An Efficient Method to Find a Shortest Path for a Car-Like Robot

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Abstract Dubins showed that any shortest path of a car-like robot consists of exactly three path segment which are either arcs of circles of radius r(denoted C), or straight line segments(denoted S). Possible six types classified into two families, i.e. CSC and CCC. CSC includes 2 types(LRL and RLR) and CSC includes 4 types(LSL, RSR, LSR, RSL). This paper proposes new formulae for CSC family to find the shortest smooth path between the initial and final configurations of the car-like robot. The formulae are used for finding connection points explicitly between C→S and S→C which are necessary for a real application. The formulae have simple forms mainly because they are transformed to origin of their original coordinates of initial and target configuration, and derived from standard forms which are representative configurations of type LSL and LSR, respectively. The formulae in this research, which are derived from the standard forms, are simple and new.

Keywords: Dubins robot, Geometric algorithm, Shortest path, Robotics

1. Introduction

The classical result by Dubins[1] gives a sufficient set of paths which always contains the optimal path for a car-like mobile robot that only moves forwards from an initial position to a target position. He showed that any shortest path of the car-like mobile robot consists of exactly three path segment which are either arcs of circles of radius, or straight line segments.

Melzak[2], Robertson[3], and Cockayne and Hall[4] studied the accessibility regions in which the robot can reach, but all these works do not concentrate on optimality of the paths. Bui et. al.[5] proposed the synthesis problem for a car-like robot only moving forward in a plane. Then Boissonnat and Bui[6] considered that the partition of the plane w.r.t. the types of the optimal paths and found the shapes of the shortest path accessibility regions for a car that only moves forwards along optimal paths.

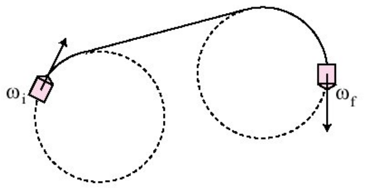
Reed and Shepp[7] showed that the initial and final configurations define a sufficient set of 48 paths which contains the optimal path. Each of these paths has at most two cusps and five segments, which are either line segment or arcs of a circle of radius. These result has been proved again by Sussman and Tang[8] and Boissonnat et. al respectively. These authors produced a more elegant proof and added more necessary conditions on the optimality of a path. They reduced the number of path types to a sufficient set of 46 paths. Boissonat[9] et. al. approached this problem using a well-known control theory. And Soueres and Laumond[10,11] proposed an alternative approach to the problem with reversals. The problem was treated by the combined Pontryagin's optimality principle with geometric reasoning.

To find the shortest path using Dubins method, it is selected among all possible paths that are calculated, but it may become a problem that it takes much computation time in applications. Shkel and Lumelsky[12] proposed the logical classification scheme to extract the shortest path from the Dubins set directly, without explicitly calculating the candidate paths. But this method has a weak point in occurrence of several number of equivalency groups, based on the angle quadrants of the corresponding pairs of the initial and final orientation angles.

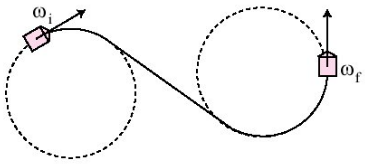
To reach a target position starting from an initial position using the Dubins method, we must know the connection point between a circle of the initial position and a line segment, and the connection point between a circle of the target position and the line segment of CSC type path respectively. Using the known initial position and the final position we can find a rotational angle of the initial position and the final angle individually. To find the two rotational angles, in this research, we propose new formulae in simple forms. After transforming a robot configuration located an arbitrary point to an origin of coordinate, the standard form of type LSL and type LSR is defined and the formulae are derived from the standard form. Comparing to Shkel and Lumelsky[12] method, the proposed method has ability to accomplish the faster calculation for searching the shortest path.

2. Dubins' car model

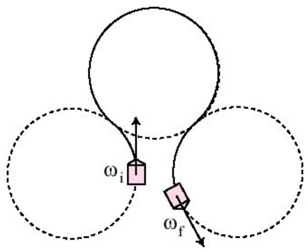
A position of a robot is represented by $w(x, y, \theta)$. The coordinates of the robot are represented by (x, y) and the direction is represented by θ. Dubins[1] showed that any path from an initial position to a target position can have 6 admissible paths. The shortest path consists of exactly three path segments and presents a sequence CCC or CSC, where C for an arc of circle will be denoted R for right-arc and L left-arc, and S for a straight line segment. CSC includes 2 types(LRL and RLR) and CSC include 4 types(LSL, RSR, LSR, and RSL). Fig 1(a) shows RSR type, (b) for RSL, and (c) for LRL. Others not shown in this figure are represented by symmetry of the 3 types.



(a) Type RSR



(b) Type RSL



(c) Type LRL

(Fig.1) Examples of Dubins shortest paths

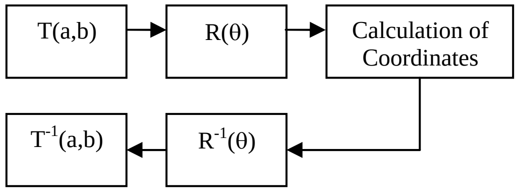
3. Calculation of coordinates for a shortest path

3.1 The coordinates transformation

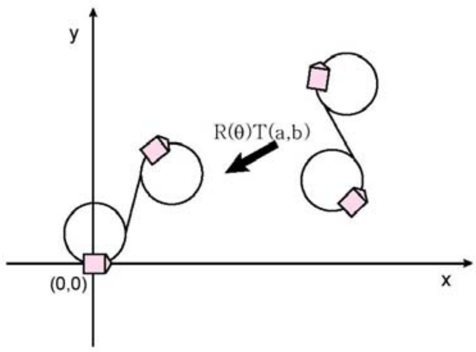
The first step of calculating a shortest path for Dubins robot is to transform an arbitrary position to an origin. The position translated by T(a,b) , and then rotated by R(θ) as in Fig 2. After all process of calculation, the position is transformed to the original position in a reverse order.

Standard forms have type LSL and type LSR. Type RSR is symmetrical to type LSL, and type LSR is symmetrical to type RSL about x-axis. Calculation of Type LSL and type LSR can be applied to type RSR and type RSL in symmetrical forms, which are explained in Sec. 3.1 and 3.3.

After transformation the arbitrary position to the origin, the angle of the target configuration is set to π for type LSL, and 0 for type LSR. These cause the angle differences between the original and the standard form. Calculations of the difference angles are very essential for the proposed formulae. In Sec 3.4 and 3.5, the details of the formulae are shown.



(Fig.2) Process of the coordinates transformation



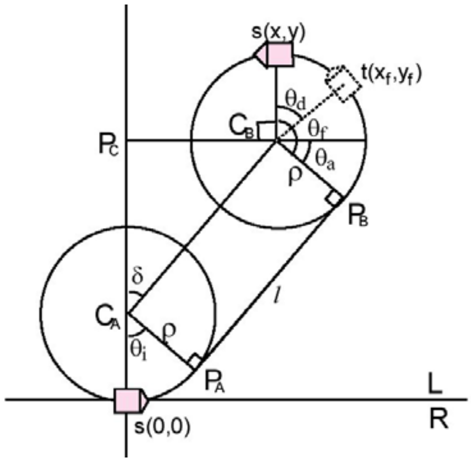
(Fig.3) Examples of Dubins shortest paths

3.2 The case of type LSL

The initial (at the original point) and the final (after reaching at the final point) forms are located at an arbitrary position and direction. These forms are transformed to the forms in Fig. 4 by the coordinate transformation. Type LSL consists of rotation-linear motion-rotation. When a robot rotates clockwise by θ_i on the basis of the center C_A of a circle with the rotational radius of ρ there exists the cross point P_A of the circle and a straight line. And P_B is the cross point of the tangent at P_A and the target circle which revolves round C_B as a center. The length of the line $\overline{P_A P_B}$ is l. The robot reaches to the standard form by revolving round C_B by θ_f. The goal is to obtain the coordinates of P_A and P_B by calculating θ_i and θ_f since the standard form s which is the result of rotation by θ_d from the position t.

The length l equals to the distance between $C_A(0,\rho)$ and $C_B(x, y-\rho)$ as the following equation.

$$l = \sqrt{x^2 + \left(y - 2\rho\right)^2} \qquad (1)$$



(Fig.4) The standard form of type LSL

The angle δ between C_A and C_B can be calculated as follows.

$$\delta = \tan^{-1}\left(\frac{|x|}{|y-2\rho|}\right) \tag{2}$$

Then, θ_i which revolves counterclockwise round C_A as a center is obtained.

$$\theta_i = \pi - \left(\delta + \frac{\pi}{2}\right) \tag{3}$$

The line $\overline{C_A C_B}$ runs parallel to $\overline{P_A P_B}$ and crosses at the right angles with $\overline{P_B C_B}$. Consequently the angle $\angle C_A P_C C_B$ makes $(\pi - \delta)$. And then the angle θ_a is obtained as follows.

$$\theta_a = \pi - (\pi - \delta) - \frac{\pi}{2} = \delta - \frac{\pi}{2} \tag{4}$$

According to the equation (4), the angle θ_f can be determined.

$$\theta_f = \theta_a + \frac{\pi}{2} = \delta - \frac{\pi}{2} + \frac{\pi}{2} = \delta \tag{5}$$

The coordinates of P_A is obtained by revolving the original point counterclockwise round $C_A(0,\rho)$ as a center.

$$x_A = \rho \sin(\theta_i)$$
$$y_A = \rho \cos(\theta_i) \tag{6}$$

In the same manner, the coordinates of P_B is obtained by rotating $s(x,y)$ clockwise by θ_f centering around $C_B(x, y-\rho)$.

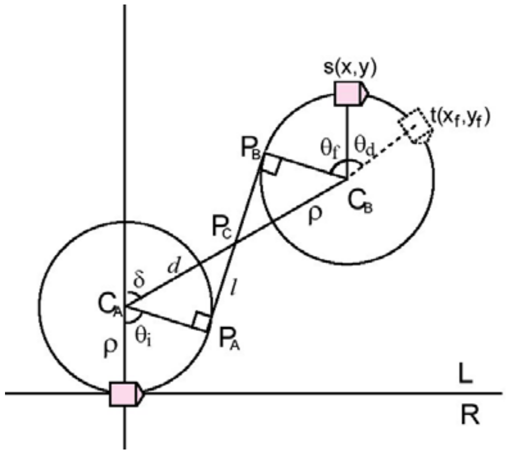
$$x_B = x + \rho \sin(\theta_f)$$
$$y_B = y - \rho + \rho \cos(\theta_f) \tag{7}$$

3.3 The case of type LSL

Fig.5 represents the coordinates transformation an arbitrary point to the origin.

The initial position of type LSR is $P_i(0,0)$, and the target position is $t(x_f, y_f)$.

To transform to the standard forms, rotation of $t(x_f, y_f)$ by θ_d reaches $s(x,y)$.



(Fig.5) The standard form of type LSR

The direction of the initial position and the standard position is set to 0 degree, respectively.

The rotational radius of the initial position is ρ and it is rotated counterclockwise by θ_i about C_A.

The rotation of the standard position $s(x,y)$ counterclockwise by θ_f reaches the point P_B.

We assume the length of $\overline{P_A P_B}$ is $2l$, the length of $\overline{P_A P_C}$ is l, and the distance $\overline{C_A P_C}$ is d. The distance d is represented by $C_A(0,\rho)$ and $C_B(x, y-\rho)$ as follows.

$$d = \sqrt{x^2 + (y-2\rho)^2} / 2 \tag{8}$$

The distance l is represented by the distance d as follows.

$$l = \sqrt{d^2 - \rho^2} \tag{9}$$

The angle β is represented as follows.

$$\beta = \tan^{-1}\left(\frac{\rho}{l}\right) \tag{10}$$

The angle δ can be calculated as follows.

$$\delta = \tan^{-1}\left(\frac{x}{y-2\rho}\right) \tag{11}$$

Because the angle α is $\alpha = \pi/2 - \beta$, θ_i can be represented as follows.

$$\theta_i = \pi - \alpha - \delta = \frac{\pi}{2} + \tan^{-1}\left(\frac{\rho}{l}\right) - \tan^{-1}\left(\frac{x}{y - 2\rho}\right) \qquad (12)$$

Because the circles of the initial point A and the target point B is symmetrical to the point P_C, the angle θ_i is identical to θ_f.

$$\theta_f = \theta_i \qquad (13)$$

The point P_A is calculated by rotating the origin by θ_i about C_A.

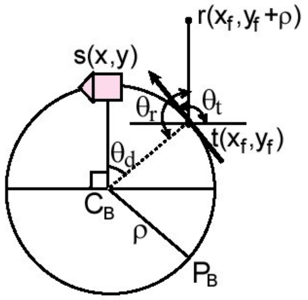
$$x_A = \rho \sin(\theta_i)$$
$$y_A = \rho \cos(\theta_i) \qquad (14)$$

The point P_B is calculated by rotating $s(x, y)$ counterclockwise by θ_f about C_B as follows.

$$x_B = x + \rho \sin(\theta_f)$$
$$y_B = y - \rho + \rho \cos(\theta_f) \qquad (15)$$

3.4 Conversion to the standard form-type LSL

In type LSL, the standard form of the target point is determined by setting $\theta_s = \pi$ at the position $s(x, y)$. Consequently the robot toward θ_t must be revolved counterclockwise by θ_d on the basis of C_B. Only $t(x_f, y_f)$ and θ_t are known values. The coordinates of C_B are calculated by using these values. Then the coordinate $s(x, y)$ is obtained.



(Fig.6) Conversion to type LSL standard form

The position $r(x_f, y_f + \rho)$ at intervals of ρ from $t(x_f, y_f)$ in the direction of y is calculated. And then that point is located at C_B by rotating process. The angle θ_r has the same value as θ_t which is the angle of the tangent at $t(x_f, y_f)$ (Eq. (16)).

$$\theta_r = \theta_t \qquad (16)$$

Then, the center of a circle C_B based on the rotation angle θ_r can be obtained as following equations.

$$x_{C_B} = x_f - \rho \sin(\theta_t)$$
$$y_{C_B} = y_f + \rho \cos(\theta_t) \qquad (17)$$

The point $s(x, y)$ is located at intervals of $+\rho$ from C_B in the direction of y. Therefore the coordinates of that point are as the equation (18).

$$x = x_f - \rho \sin(\theta_t)$$
$$y = \rho + y_f + \rho \cos(\theta_t) \qquad (18)$$

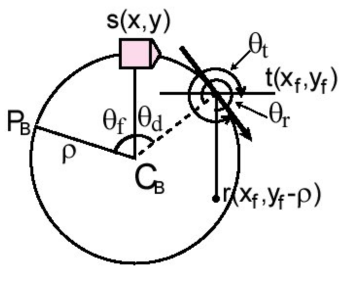
3.5 Conversion to the standard form-type LSR

In type LSR, the standard form of the target point is determined by setting $\theta_s = 0$ at the position $s(x, y)$. The position $r(x_f, y_f - \rho)$ at intervals of $-\rho$ from $t(x_f, y_f)$ in the direction of y is obtained. Then the point is moved to the point C_B by rotating the point by θ_r. The coordinates of C_B rotated by θ_r are as the equation (19) because the angle θ_r is the same as the angle θ_t.

$$x_{C_B} = x_f - \rho \sin(\theta_t)$$
$$y_{C_B} = y_f + \rho \cos(\theta_t) \qquad (19)$$

The point $s(x, y)$ is located at intervals of $+\rho$ from C_B in the direction of y. Therefore the coordinates of $s(x, y)$ are as the equation (20).

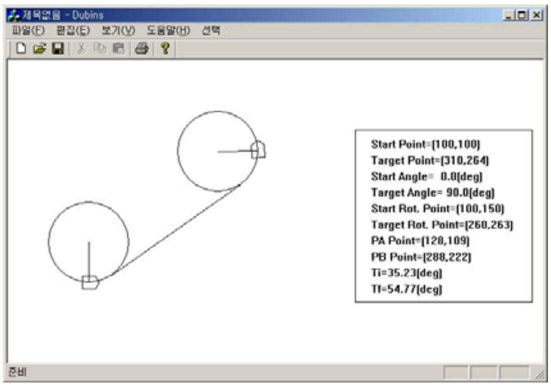
$$x = x_f - \rho \sin(\theta_t)$$
$$y = \rho + y_f + \rho \cos(\theta_t) \qquad (20)$$



(Fig.7) Conversion to type LSR standard form

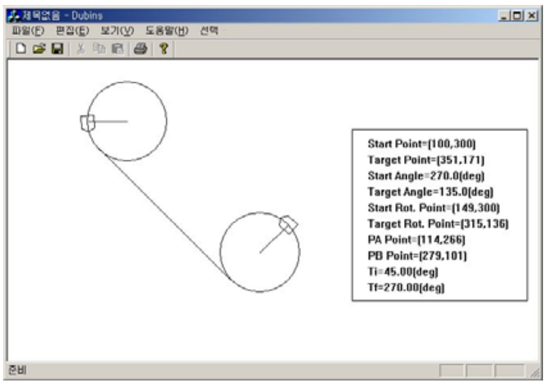
4. Simulation

Fig. 8 and 9 represent the results of simulation for type LSL. In Fig. 8, the coordinates of the initial point of a robot are (100,100) and those of the target point are (310,264). The angles at the initial point and at the target point are 0 and 90 respectively. The units of all coordinates are pixel. The coordinates of C_A and C_B which are the centers of rotations are (100,150) and (260,263), respectively. θ_i and θ_f obtained by the equation (4) and (5) are the degree of 35.23 and 54.77 respectively. The cross point of the initial circle obtained by applying these rotational angles to the equation (6) and (7) and a straight line P_A is (128,109). And that of the target circle and a straight line P_B is (288,222).



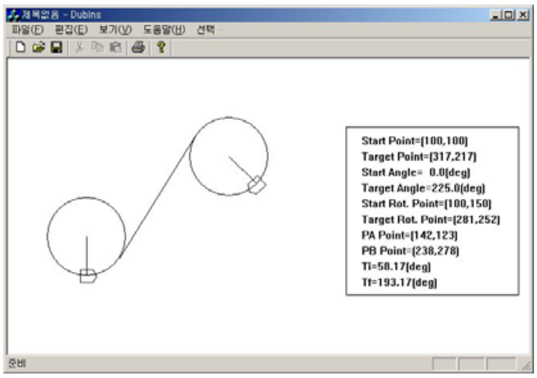
(Fig.8) Case 1 for type LSL

In Fig. 9, the position and the direction of the initial point of a robot are (100,300) and the degree of 270 respectively. And those of the target point are (351,171) and the degree of 135. The centers of rotations of the initial point are C_A=(149,300) and C_B=(315,136). The cross point of the initial circle and a straight line P_A is (114,266) and that of the target circle and a straight line P_B is (279,101). θ_i and θ_f have the degrees of 45 and 270 respectively.



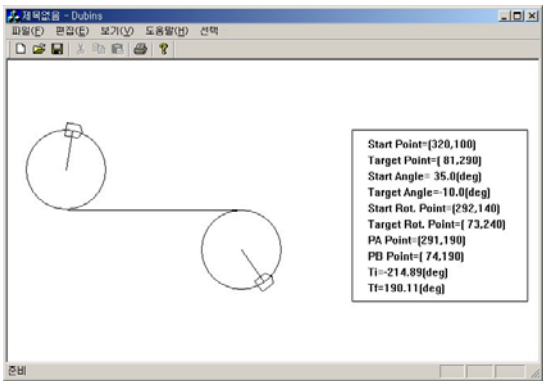
(Fig.9) Case 2 for type LSR

The results of simulation for type LSR are represented in Fig. 10 and 11. In Fig. 10, the position of the initial point of a robot is (100,100) and the angle is the degree of zero. The position and the direction of the target point are (317,217) and the degree of 225. The centers of rotation of the initial point C_A and C_B are (100,150) and (281,252). The cross point of the initial circle and a straight line P_A is (142,123). And that of the target circle and a straight line P_B is (238,278). θ_i and θ_f have the degrees of 58.17 and 192.17 respectively.



(Fig.10) Case 1 for type LSR

In Fig. 11, the position of the initial point of a robot is (320,100) and the angle is the degree of 35. The position and the direction of the target point are (81,290) and the degree of -10. The centers of rotation of the initial point C_A and C_B are (292,140) and (73,240). The cross point of the initial circle and a straight line P_A is (291,190). And that of the target circle and a straight line P_B is (74,190). θ_i and θ_f have the degrees of -214.89 and 190.11 respectively.



(Fig.11) Case 2 for type LSR

5. Conclusions

This paper is about Dubins' car-like robot in which the shortest path between the initial form and the final form can be accomplished. We derived the formulae for calculating the rotational angles at the initial form and the final form to obtain the coordinates of the connection points between C→S and S→C in the type CSC by using the known initial and the final form values. Our method for deriving the formulae is novel and simple in comparison with other researches[10-12]. Introducing the standard forms makes it possible to derive these simple formulae. And we also proposed the method for the coordinates transformation from a final form to a standard form.

Although the fundamental types in Dubins' method are classified into the type CSC and type CCC, the range in our research was limited to type CSC. Some other methods are required for the type CCC because the radius of the initial form and that of the final form are overlapped in the type. In addition, a study for grafting our results on the Reeds and Shepp's method in which moving forward and backward is possible must be performed as a further study.

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