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Encoding Ternary Information using a Chaotic Neural Network

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Abstract We analyzed a model of a chaotic neural network consisting of a chaotically forcing neuron and two stable neurons in the previous study. In the study, we showed that the dynamics of a chaotically forcing neuron is embedded in the form of a code sequence on a fractal attractor of a two-neuron response system. As an engineering application on the information processing, we show that a desirable message can be encoded into an attractor space by using a chaotic neural network in the present study.

Keyword: Chaotic neural network, Dynamics of a chaotically forced neuron, Attractor space

1. Introduction

Many studies on the relations between chaos and fractals[4,13,6] have been done in recent years. Rossler et al.[9] reported some models representing chaos-driven contraction mapping. Their paper describes a hierarchy of models exhibiting fractal attractors including strange nonchaotic attractors found by Grebogi et al. and singular-continuous nowhere differentiable(SCND) attractors, too. Tsuda[11] have also found the SCND attractor in a neural system consisting of chaotic neuron models proposed by Aihara et al.[1]. On the other hand, a lot of research on the use of chaos for nonlinear digital communications, especially for the encoding of digital information, has been reported[5,3,7]. Based on these researches, we can expect that if a dynamical system has well-defined symbolic dynamics, the encoding of digital information is accomplished using the principle of controlling chaos[8]. Furthermore it has been showed by Hayes et at.[5] that a chaotic system can be manipulated to generate controlled chaotic time series whose symbolic representation corresponds to the digital information via arbitrarily small time-dependent perturbations.

The purpose of the present paper is to further develop fractal symbolic encoding in a chaotically-forced contraction system that exhibits fractal attractor. We generate a chaotic time series corresponding to a ternary desirable message by applying proper perturbations to the initial values of a chaotic neuron at each time[7]. At this time, we use two peaks of the return map of a chaotic neuron to assign a symbolic representation to the signal. And then, we observe a 2-dim. fractal attractor of two almost linear neurons forced by the chaotic orbits obtained by the time series for the desirable message. To clarify the encoding property, we introduce hard-limit functions, or Heaviside functions as transfer functions from the forcing neuron to the response system, thereby the system is converted to an IFS(Iterated Function System)-like model which is composed of not affine but rather nonlinear transforms. According to Barnsley[2], if the IFS is totally disconnected and if the points on the attractor are distributed sparsely, it is possible to improve memories with a very high storage capacity and robustness against noise. Although the transformation of the proposed system is nonlinear rather than affine, and it may not be completely invertible, a kind of coding of information may also be possible [11],[12].

2. Encoding code sequences using a chaotic neuron

2.1 Chaotic neuron model

We use a chaotic neuron model proposed by Aihara[1] to encode the desired code sequence into a trajectory of the chaotic neuron. The equation of the chaotic neuron is as follows.

$$x_{n+1} = f_1(-\alpha_1 \sum_{r=0}^{n} k_1^r x_{n-r} + I_0) \tag{1}$$

Where α_1 is a positive parameter, I_0 is the strength of the external input to the neuron x , k_1 is a decay parameter with $0 < k_1 < 1$, and the function $f_1(x)$ is the following sigmoidal function:

$$f_1(x) = \frac{1}{1+e^{-x/\varepsilon_1}}, \tag{2}$$

where ε_1 is a steepness parameter. We can represent the dynamics of the chaotic neuron as follows by defining new variables X_{n+1}:

$$X_{n+1} = k_1 X_n - \alpha_1 f_1(X_n) + I \tag{3}$$

where $I = I_0(1-k_1)$ which is used to control the chaotic dynamics of the neuron X.

Fig. 1 shows a chaotic neuron map and an example of generating a code sequence.

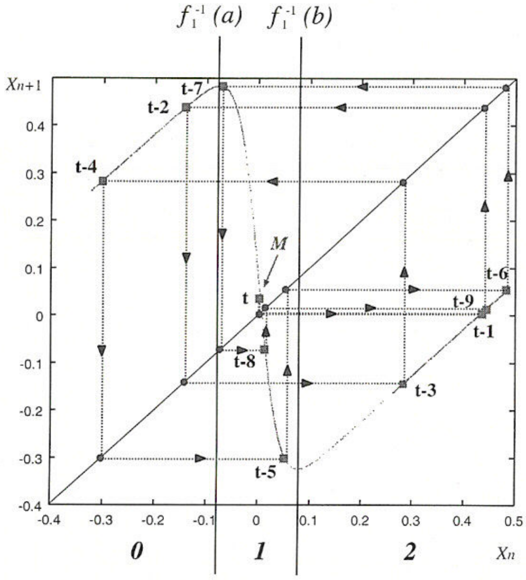
2.2 Encoding code sequences

We show an example of encoding digitized information into a trajectory of a chaotic neuron map. We use the algorithm of Lai [7] for the logistic map. The steps of encoding ternary codes are as follows.

1. Choose an initial value.

2. Determine m symbols corresponding to m points on the trajectory starting from the initial value x_0.

3. Examine whether the mth symbol is identical to the first message bit.
   - If so, iterate the process from x_0 to obtain x_1 and determine the $(m+1)$th symbol from x_0. Then examine whether it is identical to the second message bit.
   - Otherwise, apply some perturbation to x_0 for the m th symbol from x_0 to make accordance with the first message bit.
4. Continue this procedure by the time that all message bits are encoded into a chaotic trajectory.



Code sequence of $M : 1\ 2\ 0\ 2\ 0\ 1\ 2\ 1\ 1\ 2 \ldots$

Fig. 1. Definition of a code sequence based on a chaotic neuron map

The coding function[5] is used for calculating perturbations. The procedure obtaining the coding function is as follows[7].

1. Divide the unit interval in x into N bins of size $\delta x = 1/N$, where $\delta x << 1/3^m$ and s is the maximally allowed perturbation.
2. Determine the symbol sequence of length $m : a_1 a_2 \mathrm{L}\ a_m (a_i \in 0,1,2)$ by choosing a point from each bin and performing Determine the symbol sequence of length $m : a_1 a_2 \mathrm{L}\ a_m$ $(a_i \in 0,1,2)$ by choosing a point from each bin and performing m iterations.

The symbolic value R can be calculated as follows:

$$R = \sum_{i=1}^{m} a_i / 3^i, a \le R \le 1. \qquad (4)$$

Fig. 2 shows the coding function for a chaotic neuron map.

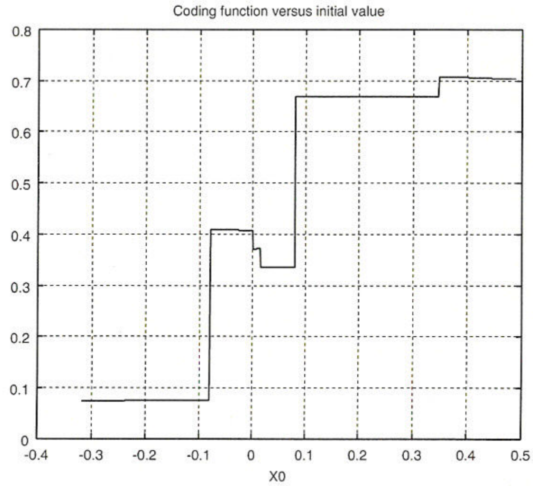


Fig. 2. The coding function for the chaotic neuron map

at $k = 0.98, \varepsilon = 0.02$

The values of perturbations are determined by the coding function as following procedures.

1. Let the m-bit symbol sequence a_m generated from x_0 be $a_1 a_2 \mathrm{L}\ a_{m-1} a_m$ and let the first message bit to be encoded be b_1 .
2. Calculate $\delta R = (a_m - b_1)/3^m$ by comparing generated symbol sequence $a_1 a_2 \mathrm{L}\ a_{m-1} a_m$ with the desirable symbol sequence $a_1 a_2 \mathrm{L}\ a_{m-1} b_1$.
3. Compute the perturbation δx from the coding function $R(x)$.

An example of encoding the sequence "21011202 10202102 11212020 20111201" into a chaotic trajectory is shown in Fig. 3. Fig. 4 shows the perturbation δx applied to the initial value.

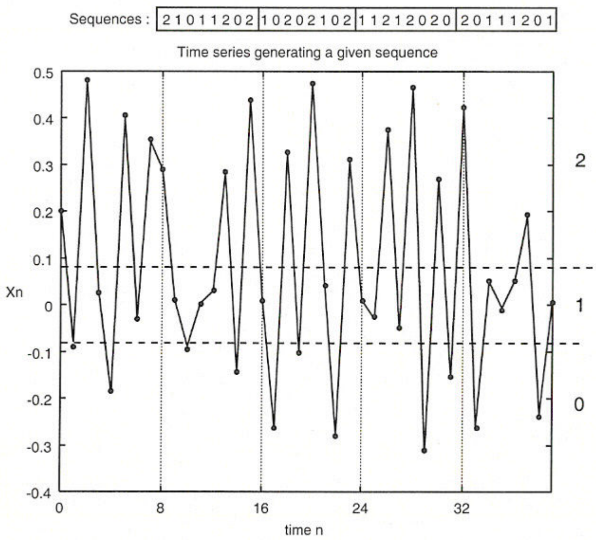


Fig. 3. Time series for the perturbed initial values

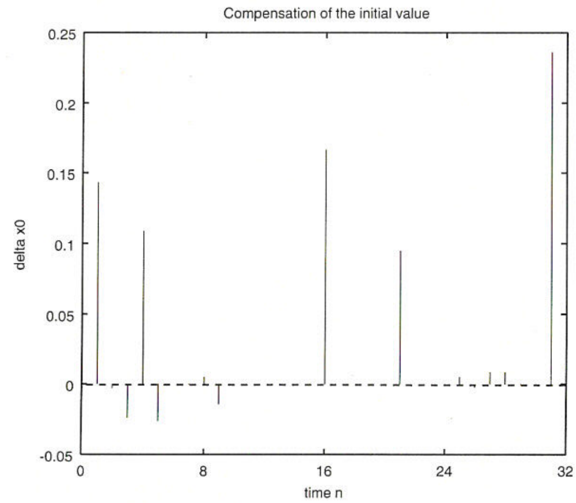


Fig. 4. The perturbation δx applied to the initial value

3. Fractal encoding

3.1 Chaotic neural network model

We have considered the nonlinear dynamics of a type of chaotic neural network[1], which consists of a chaotically forcing neuron and two almost linear neurons[10]. The latter stable neurons are forced by the chaotic neuron described in subsection 2.1 through transfer functions as shown in Fig. 5. In this model, we use two hard-limit functions, or Heaviside functions as transfer functions from the chaotic neuron X to the static neurons Y and Z to encode the dynamics of the chaotic neuron. The equations of the two stable neurons are as follows:

$$y_{n+1} = f_2(-\alpha_2 \sum_{r=0}^{n} k_2^r y_{n-r} + w_{yz} \sum_{r=0}^{n} k_2^r z_{n-r} + w_{yx} \sum_{r=0}^{n} k_2^r h_1(x_{n-r})) \quad (5)$$

$$z_{n+1} = f_3(-\alpha_3 \sum_{r=0}^{n} k_3^r z_{n-r} + w_{zy} \sum_{r=0}^{n} k_3^r y_{sn-r} + w_{zx} \sum_{r=0}^{n} k_3^r h_2(x_{n-r})), \quad (6)$$

where α_i is a positive parameter, k_i is a decay parameter with $0 < k_i < 1 (i = 2,3)$; w_{vu} is the connection weight from neuron u to v with $w_{zy} < 0$, and $w_{yz}, w_{yz}, w_{zz} > 0$; and the function $f_i(x)(i = 2,3)$ is the sigmoidal function.

The hard-limit transfer functions $h_i(x)(i = 1,2)$ are defined as follows:

$$h_1(x) = \begin{cases} 0 & (x < a) \\ 1 & (x \geq a) \end{cases} \quad (7)$$

$$h_2(x) = \begin{cases} 0 & (x < b) \\ 1 & (x \geq b) \end{cases} \quad (8)$$

where a and b are the threshold parameters assumed to be $a < b$ in this paper. The functions $h_i(x)(i = 1,2)$ represent wave-shaping effect of axons producing all-or-none behaviors of propagating action potentials[1].

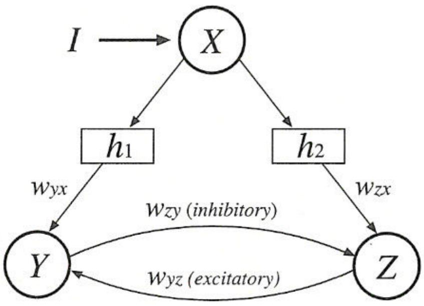


Fig. 5 Network configuration of a chaotically forced contracting system

Then we can represent the dynamics of the network except for X as follows:

$$Y_{n+1} = k_2 Y_n - \alpha_2 f_2(Y_n) + w_{yz} f_3(Z_n) + w_{yx} h_1(f_1(X_n)) \quad (9)$$

$$Z_{n+1} = k_3 Z_n - \alpha_3 f_3(Z_n) + w_{zy} f_2(Y_n) + w_{zx} h_2(f_1(X_n)). \quad (10)$$

3.2 Fractal attractor

We next consider the relation between the structure of a code and that of an attractor. We divide the region of the state in the chaotic neuron map into three subintervals and label the symbols 0, 1, and 2 for each one as shown in Fig. 1. We then express the dynamical series $\{X_n\}$ as a code sequence consisting of the symbols 0, 1, and 2. For example, the point M on the chaotic map in Fig. 1, which corresponds to the point 0 on the attractor shown Fig. 6, can be labeled as $1202012112\ldots$. Fig. 6 shows the hierarchical relation between the codes generated by the symbolic dynamics of the forcing chaotic neuron and the structure of the attractor. It has been shown that the attractors on $Y-Z$ space have self-similar fractal structures where the hierarchy of the structure of the attractor corresponds to the hierarchy of the symbolic code generated by the forcing chaotic neuron in the previous study[10]. Fig. 7 shows the attractor driven by the chaotic time series of Fig. 3.

4. The relation between the distance on a code space and that on an attractor space

4.1 The distance space

The distance function on a set X has the following properties.

- $d(x,y) \geq 0$ if $x \neq y$; if and only if $x \neq y$
- $d(x,y) = d(y,x)$
- $d(x,y) \leq d(x,z) + d(z,y)$ for any $z \in X$

$d(x,y)$ represents the distance between x and y.

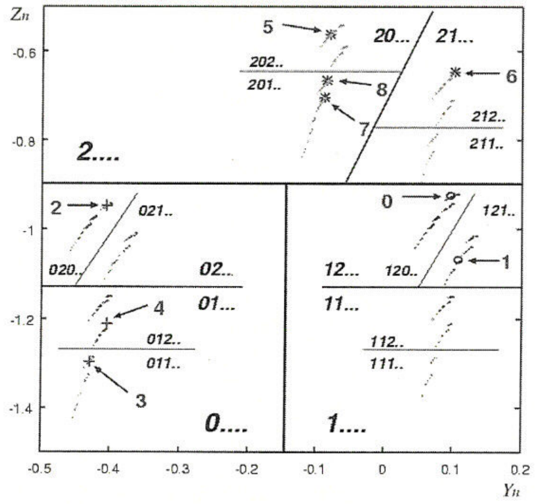


Fig. 6. Hierarchy of codes on the fractal attractor for I=0.58

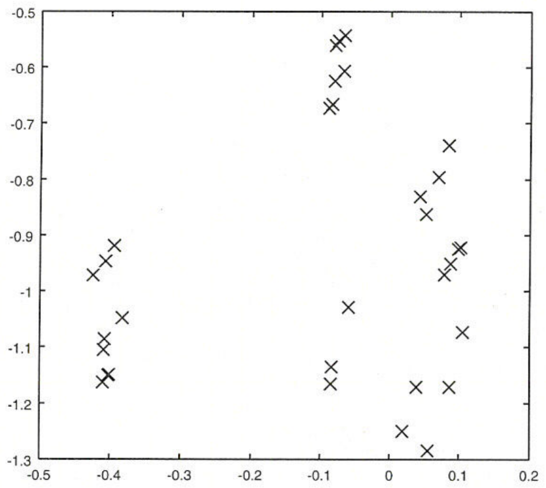


Fig. 7 Attractor for the chaotic time series of Fig. 3

(X,d) is referred to as a distance space. For example, the Hausdorff distance $D(A,B)$ between A and $B(A,B \in H)$ can be defined as following equations:

$$D(A,B) = \max\{d(A,B),\ d(B,A)\} \quad (11)$$

$$d(A,B) = \inf\{\varepsilon > 0 \ :\ A \subset N_\varepsilon(B)\} \quad (12)$$

$$d(B,A) = \inf\{\varepsilon > 0 \ :\ B \subset N_\varepsilon(A)\} \quad (13)$$

$$N_\varepsilon(A) = \{x \in R^2 \ :\ d(x,a) \le \varepsilon,\ a \in A\} \quad (14)$$

where (H,D) is the distance space.

4.2 Continuous mapping

Let the mapping $f : X \to Y$ be a mapping from a distance space (X,d_1) to a distance space (Y,d_2). The mapping f is continuous on $x \in X$ in case that the following condition is satisfied.

For $\forall \varepsilon > 0$, there exists $\delta > 0$ such that $d_1(x,y) < \delta$ implies $d_2(f(x),f(y)) < \varepsilon$.

4.3 Calculation of the distance on a code space using symbolic dynamics

A code sequence space can be defined as follows.

$$\sum\nolimits_3^+ = \{x = \{x_n\}_{n=0}^{\infty} : x_n = 0,1,2\} \quad (15)$$

And then, a distance function on $\sum_3^+$ for $x = \{x_n\},\ y = \{y_n\},\ x,y \in \sum_3^+$ is defined as the following equation.

$$d(x,y) = \sum_{i=0}^{\infty} \frac{|x_i - y_i|}{3^i} \quad (16)$$

Table 1-3 represent the examples of the distance between the codes of some points on the Fig. 6 calculated by the above definition.

Table 1. Distance on the code space(Region 0)

	4	6	8	13	16
4	0	0.5933	0.1836	0.5950	0.5951
6	0.5933	0	0.5537	0.0023	0.0022
8	0.1836	0.5537	0	0.5515	0.5514
13	0.5950	0.0023	0.5515	0	0.0001
16	0.5951	0.0022	0.5514	0.0001	0

Table 2. Distance on the code space(Region 1)

	0	5	7	11	12	17
0	0	0.6605	0.1735	0.6605	0.6685	0.0192
5	0.6605	0	0.4879	0.0000	0.0021	0.6523
7	0.1735	0.4879	0	0.4879	0.4863	0.1680
11	0.6605	0.0000	0.4879	0	0.0021	0.6523
12	0.6685	0.0021	0.4863	0.0021	0	0.6530
17	0.0192	0.6523	0.1680	0.6523	0.6530	0

Table 3. Distance on the code space(Region 2)

	1	3	9	10	14	15
1	0	0.2172	0.3407	0.0023	0.0000	0.0007
3	0.2172	0	0.5423	0.2167	0.2172	0.2179
9	0.3407	0.5423	0	0.3401	0.3407	0.3413
10	0.0023	0.2167	0.3401	0	0.0023	0.0016
14	0.0000	0.2172	0.3407	0.0023	0	0.0007
15	0.0007	0.2179	0.3413	0.0016	0.0007	0

4.4 The calculated Hausdorff distances on the attractor for the above distance between the codes are showed in Table 4-6.

Table 4. Distance on the attractor space(Region 0)

	4	6	8	13	16
4	0	0.1792	0.0554	0.1793	0.1791
6	0.1792	0	0.1368	0.0020	0.0022
8	0.0554	0.1368	0	0.1368	0.1366
13	0.1793	0.0020	0.1368	0	0.0003
16	0.1791	0.0022	0.1366	0.0003	0

Table 5. Distance on the attractor space(Region 1)

	0	5	7	11	12	17
0	0	0.2254	0.1636	0.2254	02248	0.0172
5	0.2254	0	0.0618	0.0000	0.0034	0.2082
7	0.1636	0.0618	0	0.0618	0.0612	0.1464
11	0.2254	0.0000	0.0618	0	0.0034	0.2082
12	02248	0.0034	0.0612	0.0034	0	0.2076
17	0.0172	0.2082	0.1464	0.2082	0.2076	0

Table 6. Distance on the attractor space(Region 2)

	1	3	9	10	14	15
1	0	0.1251	0.1547	0.0063	0.0001	0.0005
3	0.1251	0	0.2559	0.1314	0.1251	0.1252
9	0.1547	0.2559	0	0.1567	0.1547	0.1542
10	0.0063	0.1314	0.1567	0	0.0063	0.0062
14	0.0001	0.1251	0.1547	0.0063	0	0.0006
15	0.0005	0.1252	0.1542	0.0062	0.0006	0

The above tables show that the relative distances on the code space and those of the attractor space are almost the same. It can be said that the attractor space contains the information of code space substantially.

5. Conclusions

In out previous study[10], the network composed of a chaotic neuron and two linearly static neurons has been proposed and implemented as a hardware system with analog discrete devices to investigate whether or not the fractal encoding is actually realized. An attractor structure with three regions 0, 1, and 2 has been clearly demonstrated in the study. The robustness of the fractal attractor for noise to a certain degree also has been observed. Furthermore, the LSI chip for the network has been designed and fabricated. From the viewpoint of engineering, it is interesting to apply the concept of encoding on the fractal attractor to practical information processing. In this respect, we showed that a desirable message can be encoded into an attractor space by using a chaotic neural network in the present study.

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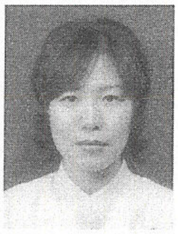

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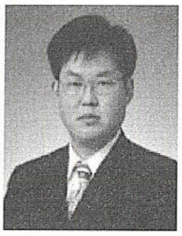

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