International Journal of Multimedia and Ubiquitous Engineering
Vol. 3, No. 2, April, 2008

BER-Minimizing Limited Feedback for Transmit Beamforming in MISO-OFDM Systems*

Lei Zhang, Gang Wu, and Shaoqian Li
National Key Lab of Communication, University of Electronic Science and Technology of China, Chengdu, P.R.China
Email: zhanglei@uestc.edu.cn

Abstract

To implement transmit beamforming, which is an effective way of leveraging the diversity gains, for multiple-input single-output orthogonal frequency division multiplexing (MISO-OFDM) systems, (partial) channel state information is required at the transmitter. In this paper, we present a new limited feedback scheme in MISO-OFDM systems with transmit beamforming. By dividing the total subcarriers of an OFDM symbol into a series of clusters, from the codebook we choose the "proper" one, which minimizes the average bit error rate (BER) per subcarrier of the considered cluster, as the beamforming vector for this cluster. The scheme can reduce the feedback overhead remarkably while obtain good BER performance. Further, a simple sub-optimal algorithm exploiting the correlation of channel frequency responses at different subcarriers is proposed to avoid exhaustive search in the optimal solution. The complexity analysis and simulations show that the proposed method has low computational complexity and provides improvement over the existing schemes.

1. Introduction

Multiple-input single-output (MISO) systems, which use multiple transmit antennas and a single receive antenna, can provide transmit diversity to mitigate fading. When channel state information (CSI) is available at the transmitter, beamforming is a simple approach to achieve the full diversity order as well as additional array gain [1]. In many situations, however, the transmitter can only obtain partial CSI through limited feedback from the receiver, especially for frequency division duplexing systems. Usually, at the beginning of each block, the receiver chooses a "proper" codeword as the desired beamforming vector (BFV) from the pre-designed codebook according to the channel instantiation and sends the corresponding index to the transmitter using a finite number of bits. Then the transmitter performs beamforming according to the received index [2,3].

For frequency-selective fading, orthogonal frequency division multiplexing (OFDM) can be used to convert a broadband channel into a series of narrowband channels [4]. Hence, the beamforming techniques with limited feedback proposed for flat fading channels can be performed independently for each subcarrier. Unfortunately, the feedback requirements generally grow in proportion to the number of subcarriers. To reduce the feedback overhead of MISO-OFDM, previous approaches have exploited the correlation of channel frequency responses at different subcarriers. And they mainly include: a) clustering, such as traditional clustering [5,6], Karcher mean (KM) clustering [7] and Clustering by varying codebook size [8], and b) interpolation, such as phase rotation (PR) interpolation [5,6] and Geodesic interpolation [7].

This paper focuses on solving the problem of selecting the best codeword from the designed codebook as the optimal BFV for the considered cluster. Our main contributions include:

* This work is supported by the National Natural Science Foundation of China under Grant No. 60496313.

- Adopt the average uncoded bit error rate (BER) per subcarrier as the optimization object. The BER criterion has also been used for precoded multiple-input multiple-output (MIMO) system in flat fading channels [9]. Hence, the new criterion outperforms those based on indirect performance indicators [5–7] in terms of improving BER performance of an uncoded system.
- Propose a simple sub-optimal algorithm to avoid exhaustive search. It reduces the computational complexity remarkably with a slight performance loss compared to the optimal solution.

Naturally, the proposed method can be integrated with that of [8]. For simplicity, it is not be involved herein. Furthermore, the proposed method can be easily extended to the scenario of MIMO-OFDM systems with linear receiver.

The remainder of this paper is organized as follows. The system model is described in Section 2. In Section 3 we propose the BER-minimizing limited feedback scheme. Section 4 presents the complexity analysis. Simulation results are discussed in Section 5. Section 6 concludes this paper.

Notation: $k=n:l:m$ denotes the integer series with the first value n, the last value m and the interval l, and ceil(x) rounds the value of x to the nearest integers towards infinity.

2. System model

A MISO-OFDM system with transmit beamforming, using M_t transmit antennas and N subcarriers is illustrated in Fig. 1. At the transmitter, the symbol $s(k)$ at the k-th subcarrier is multiplied by an $M_t\times 1$ BFV $\mathbf{w}(k)$. Assuming that the cyclic prefix (CP) is longer than the length of impulse response of the channel, OFDM transforms a broadband frequency selective channel into N narrowband flat fading subchannels. So the channel in the k-th subcarrier can be described by an $1\times M_t$ vector $\mathbf{h}(k)$ whose entries represent the channel gains experienced by subcarrier k from the transmit antennas to the receive antenna. Hence, the received signal at the k-th subcarrier can be expressed as

$$r(k) = \mathbf{h}(k)\mathbf{w}(k)s(k) + n(k) \tag{1}$$

where $n(k)$ is additive white Gaussian noise with zero mean and unit variance. We allocate the equal transmit power P to each subcarrier, and restrict $\mathbf{w}(k)$ to unit norm to avoid the overall transmitting power enhancement.

We consider the block-fading channel, which means that the channel is constant in a frame and changes independently from one frame to another. Further, we assume that the channel vector $\mathbf{h}(k)$ is perfectly known by the receiver while not available to the transmitter; however, there exists a low-rate, error-free and zero-delay feedback link from the receiver to the transmitter. First, the BFV for each subcarrier is chosen from a pre-designed codebook $\Omega=(\mathbf{w}_1,\ldots,\mathbf{w}_L)$, which is known to both the transmitter and receiver, according to some optimization criteria. Then the index of the selected coderword is sent to the transmitter with $B=\log_2 L$ bits. If all N indices are to be fed back, the total requirements will be NB bits. This load is too large to be implemented in practice, but, fortunately, can be reduced by the approach developed later on.

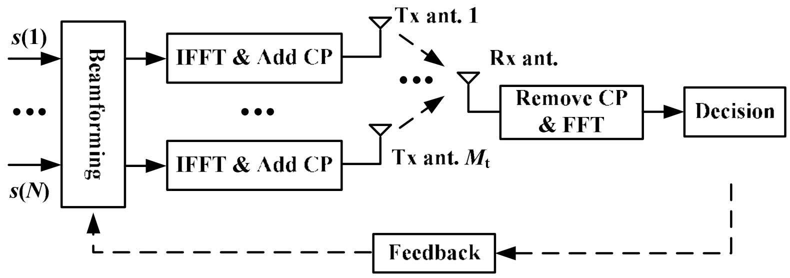

Figure 1. Block diagram of a MISO-OFDM system with transmit beamforming.

3. BER-minimizing limited feedback

3.1. BER-minimizing BFV selection

The basic idea of both the clustering and interpolation approaches in [5–7] is to divide the total subcarriers into some groups, and only the index of one selected BFV in each group need to be fed back. However, all the methods in [5–7] are based on vector/matrix that is indirect (though good) performance indicator. As it is well-known, the uncoded BER is usually used to evaluate the performance for the considered system. So we can directly adopt BER as the BFV selection criterion in each subcarrier-group (cluster).

We first investigate the relationship between BER p and signal to noise ratio (SNR) u in an AWGN channel, denoted by $p(u)$. Define the Gaussian-Q function as

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-t^2/2)dt. \tag{2}$$

We consider rectangular or square quadrature amplitude modulation (QAM) constellations with size M. So $p(u)$ can be written as a finite sum of Gaussian-Q functions [9]

$$p(u)=\sum_i b_i Q\left(a_i\sqrt{u}\right) \tag{3}$$

where the constant pair $\{a_i, b_i\}$ need to be figured out for each constellation in use. The simple examples are 2-QAM and 4-QAM, where only one term in the summation: $a_1=2^{1/2}$, $b_1=1$ for 2-QAM and $a_1=b_1=1$ for 4-QAM [9].

On the other hand, the signal model in (1) is identical to that of a narrowband MISO system, thus the SNR for subcarrier k can be written as

$$u(k)=P\left|\mathbf{h}(k)\mathbf{w}(k)\right|^2. \tag{4}$$

With clustering, the N subcarriers are divided into clusters of size K. Without loss of generality, we hereafter consider the n-th ($0\le n\le N/K-1$) cluster containing the subcarriers from nK to $(n+1)K-1$.

Here we take example for 4-QAM to describe our BER criterion. From (4), the average BER over the n-th cluster is given by

$$p_n=\frac{1}{K}\sum_{k=nK}^{(n+1)K-1}p\left(u(k)\right)=\frac{1}{K}\sum_{k=nK}^{(n+1)K-1}Q\left(\sqrt{u(k)}\right). \tag{5}$$

Given K, to minimize p_n, the optimal BFV should be selected according to

$$\boldsymbol{w}(n) = \arg\min_{\boldsymbol{w}\in \boldsymbol{W}} \sum_{k=nK}^{(n+1)K-1} Q\left(\sqrt{P\left|\boldsymbol{h}(k)\boldsymbol{w}\right|^2}\right) \tag{6}$$

Then applying Chernoff bound $Q(x) \leq \exp(-x^2/2)$, we have

$$Q\left(\sqrt{P\left|\mathbf{h}(k)\mathbf{w}_l\right|^2}\right) \leq \exp\left(-\frac{P}{2}\left|\mathbf{h}(k)\mathbf{w}_l\right|^2\right). \tag{7}$$

Thus, the optimization problem in (6) can be simplified as

$$\mathbf{w}_n = \arg\min_{\mathbf{w}_l\in \mathrm{W}} \sum_{k=nK}^{(n+1)K-1} \exp\left(-\frac{P}{2}\left|\mathbf{h}(k)\mathbf{w}_l\right|^2\right). \tag{8}$$

In general, it is difficult to find the close-form solution for (8). Since our codebook size is finite, it can be accomplished by a brute force search, as used for KM clustering [7] and the found $\mathbf{w}(n)$ is named BER-minimizing mean vector (BMMV).

3.2. Suboptimal algorithm

When K and L are quite large, however, the exhaustive search in (8) could be very complex. So we should develop a simpler solution for (8). Generally, due to the correlation of channel frequency responses at adjacent subcarriers, we have the following two self-evident facts for the n-th cluster:

1) The BMMV and the channel vector at the center subcarrier are strongly correlated. This is because the optimal BFV for each subcarrier is the normalized Hermitian transpose of the corresponding channel vector, and the correlation between the BMMV and the optimal BFV of the center subcarrier is high.

2) A small number of neighboring subcarriers have about the same channels.

Based on the above two facts, we propose a simpler search algorithm for (8) as follows:

Step 1: Construct a sub-codebook Ω_s composed of L_s ($L_s<L$) codewords in Ω which have the maximum correlation between themselves and the channel vector at the center subcarrier, $\mathbf{h}(nK+K/2)$, i.e.,

$$\mathbf{W}_s = \left\{\mathbf{w}_{s,1}, \mathrm{K}\ , \mathbf{w}_{s,L_s}\right\} = \arg\max_{\mathbf{w}\in \mathbf{W}} \left|\mathbf{h}(nK + K/2)\mathbf{w}\right| \tag{9}$$

Step 2: Select the codeword in sub-codebook Ω_s according to

$$\mathbf{w}(n) = \arg\min_{\mathbf{w}\in \mathbf{W}_s} \sum_{k=nK:K_s:(n+1)K-1} \exp\left(-\frac{P}{2}\left|\mathbf{h}(k)\mathbf{w}\right|^2\right), K_s < K \tag{10}$$

as the BFV for the n-th cluster.

By these two steps, we lessen the 2-dim space about codeword and subcarrier for search, and consequently reduce the complexity remarkably. Our simulations, though, show that the cost function in (10) performs very close to the grid search in (8) as long as the parameters L_s and K_s are properly chosen.

4. Complexity analysis

In the computational complexity analysis, we roughly calculate the number of complex multiplications for all the mentioned methods. We just consider the computation of BFV selection at the receiver for all the methods (phase selection is also included for PR interpolation) and BFV reconstruction for PR interpolation and Geodesic interpolation, but ignore the same computation of channel estimation, receiver detection, transmit beamforming and etc. For simpleness, we directly present their complexities in Table I, where the columns labeled by "Generalization", "Example 1" and "Example 2" mean the cases of general parameters and special parameters to be used in two simulations of Section III, respectively. The ideal feedback means that the indices of BFVs are fed back for all subcarriers. Besides, cosine operation is required for KM clustering and Geodesic interpolation, and exponential operation is included in our scheme. Usually, both cosine and exponential operations can be performed by using looking up from a pre-designed table, and the consequent effects on complexity and performance are neglected here for the simpleness of discussion. It is found from Table 1 that in the practical scenarios our solution in (10) is only more complex than the traditional clustering, while simpler than all other schemes.

Table 1. Number of complex multiplications for MISO-OFDM beamforming

Scheme	Generalization	Example 1	Example 2
Ideal feedback	LNM_t	8192	2048
Traditional cluster. [5,6]	LNM_t/K	512	128
KM cluster.[7]	LNM_t	8192	2048
Geodesic interp.[7]	$(L/K+2)NM_t$	1536	1152
PR interp.[5,6]	$((L+2)/K+2)NM_t+(3\times2^Q/2+1/K-2)N$	1736	1288
Proposed, (8)	LNM_t	8192	2048
Proposed, (10)	$(\mathrm{ceil}(K/K_s)+1)L_sNM_t/K$	896	448

5. Simulation results

In the simulation, we consider a system with parameters: M_t=4, N=128, K=16 and CP length of 16. We assume that, the ITU Ped-B channel model [10] is used, the channels among different pairs of transmit and receive antenna are independent identically distributed, the receiver has perfect CSI, the feedback channel is error-free and zero-delay. The codebooks used for beamforming are given in [11]. In all results, 4-QAM constellation is used and the SNR is defined as the ratio of the total transmit power to the noise power at the receiver.

Experiment 1: For fairness, the total feedback bits per OFDM symbol are fixed as 32 for all the limited feedback methods. In all except PR interpolation, we let B=4. For PR interpolation, we use B=3 bits and Q=1 bit to send the indices of BFV and phase factor, respectively. For the proposed simple solution in (10), we let L_s=4 and K_s=3. Also, the ideal beamforming which conveys the indices of BFVs for all subcarriers using 512 (4×128) feedback bits is simulated.

Fig. 2 compares the uncoded BER performance. Among all the beamforming methods, our sub-optimal solution in (10) outperforms other existing limited feedback methods, giving 0.5–1.8 dB gain at the BER of 4×10^{-3}, and just suffers a slight degradation relative to the optimal solution in (8). This testifies that the proposed simple BFV search algorithm can improve the performance for practical modulation with quite low complexity. Naturally, the ideal feedback shows much gain over the limited feedback methods due to its vast feedback overhead.

Experiment 2: we reduce the total feedback bits to 16 for all the limited feedback schemes. We let B=2 for all except PR interpolation. For PR interpolation, we use B=1 bit and Q=1 bit to send the indices of BFV and phase factor, respectively. For the proposed simple solution in (10), we let L_s=2 and K_s=3. For comparison, we also simulate the ideal beamforming which uses 256 (2×128) feedback bits to send the indices of BFVs.

The uncoded BER performance is illustrated in Fig. 3. With these curves we can find some important points. Firstly, the proposed sub-optimal solution in (10) gives larger gain (about 1.3–5.2 dB at the BER of 4×10^{-3}) over other limited feedback schemes compared with experiment 1.

This indicates that the lower the feedback overhead, the more our scheme surpasses other limited feedback schemes. Secondly, all methods show some SNR loss compared to Fig. 2 due to less feedback bits.

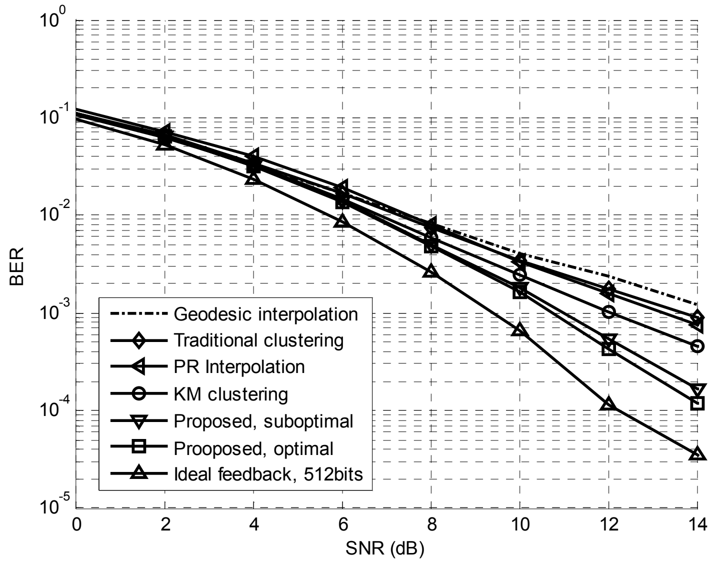

Figure 2. Comparison of BER performance when the total feedback bits equal 32.

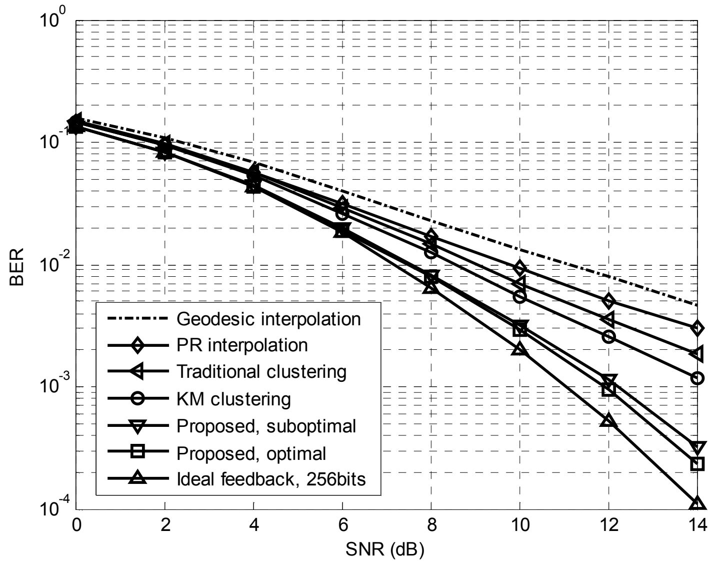

Figure 3. Comparison of BER performance when the total feedback bits equal 16.

6. Conclusions

In this paper, we investigated limited feedback for transmit beamforming in MISO-OFDM systems. Clustering subcarriers is an effective approach to reduce feedback overhead. We presented a new BFV selection criterion based on minimizing the average BER per subcarrier, and proposed a simple sub-optimal algorithm which reduces the computational complexity sharply with an acceptable performance loss compared to the optimal solution. It is shown by the simulations that the proposed scheme outperforms any other competitor in terms of uncoded BER. Further, the proposed method can be easily extended to the case of multiple receive antennas. Hence, it is a promising candidate for future wireless communications.

References

[1] J. B. Andersen, “Antenna arrays in mobile communications: gain, diversity, and channel capacity”, IEEE Antennas Propagation Magazine, vol. 42, no. 2, 2000, pp. 12–16.

[2] D. J. Love, R. W. Heath Jr., and T. Strohmer, “Grassmannian beamforming for multiple-input multiple-output wireless systems”, IEEE Trans. on Information Theory, vol. 49, nol. 10, 2003, pp. 2735–2747.

[3] D. Love and R. W. Heath Jr., “Limited feedback precoding for spatial multiplexing systems,” in Proc. IEEE Globecom’03, vol. 4, pp. 1857–1861.

[4] H. Bölcskei, D. Gesbert, and A. J. Paulraj, “On the capacity of OFDM-based spatial multplexing systems”, IEEE Trans. on Communication, vol. 50, no. 2, 2002, pp. 225–234.

[5] J. Choi and R. W. Heath, Jr, “Interpolation based transmit beamforming for MIMO-OFDM with limited feedback”, IEEE Trans. on Signal Processing, vol. 53, no. 11, 2005, pp. 4125–4135.

[6] J. Choi, B. Mondal, and R. W. Heath, Jr., “Interpolation based unitary precoding for special multipexing MIMO-OFDM with limited feedback”, IEEE Trans. on Signal Processing, vol. 54, no. 12, 2006, pp. 4730–4740.

[7] T. Pande, D. J. Love, and J. V. Krogmeier, “Reduced feedback MIMO-OFDM precoding and antenna selection”, IEEE Trans. on Signal Processing, vol. 55, no. 5, 2007, pp. 2284–2293.

[8] H. Zhang, Y. Li, V. Stolpman, and N. V. Waes, “A reduced CSI feedback approach for precoded MIMO-OFDM systems”, IEEE Trans. Wireless Commun., vol. 6, no. 1, 2007, pp. 55–58.

[9] Shengli Zhou and Baosheng Li, “BER criterion and codebook construction for finite-rate precoded spatial multiplexing with linear receivers”, IEEE Trans. on Signal Processing, vol. 54, no. 5, 2006, pp. 1653–1665.

[10] ITU-R Recommendation M.1225. Guidelines for evaluation of radio transmission technologies for IMT-2000. 1997.

[11] D. J. Love, “Grassmannian subspace packings,” (2007). [Online]. Available: http://cobweb.ecn.purdue.edu/~djlove/grass.html.

Authors

Lei Zhang

Received his B.Sc. degree in communication engineering and M.Sc. degree in communication & information system, both from University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2001 and 2004, respectively. He is currently working toward the Ph.D. degree from National Key Lab of Communication at UESTC. From Sept. 2005 to May 2007, he was an intern in Intel China Research Center (ICRC), Beijing, China. His research interests include MIMO signal processing and limited feedback techniques for precoded MIMO-OFDM systems.

Gang Wu

Received his B.Eng. and M.Eng. degrees in 1996 and 1999, respectively from Department of Radio Communication Engineering, Chongqing University of Post and Telecommunications, Chongqing, China, and his PhD degree from School of Communications and Information Engineering at University of Electronic Science and Technology of China (UESTC), Chengdu, China.

He was a Lecturer then an Associate Professor with UESTC, Chengdu, China from June 2004 to Nov. 2005. He was a Research Fellow with Positioning and Wireless Technology Centre, Nanyang Technological University, Singapore from Nov. 2005 to Feb. 2007. He is a Faculty of National Key Lab of Communication, University of Electronic Science and Technology of China. His research interests include space-time codes for multiuser MIMO-OFDMA systems, OFDM for wireless communications, Location in Impulse-Radio Ultra-Wideband (IR-UWB) systems, Frequency-domain equalization for single-carrier systems, and Multiple-user information theory and its application to Anti-Jamming application

Shaoqian Li

Received his B.S. degree in communication technology from Xidian University, Xi'an, China, in 1982, and his M.S. degree in Information and Communication Systems from University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 1984, respectively.

Since July 1984, he has been a faculty member with Institute of Communication and Information Engineering, UESTC, China, where he is a professor since 1997, and Ph.D. supervisor, since 2000. He is also the director of National Key Lab of Communication in UESTC and a member. He is the leader of *Mobile Broadband Communications (MBC) Experts Team*, Ministry of Information Industry of China, for evaluation of strategy and technologies of MBC. His research includes mobile communication systems (3G and Beyond 3G), wireless communications (CDMA, OFDM, Ultra-Wideband, and MIMO etc.), integrated circuits in wireless communications, and radio resource managements techniques.