le n-1 \\ (s_{i \bmod n} + 1) \bmod 2, & n \le i \le 2n-1 \end{cases}. \tag{4}$$

The odd-periodic auto-correlation function is defined in [13] as

$$\hat{R}(\tau) = \sum_{t=0}^{n-1} (-1)^{s_t + \hat{s}_{(t+\tau)}}. \tag{5}$$

Let $\mathbf{S}_1 = \{s_{1,i}\}$ and $\mathbf{S}_2 = \{s_{2,i}\}$ be binary sequences of length *n*. Two sequences are said to be "cyclically equivalent" if there exists an integer *i* such that $\mathbf{S}_1 = T^i \mathbf{S}_2$; otherwise, they are said to be "cyclically distinct." The periodic cross-correlation function between the two sequences is defined as

$$R_{1,2}(\tau) = \sum_{t=0}^{n-1} (-1)^{s_{1,t} + s_{2,(t+\tau) \bmod n}} \quad . \tag{6}$$

3. Almost Perfect Complementary Sequence Pairs

3.1. Correlation properties

A sequence pair $(\mathbf{S}_1, \mathbf{S}_2)$ is said to be an almost perfect complementary sequence pair (APCSP) with an ideal bipolar correlation if the pair satisfies the following correlation property:

$$R_1(\tau) + R_2(\tau) = \begin{cases} 2n, & \tau = 0 \\ -2n, & \tau = n/2 \\ 0, & \text{elsewhere} \end{cases} . \tag{7}$$

The sidelobe cancellation effect, except at the middle shift $R_1(\tau) + R_2(\tau) = 0, \tau \neq 0, n/2$, is an important factor to consider when designing the sequence. Lemma 1 and lemma 2 in [14] are closely related to the sidelobe cancellation and bipolar correlation properties, respectively.

Lemma 1: Let $\mathbf{A} = \{a_i\}$ be a binary sequence of even period. Let $\mathbf{S}_1 = \{s_{1,i}\}$ and $\mathbf{S}_2 = \{s_{2,i}\}$ be sequences obtained by decimating $\mathbf{A}$ and $T\mathbf{A}$ by 2, respectively. Then, the auto-correlation function of $\mathbf{A}$ is represented by

$$R(2\tau) = R_1(\tau) + R_2(\tau), \tag{8}$$

$$R(2\tau + 1) = R_{1,2}(\tau) + R_{2,1}(\tau + 1). \tag{9}$$

Lemma 2: Let $\mathbf{S} = \{s_i\}$ be a binary sequence of length $n = 2 \cdot l$ with $s_i = s'_{(i+l)}$. Then, we have

$$R(\tau) = -R(\tau + l) \text{ for all } \tau, \tag{10}$$

where $s'_i = (s_i + 1) \bmod 2$.

From lemma 2 we can see that if $s_i = s'_{(i+l)}$ for $\mathbf{S} = \{s_i\}$ of even period $n = 2 \cdot l$, then the auto-correlation function of the sequence has two maximum values equal in magnitude and opposite in polarity at the zero and middle shifts. Furthermore, if such a sequence has out-of-phase coefficients with a value of zero, then the sequence has ideal bipolar correlation properties. From the definition of the auto-correlation function we can observe the following property [16].

Property 1: The following sequences have the same auto-correlation function:

1) a cyclically shifted version of the original sequence

2) a time-inversed version of the original sequence and its cyclically shifted version.

3.2. Fundamental principles

Let $\mathbf{S}_1$ and $\mathbf{S}_2$ be binary sequences of even period $n = 2 \cdot l$ with $s_{1,i} = s'_{1,(i+l)}$ and $s_{2,i} = s'_{2,(i+l)}$ and let them have negative peak auto-correlation values at the middle shift in accordance with lemma 2. If $R_1(\tau) + R_2(\tau) = 0, \tau \neq 0, l$, then $(\mathbf{S}_1, \mathbf{S}_2)$ becomes the APCSP with ideal bipolar correlation. Now, assume that $(\mathbf{S}_1, \mathbf{S}_2)$ is an APCSP of length $n = 2 \cdot l$. Let $(\mathbf{S}_1, \mathbf{S}_2)$ be an element of $\mathbf{A} = \{a_i\}$ of double length $N = 2 \cdot n$ with $a_{2z} = s_{1,z}$ and $a_{2z+1} = s_{2,z}$. Since $s_{1,i} = s'_{1,(i+l)}$ and $s_{2,i} = s'_{2,(i+l)}$, we see that $a_i = a'_{i+n}, 0 \leq i \leq n-1$. Then, from equations (3), (8), and (9), we obtain $R(2\tau) = 0$ $(\tau \neq 0, n)$ and $R(2\tau+1) = N - 4K = 4l - 4K = \pm 4\lambda$, where λ is a non-negative integer. That is, the auto-correlation function of $\mathbf{A}$ becomes

$$R_{\mathrm{A}}(\tau) = \begin{cases} N, & \tau = 0 \\ -N, & \tau = n \\ 0, & \tau : \text{even}, \tau \neq 0, n \\ \pm 4\lambda, & \tau : \text{odd} \end{cases}. \tag{11}$$

If we define $\alpha = (a_0, a_1, \cdots, a_{n-1})$ which is the first half of $\mathbf{A}$, then

$$\hat{R}_\alpha(\tau) = \frac{1}{2} R_{\mathrm{A}}(\tau), 0 \leq \tau \leq n-1. \tag{12}$$

From property 1, we see that if $(\mathbf{S}_1, \mathbf{S}_2)$ is an APCSP of length *n*, then $(T^i\mathbf{S}_1, T^j\mathbf{S}_2)$ becomes another APCSP. Now, let $(T^i\mathbf{S}_1, T^j\mathbf{S}_2)$ be an element of $\mathbf{B} = \{b_i\}$ of double length $N = 2 \cdot n$ with $b_{2z} = s_{1,(z+i)\bmod n}$ and $b_{2z+1} = s_{2,(z+i)\bmod n}$. If the sum of two auto-correlation values for $\mathbf{A}$ and $\mathbf{B}$ at odd time shifts is zero, then

$$R_{\mathrm{A}}(\tau) + R_{\mathrm{B}}(\tau) = \begin{cases} 2N, & \tau = 0 \\ -2N, & \tau = n \\ 0, & \text{elsewhere} \end{cases} \tag{13}$$

and $(\mathbf{A}, \mathbf{B})$ becomes an APCSP of length $N = 2n$. Similarly, we can also apply this method to $(T^i\mathbf{S}_1, T^j\mathbf{S}_2^r)$, where $\mathbf{S}^r$ is the time-inversed version of the original sequence $\mathbf{S}$. If we define $\beta = (b_0, b_1, \cdots, b_{n-1})$ which is the first half of $\mathbf{B}$, the sum of odd-periodic auto-correlation functions of (α, β) becomes

$$\hat{R}_\alpha(m) + \hat{R}_\beta(m) = \begin{cases} n, & m = 0 \\ 0, & m \neq 0 \bmod n \end{cases}. \tag{14}$$

Thus, (α,β) forms a pair of odd-periodic complementary sequences.

(0,0,1,1) is the only one cyclic distinct sequence with ideal bipolar correlation properties. If we set $\mathbf{S}_1=\mathbf{S}_2=(0,0,1,1)$, then $(\mathbf{S}_1,\mathbf{S}_2)$ becomes the APCSP of period 4. The sequence $\mathbf{A}=(0,0,0,0,1,1,1,1)$ for $a_{2z}=s_{1,z}$ and $a_{2z+1}=s_{2,z}$ shows auto-correlation values of {8,4,0,-4,-8,-4,0,4}. Similarly, another APCSP $(\mathbf{S}_1,T^3\mathbf{S}_2)$ belongs to $\mathbf{B}=(0,1,0,0,1,0,1,1)$ whose auto-correlation values are {8,-4,0,4,-8,4,0,-4}. We can see that $(\mathbf{A},\mathbf{B})$ becomes the APCSP of length 8 since upon adding the two auto-correlation values, we obtain the ideal bipolar correlation values {16,0,0,0,-16,0,0,0} owing to the sidelobe cancellation, except at the middle shift. The auto-correlation values are presented in Table 1. The APCSP is written in the hexadecimal form. The * mark on an APCSP indicates that the sequences of the APCSP have the lowest out-of-phase auto-correlation coefficients, i.e., zeros and ± 4's, except at the middle shift. Such a sequence pair with the lowest out-of-phase auto-correlation coefficients is called the basic APCSP (B-APCSP). Otherwise it is called the extended APCSP (E-APCSP).

Table 1. Auto-correlation values of APCSPs of length 8

Auto-correlation values	Sequences
8 4 0 -4 -8 -4 0 4	0F*
8 -4 0 4 -8 4 0 -4	4B*

3.3. Generation method

From an APCSP $(\mathbf{S}_1,\mathbf{S}_2)$ of period *n*, we can recursively generate an APCSP $(\mathbf{A},\mathbf{B})$ of period $N=2\cdot n$ by using the following method:

Step 1: Choose an APCSP $(\mathbf{S}_1,\mathbf{S}_2)$ of length $n=2^{k-1}$, where $k\geq 3$.

Step 2: $\mathbf{A}=\{a_i\}$ is defined by $a_{2z}=u_{1,z}$ and $a_{2z+1}=v_{1,z}$,

, where $\mathbf{U}_1=\{u_{1,i}\}\in\{T^i\mathbf{S}_1\cup T^j\mathbf{S}_1, 0\leq i,j\leq n-1\}$

and $\mathbf{V}_1=\{v_{1,i}\}\in\{T^i\mathbf{S}_2\cup T^j\mathbf{S}_2, 0\leq i,j\leq n-1\}$.

Step 3: $\mathbf{B}=\{b_i\}$ is defined by $b_{2z}=u_{2,z}$ and $b_{2z+1}=v_{2,z}$

, where $\mathbf{U}_2=\{u_{2,i}\}\in\{T^i\mathbf{S}_1\cup T^j\mathbf{S}_1, 0\leq i,j\leq n-1\}$

and $\mathbf{V}_2=\{v_{2,i}\}\in\{T^i\mathbf{S}_2\cup T^j\mathbf{S}_2, 0\leq i,j\leq n-1\}$.

Step 4: If (13) is satisfied, then $(\mathbf{A},\mathbf{B})$ becomes an APCSP. Otherwise go to step 1.

Tables 2 and 3 show APCSPs obtained by using the new iterative algorithm. For example [0F,4B]* of length 8 generates [10EF,45BA]* and [41BE,14EB]* of length 16. Similarly [10EF,45BA]* of length 16 generates B-APCSPs and E-APCSPs of length 32. Using the same method, we can generate many E-APCSPs of lengths *N* = 64 and 128 from APCSPs with half lengths of *n* = 32 and 64, respectively. Since the sequences in the conventional method are limited to B-APCSPs whose auto-correlation function is (11) with $\lambda=0$ or $\lambda=1$, the complementary sequence pairs in [14] are a special case of B-APCSPs of lengths 8, 16, and 32 in Table 2. The sequences in the tables are all cyclically distinct, and sequences with the same auto-correlation values are excluded to reduce the number of APCSPs.

Table 2. APCSPs of *N* = 16, 32, 64 generated from BCSPs of *n* = 8, 16, 32, respectively

N = 16 from [0F,4B]*	[410A1664BEF5E99B,145F4331EBA0BCCE]
[10EF,45BA]*	[040A5330FBF5ACCF,515F0665AEA0F99A]
[41BE,14EB]*	[100B4661EFF4B99E,455E1334BAA1ECCB]
N = 32 from [10EF,45BA]*	[400F1325BFF0ECDA,155A4670EAA5B98F]
[0314FCEB,5641A9BE]*	[001E4634FFE1B9CB,554B1361AAB4EC9E]
[0651F9AE,5304ACFB]*	[005B1271FFA4ED8E,550E4724AAF1B8DB]
[1211EDEE,4744B8BB]	[014E4365FEB1BC9A,541B1630ABE4E9CF]
[4245BDBA,1710E8EF]	[051B0735FAE4F8CA,504E5260AFB1AD9F]
[1345ECBA,4610B9EF]	[144E1675EBB1E98A,411B4320BEE4BCDF]
[4715B8EA,1240EDBF]	[511A5375AEE5AC8A,044F0620FBB0F9DF]
[1654E9AB,4301BCFE]	[155B4331EAA4BCCE,400E1664BFF1E99B]
[5351ACAE,0604F9FB]	[554F0665AAB0F99A,001A5330FFE5ACCF]
[1351ECAE,4604B9FB]	[551E1334AAE1ECCB,004B4661FFB4B99E]
[4745B8BA,1210EDEF]	[545A4670ABA5B98F,010F1325FEF0ECDA]
[1714E8EB,4241BDBE]	[514B1360AEB4EC9F,041E4635FBE1B9CA]
[5651A9AE,0304FCFB]	[450E4720BAF1B8DF,105B1275EFA4ED8A]
[5344ACBB,0611F9EE]	[141B1620EBE4E9DF,414E4375BEB1BC8A]
[4710B8EF,1245EDBA]	[504E5221AFB1ADDE,051B0774FAE4F88B]
[1640E9BF,4315BCEA]	[411B4224BEE4BDDB,144E1771EBB1E88E]
[5301ACFE,0654F9AB]	[044F0230FBB0FDCF,511A5765AEE5A89A]
N = 64 from [0314FCEB,5641A9BE]*	[111E0261EEE1FD9E,444B5734BBB4A8CB]
[111E1221EEE1EDDE,444B4774BBB4B88B]	[445A0325BBA5FCDA,110F5670EEF0A98F]
[445A4225BBA5BDDA,110F1770EEF0E88F]	[114A0634EEB5F9CB,441F5361BBE0AC9E]
[114B0234EEB4FDCB,441E5761BBE1A89E]	[450A1271BAF5ED8E,105F4724EFA0B8DB]
[450E0271BAF1FD8E,105B5724EFA4A8DB]	[140A4364EBF5BC9B,415F1631BEA0E9CE]
[141A0364EBE5FC9B,414F5631BEB0A9CE]	[500B0731AFF4F8CE,055E5264FAA1AD9B]
[504A0731AFB5F8CE,051F5264FAE0AD9B]	

Table 3. APCSPs of *N* = 128 generated from APCSP of *n* = 64

N = 128 from [111E1221EEE1EDDE,444B4774BBB4B88B]
[121212ED121D1D12EDEDED12EDE2E2ED,474747B847484847B8B8B847B8B7B7B8]
[424243BC425C5C43BDBDBC43BDA3A3BC,171716E917090916E8E8E916E8F6F6E9]
[030306F903595906FCFCF906FCA6A6F9,565653AC560C0C53A9A9AC53A9F3F3AC]
[060613EC074D4C13F9F9EC13F8B2B3EC,535346B952181946ACACB946ADE7E6B9]
[121247B8171D1847EDEDB847E8E2E7B8,474712ED42484D12B8B8ED12BDB7B2ED]
[424316E8565C4917BDBCE917A9A3B6E8,171643BD03091C42E8E9BC42FCF6E3BD]
[030653A953590C56FCF9AC56ACA6F3A9,565306FC060C5903A9ACF903F9F3A6FC]
[061346AD474C1953F9ECB952B8B3E6AC,534613F812194C06ACB9EC07EDE6B3F9]
[124712BD17184D47EDB8ED42E8E7B2B8,471247E8424D1812B8EDB817BDB2E7ED]
[431642FC56491D17BCE9BD03A9B6E2E8,164317A9031C4842E9BCE856FCE3B7BD]
[065303F9530C5C56F9ACFC06ACF3A3A9,530656AC06590903ACF9A953F9A6F6FC]
[134607ED46195953ECB9F812B9E6A6AC,461352B8134C0C06B9ECAD47ECB3F3F9]
[471217BD124D4D47B8EDE842EDB2B2B8,124742E847181812EDB8BD17B8E7E7ED]
[164256FC431D1D16E9BDA903BCE2E2E9,431703A916484843BCE8FC56E9B7B7BC]
[530353F9065C5C53ACFCAC06F9A3A3AC,065606AC53090906F9A9F953ACF6F6F9]
[460747EC13595946B9F8B813ECA6A6B9,135212B9460C0C13ECADED46B9F3F3EC]
[121717B8474D4D12EDE8E847B8B2B2ED,474242ED12181847B8BDBD12EDE7E7B8]
[425656E9171D1C43BDA9A916E8E2E3BC,170303BC42484916E8FCFC43BDB7B6E9]
[035353AC565C5906FCACAC53A9A3A6F9,560606F903090C53A9F9F906FCF6F3AC]
[074746B953594C13F8B8B946ACA6B3EC,521213EC060C1946ADEDEC13F9F3E6B9]

[171712ED474D1847E8E8ED12B8B2E7B8,424247B812184D12BDBDB847EDE7B2ED]
[565643BD171C4917A9A9BC42E8E3B6E8,030316E842491C42FCFCE917BDB6E3BD]
[535306FC56590C56ACACF903A9A6F3A9,060653A9030C5903F9F9AC56FCF3A6FC]
[474613F9534C1952B8B9EC06ACB3E6AD,121346AC06194C07EDECB953F9E6B3F8]
[171247ED47184D42E8EDB812B8E7B2BD,424712B8124D1817BDB8ED47EDB2E7E8]
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[171247ED424D1842E8EDB812BDB2E7BD,424712B817184D17BDB8ED47E8E7B2E8]
[564317BD031C4903A9BCE842FCE3B6FC,031642E856491C56FCE9BD17A9B6E3A9]
[530656FC06590C06ACF9A903F9A6F3F9,065303A9530C5953F9ACFC56ACF3A6AC]
[461353F8134C1812B9ECAC07ECB3E7ED,134606AD46194D47ECB9F952B9E6B2B8]
[124747E847184842EDB8B817B8E7B7BD,471212BD124D1D17B8EDED42EDB2E2E8]
[431717A916490903BCE8E856E9B6F6FC,164242FC431C5C56E9BDBD03BCE3A3A9]

From (13) and (14), we see that using the proposed method, we can also construct odd-periodic complementary sequences without using q-ary m-sequences. Thus the proposed method can extend the number of odd-periodic complementary sequences.

The APCSPs can be used to improve the synchronization time performance using the synchronization circuit of [15]. On the transmitter side, an APCSP (**A**, **B**) is assigned to a pair of I and Q channels of a quadrature phase shift keying (QPSK) modulator. On the side of the receiver, the pair of I and Q channels correlates the received signal with **A** and **B**. The addition of the two correlation functions contains two peak values equal in magnitude and opposite in polarity at zero and middle shifts. This correlation circuit can be used to double-check the synchronization timing and thus reduce the synchronization search time.

4. Conclusions

A sequence with ideal bipolar correlation property has two maximum values equal in magnitude and opposite in polarity at the zero and middle shifts. Furthermore, all the out-of-phase coefficients of the sequence are zero. Thus the sequence is optimal for synchronization since its ideal bipolar correlation properties help double-check the synchronization timing and improve the synchronization performance. However (0,0,1,1) is the only one cyclic distinct sequence with this property.

In this paper, we have proposed a new algorithm for the generation of APCSPs with ideal bipolar correlation properties. APCSPs can be classified into two categories: B-APCSPs and E-APCSPs. APCSPs of length $N = 2n$ can be easily generated by using one APCSP with a period (n) that is a multiple of 4. The APCSPs can be used to improve the synchronization time performance. On the transmitter side, an APCSP (**A**,**B**) is assigned to a pair of I and Q channels of a quadrature phase shift keying (QPSK) modulator. On the side of the receiver, the pair of I and Q channels correlates the received signal with **A** and **B**. The addition of the two correlation functions contains two peak values equal in magnitude and opposite in polarity at zero and middle shifts. This correlation circuit can be used to double-check the synchronization timing and thus reduce the synchronization search time.

The relationship between a pair of odd-periodic complementary sequences and a pair of bipolar complementary sequences has been discussed. Using the proposed method, we can also generate odd-periodic complementary sequences without using q-ary m-sequences. Thus the proposed method can extend the number of odd-periodic complementary sequences.

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