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Three-dimensional Chained Nonholonomic Systems Stabilization Control via Dynamic Feedback

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Abstract

Three kinds of global universal controllers are proposed for nonholonomic systems, namely, the universal exponential regulators, the universal K-exponential controller and the universal practical controller. With help of an introduced state and the dynamic feedback technique, a controller with special structure is constructed to obtain an augmented closed-loop error system. The error system tends to continuous oscillation. So the controller structure is modified to loosen the control objective to practical stability and the error can converge to a neighborhood of origin as small as possible. Thus, oscillation and peaking phenomena are avoided and engineering precision is obtained. Detailed simulations on the three-dimension chained system are carried out, and the results show the effectiveness of the proposed controllers.

Keywords*: Nonholonomic systems; Universal Controller; Point Stabilization*

1. Introduction

In recent years, with the development of the mobile robot technology and aerospace technology, the control problems of nonholonomic systems get the attention of the scholars around the world. Because the constraint equations of this kind of system always contain differential terms and nonholonomic constraint equation [1], it's called the nonholonomic system.

In engineering practice, there are a lot of nonholonomic systems, the causes of this phenomenon are various, some of which are caused due to the moving characteristics of the object itself, such as for pure roll, no sliding movement of the ball and sliding on the ice skate, wheeled mobile robot [2-4], *etc.*; Also some of them considers actual situation and are artificially added, such as flexible manipulators[5], intelligent vehicle [6], the surface ship [7], *etc.*; Some of them are caused by under-actuated, such as satellites in space and the space shuttle, space robot [8], *etc*. These systems have common characteristics: they are constrained by the nonholonomic constraint equation, namely in the constraint equations of the nonholonomic system contain at least one differential item, and unintegrable constraint equation. Because the approximate linearization of nonholonomic systems are uncontrolled, the continuous static state feedback control law cannot be applied to realize the whole state of stabilization [9-10]. It has important theoretical significance for noholonomic system control.

Based on such typical nonholonomic wheeled mobile robot system to study the point stabilization and trajectory tracking control problem, when the control task switching between point stabilization and trajectory tracking, require the system to switch to the corresponding controller, and this kind of switch is likely to make the system produce shock, the components damage [11-13]. In order to avoid this kind of switch, scholars put forward the unified controller, the controller of processing a variety of control tasks at the same time.

At present, the main content of the control problem of nonholonomic systems research including object model establishment and transform, motion planning, fixed point stabilization and trajectory tracking and optimization control and so on.

Many industrial controls contain both fixed point stabilization and trajectory tracking [14-18]. Traditional switcher will cause the following problems:

(1) Switching can cause the shock, it is harm to the system;

(2) It is hard to determine the switch time exactly, the delay features of the practical system will cause the system worsen. It is necessary to study and apply to the unity of the fixed point stabilization and trajectory tracking controller.

The content of reference [19] introduce the dynamic feedback technology (but not for the linearization) for the second order nonholonomic system, the global continuous time invariant dynamic feedback controller is designed, without any switching, obtained the exponential convergence speed, but didn't get the asymptotic stability, when $t \to \infty$, systems tend to be more persistent oscillation.

According to the content of reference [20], a small constant $\delta > 0$ can be introduced for three-dimensional chain systems in order to improve the structure of controller except any switches [21-22]. It can eliminate persistent oscillation and obtain the exponential of convergence speed.

2. Global Universal Control

2.1. Universal Exponential Regulator

The mathematical model of three-dimension chain systems is as follows:

$$\dot{x}_1 = u_1, \dot{x}_2 = u_2, \dot{x}_3 = x_2 u_1 \tag{1}$$

Where $X = [x_1, x_2, x_3]^T$ is the system state, (u_1, u_2) is control input. The reference mode of system (1) is

$$\dot{x}_{1r} = u_{1r}, \dot{x}_{2r} = u_{2r}, \dot{x}_{3r} = x_{2r} u_{1r} \tag{2}$$

Where $X_r = [x_{1r}, x_{2r}, x_{3r}]^T$ is the system state, (u_{1r}, u_{2r}) is control input.

Where $X_r = [x_{1r}, x_{2r}, x_{3r}]^T$ is the system state, (u_{1r}, u_{2r}) is control input.

2.2. Universal K-exponential Controller

To obtain asymptotic stability, choose K–class function $k_0(o)$ as the initial value of auxiliary dynamic variables x_{2d}, where $o \,\square\, \|\boldsymbol{X}_e(0)\|$.

Theorem 1 If the system is $k_2 > 2k_1 > 0, k_3 > k_1$, the initial value of x_{2d} is

$$x_{2d}(0) = \sqrt{-x^2{}_{1\varepsilon}(0) + \sqrt{x^4{}_{1\varepsilon}(0) + o^{4\varepsilon}}} \le o^{\varepsilon}, 0 < \varepsilon < 1 \tag{3}$$

The continuous control law is as follows:

$$\begin{aligned} & x_e(0) = 0, u_1 = u_{1r}, u_2 = u_{2r} \\ & u_1 = u_{1r} - k_1 x_{1e} - k_3 x_{2d}\, z/V \\ & u_2 = u_{2r} + \dot{x}_{2d} - k_2 e_2 \\ & \dot{x}_{2d} = -k_1 x_{2d} + k_3 x_{1e}\, z/V \end{aligned} \tag{4}$$

(u_1, u_2) is in bounded and the X_ε is K global exponential stability.

Demonstrate

1) When $X_\varepsilon(0) = 0, X_\varepsilon(t) = 0 (\forall t \ge 0)$, so (u_1, u_2) is in bounded.

2) Suppose $\boldsymbol{X}_e(0) \neq 0$ .By the equation (3) and the $a^2 + b^2 \leq (a+b)^2$ (if $ab > 0$) we known,

$$V(0) = \frac{1}{2}\left(x_{1s}^2(0) + x_{2d}^2(0)\right) \leq \frac{1}{2}\left(o + o^s\right)^2 \tag{5}$$

$$V(0) = \frac{1}{2}\left(x_{1s}^2(0) + x_{2d}^2(0)\right) = \frac{1}{2}\sqrt{x_{1s}^4(0) + o^{4s}} \geq \frac{1}{2}o^{2s} \tag{6}$$

For short, define $O = o + o^{\varepsilon}$, time derivative of the closed-loop system V satisfies:

$$\begin{aligned} &\dot{V} = -2k_1 V \Rightarrow 0 < V(t) = V(0)e^{-2k_1 t} \\ &\Rightarrow \max\left(|x_{1\varepsilon}(t)|, |x_{2d}(t)|\right) \leq \sqrt{2V(t)} \leq Oe^{-2k_1 t} \end{aligned} \tag{7}$$

From equation (3) and $k_2 > 2k_1$,

$$|e_2(t)| = |e_2(0)|e^{-k_2 t} = |x_{2\varepsilon}(0) - x_{2d}(0)|e^{-k_2 t} \leq Oe^{-2k_1 t} \tag{8}$$

From equation (7) and (8),

$$|x_{2\varepsilon}(t)| \leq |e_2(t)| + |x_{2d}(t)| \leq 2Oe^{-k_1 t} \tag{9}$$

According to the equation $h = e_2 x_{2d}/V$ and equation (6)-(9),

$$|e_2(t)|\max\left(|x_{1\varepsilon}(t)|, |x_{2d}(t)|\right) \leq O^2 e^{-3k_1 t} \tag{10}$$

$$|h(t)| \leq O^2 e^{-3k_1 t} \Big/ \left(\frac{1}{2}o^{2\varepsilon}e^{-2k_1 t}\right) \leq 2\left(o^{1-\varepsilon} + 1\right)e^{-k_1 t} \tag{11}$$

There exists strict increasing function $c_2(o)$ which satisfies

$$\lim_{t \to \infty}|h(t)| = 0, \int_0^{\infty}|h(t)|dt \leq c_2(o) \tag{12}$$

According to the definition f and equation from (7)-(10),

$$\begin{aligned} |f| &\leq |x_{2d}u_{1r}| + 2|x_{1\varepsilon}u_{2r}| + (k_2 - k_1)|e_2 x_{1\varepsilon}| + (|e_2| + |x_{2\varepsilon}|)|u_{1r}| \\ &\leq 3MOe^{-k_1 t} + (k_2 - k_1)O^2 e^{-3k_1 t} + MO\left(e^{-k_1 t} + 2\right)e^{-k_1} \\ &\leq k_0(o)e^{-k_1 t} \end{aligned} \tag{13}$$

Where $\kappa_0(o) = 6MO + (k_2 - k_1)O^2$ is K-class function. By the definition of z , we know:

$$\begin{aligned} |z(0)| &\leq 2|x_{3\varepsilon}(0)| + |x_{1\varepsilon}(0)||x_{2\varepsilon}(0)| + 2|x_{2\gamma}(0)||x_{1\varepsilon(0)}| \\ &\leq 2o + 2|x_{2\gamma}(0)|o + o^2 \end{aligned} \tag{14}$$

Consider the fourth system equation, both f and h converge to zero with the exponential of k_1 and $k_2 - k_1$ respectively.

From equation (12)-(14), there is constant (c_4, c_5, c_6) satisfies the following equation:

$$|z(t)| \leq \left(c_4|z(0)| + c_5\kappa_0(0)\right)e^{c_6 c_2(0)}e^{-\kappa_1 t}$$

Substitute equation (14) to the above equation, there is the K-class function $\kappa_1(\cdot)$ satisfies

$$|z(t)| \leq \kappa_1(o)e^{-\kappa_1 t} \tag{15}$$

Where $\kappa_1(o) = \left[c_4\left(2o + 2|x_{2r}(0)|o + o^2\right) + c_5\kappa_0(o)\right]e^{c_6 c_2(o)}$ is K-class function.

From the equation (7),

$$|x_{2r}(t)x_{1\varepsilon}(t)| \leq \left(Mt + |x_{2r}(0)|\right)Oe^{-\kappa_1 t} \leq \left(c_3 + |x_{2r}(0)|\right)Oe^{-\kappa_1 t} \tag{16}$$

According to the definition of z, and equation (7), (9), (15), (16), there is the following equation:

$$\begin{aligned}|x_{3e}(t)| &\le \frac{1}{2}\left(|z(t)|+|x_{1\varepsilon}(t)||x_{2\varepsilon}(t)|+2|x_{2r}(t)x_{1\varepsilon}(t)|\right)\\ &\le \frac{1}{2}\left(\kappa_1(o)e^{-\kappa_1 t}+O^2e^{-2\kappa_1 t}+2\left(c_3+|x_{2r}(0)|Oe^{-\kappa_4 t}\right)\right)\\ &\le \kappa_2(o)e^{-\kappa_4 t}\end{aligned} \tag{17}$$

Where $\kappa_2(o)=\frac{1}{2}\left[\kappa_1(o)+O^2+2\left(c_3+|x_{2r}(0)|\right)O\right]$ is K-class function.

From equation (7), (9) and (17) ,there is

$$\|X_\varepsilon(t)\| \le |x_{1\varepsilon}(t)|+|x_{2\varepsilon}(t)|+|x_{3\varepsilon}(t)| \le \kappa_3(o)e^{-\kappa_4 t} \tag{18}$$

Where $\kappa_3(o)=3O+\kappa_2(o)$ is K-class function. So $\boldsymbol{X}_e(t)$ is K-exponential stable.

3) Suppose $\boldsymbol{X}_e(0)\neq 0$, to prove (u_1,u_2) is in bounded. By the equation (4),(6),(7), (15),

$$\begin{aligned}|u_1(t)| &\le |u_1(t)|+k_1|x_{1\varepsilon}(t)|+k_1|x_{2d}(t)||z(t)|/V\\ &\le M+k_1Oe^{-k_1 t}+\frac{k_1Oe^{-k_1 t}\kappa_1(o)e^{-k_1 t}}{\frac{1}{2}o^{2\delta}e^{-2k_1 t}}\end{aligned}$$

Above all, both O and $\kappa_1(o)$ are the same order infinitesimal of o^{ε} .So $|u_1(t)|$ is in bounded. So $|\dot{x}_{2d}(t)|$ is in bounded in the same way, and $|u_2(t)|$ is also in bounded.

By the way, (u_1,u_2) will not converge to zero except fixed-point stabilization.

4) Suppose $\boldsymbol{X}_e(0)\neq 0$, to prove (u_1,u_2) is continuous. According to equation (1)-(3), only have to prove that when $\boldsymbol{X}_e(0)\to 0$ ($o\to 0$), there is $(u_1,u_2)\to(u_{1r},u_{2r})$. From definition z there is the following equation:

$$\begin{aligned}&\lim_{o\to 0} z = 2x_{3\varepsilon}(0)-x_{1\varepsilon}(0)x_{2\varepsilon}(0)-2x_{2r}(0)x_{1\varepsilon}(0)\\ &\Rightarrow \left|\lim_{o\to 0} z\right| \le \left(2+|x_{2\varepsilon}(0)|+2|x_{2r}(0)|\right)o\end{aligned}$$

From (6) and (14), there is the following equation:

$$\left|\lim_{o\to 0} x_{2d}(0)z(0)/V(0)\right| \le \lim_{o\to 0} o^{\delta}\left(2+|x_{2\varepsilon}(0)|+2|x_{2r}(0)|\right)o/\left(\frac{1}{2}o^{2\delta}\right)$$

From (4), it can obtain:

$$\lim_{o\to 0} u_1(0)=u_{1r}(0)-\lim_{o\to 0}\left(k_1x_{1\varepsilon}(0)+k_3x_{2d}(0)z(0)/V(0)\right)=u_{1r}(0)$$

So u_1 is continuous. It can be proved that u_2 is also continuous in the same way.

2.3. Universal Practical Controller

For universal practical control, the system tracking error converges to a small neighborhood of origin. Because the neighborhood can be arbitrarily small, therefore, for the engineering application, it can ensure accuracy.

Theorem 2 If $k_2>2k_1>0, k_3>k_1$, error bound is $\delta>0$, choose $x_{2d}(0)\neq 0$, from the smooth control law:

$$\begin{cases} u_1=u_{1r}-k_1x_{1\varepsilon}-k_3x_{2d}z/(V+k_4) \\ \quad u_2=u_{2r}+\dot{x}_{2d}-k_2e_2 \qquad ,0\le k_4\le(2k_3-k_1)\delta/k_1 \\ \dot{x}_{2d}=-k_1x_{2d}+k_3x_{1\varepsilon}z/(V+k_4)\end{cases} \tag{19}$$

State trajectory $\boldsymbol{X}_e(t)$ is ultimately global bounded, control input (u_1,u_2) is in bounded,. $\lim_{t\to\infty}(u_1(t),u_2(t))=(u_{1r}(t),u_{2r}(t))$

Demonstrate A closed loop system is as follows:

$$\begin{cases} \dot{x}_{1\varepsilon} = -k_1 x_{1\varepsilon} - k_3 x_{2d} z/(V+k_4) \\ \dot{x}_{2d} = -k_1 x_{2d} - k_3 x_{1\varepsilon} z/(V+k_4) \\ \dot{e}_2 = -k_2 e_2 \\ \dot{z} = -2k_3 V z/(V+k_4) - k_3 h z + f \end{cases} \tag{20}$$

Where $h = e_2 x_{2d}/(V+k_4)$, V and f are defined in (9) and (10) respectively, time derivative $\dot{V}$ satisfies $\dot{V} = -2k_1 V \Rightarrow V(t) = V(0)e^{-2k_1 t}$, so $(x_{1\varepsilon}(t), x_{2d}(t))$ converges to zero with the speed of k_1 exponential. Finally, consider the definition of z, $|x_{2r}(t)x_{1\varepsilon}(t)|$ is exponential convergence, in the same way, it is practical stability.

Because $V+\delta$ has lower bound, so both $x_{2d}z/(V+k_4)$ and $x_{1\varepsilon}z/(V+k_4)$ can converge to zero, $(u_1, u_2)$is in bounded, and $\lim_{t\to\infty}(u_1(t), u_2(t)) = (u_{1r}(t), u_{2r}(t))$.

From demonstrate of Theorem 2, because the numerator of $x_{2d}z/(V+k_4)$ and $x_{1\varepsilon}z/(V+k_4)$ are both exponential convergent, the denominator has lower bound ($\delta > 0$), so it avoid the peaking phenomenon in the theorem 1 completely.

3. The Simulation Verification

Considering the unicycle mobile robot kinematics model as follows:

$$\dot{x} = v\cos\theta, \dot{y} = v\sin\theta, \dot{\theta} = \omega$$

From a simple transformation,

$$\begin{aligned} & x_1 = \theta, x_2 = x\cos\theta + y\sin\theta, x_3 = y\sin\theta - x\cos\theta \\ & u_1 = \omega, u_2 = v - \omega x_3 \end{aligned} \tag{21}$$

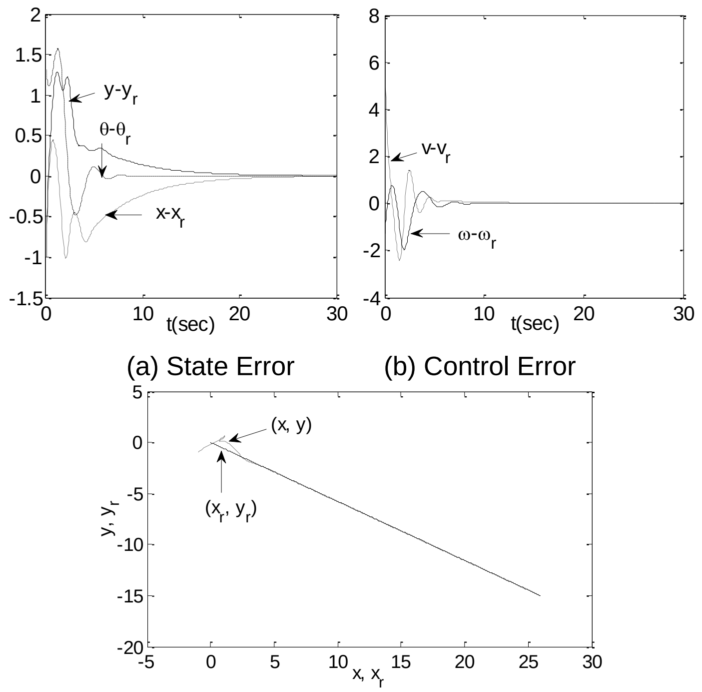
It can obtain a three-dimension chain system $\dot{x}_1 = u_1, \dot{x}_2 = u_2, \dot{x}_3 = x_2 u_1$.

Suppose that the system initial state is $(x, y, \theta) = (-1, -1, \pi/4)$,initial state of reference system is $(x_r, y_r, \theta_r) = (0, 0, -\pi/6)$. Choose the controller parameters as

$$k_1 = 0.2, k_2 = 2, k_3 = 1.5, k_4 = 1.4, \delta = 0.1.$$

3.1. Straight Line Tracking

Target trajectory is a straight line, using the controller mentioned in Theorem 2. The results of the simulation are shown in Figure 1, the controller mentioned in this section can track the reference line better and faster.



(a) State Error (b) Control Error

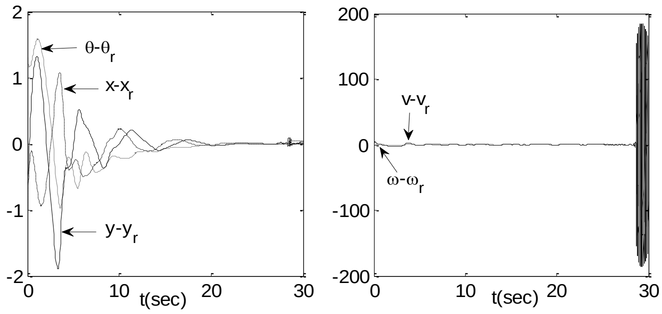
(c) Geometrical Locus

Figure 1. Straight Line Tracking Simulation Time 30 Seconds

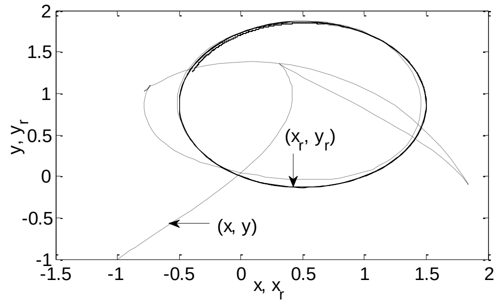
It seems that the peaking phenomenon does not appear in Figure 1, in fact that is wrong. Only when $V(t)$ is closed to zero, the peaking phenomenon will appear. The larger the k_1 is, the sooner peaking phenomenon appears; When k_1 is constant, the peaking phenomenon will appear finally as long as the simulation time is long enough.

3.2. Circumference Tracking

Target trajectory is the circumference of radius 1, using the controller that mentioned in Theorem 2. The results of the simulation are shown in Figure 2. From Figure 2(b), the severe peaking phenomenon is appeared at about 27 seconds. In Figure 2(a) there is peaking phenomenon too. Therefore, state error can't really converge to zero, asymptotic exponential convergent properties cannot be achieved practically.



(a) State Error (b) Control Error

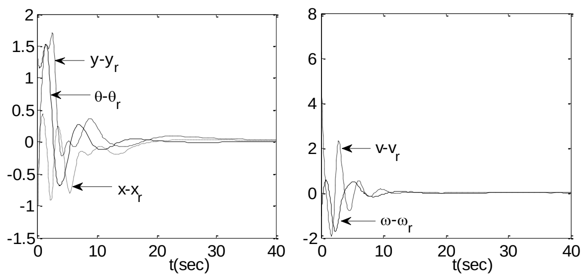


(c) Geometrical Locus

Figure 2. Circumference Tracking

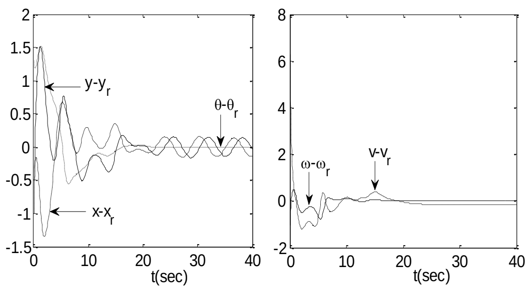
3.3. The Universal Practical Controller

The universal practical controller is mentioned in Theorem 2, simulations on straight line tracking and circular trajectory are shown in Figure 3 and Figure 4.



(a) State Error (b)Control Error

Figure 3. The Practical Tracking Straight Trajectory



(a) State Error (b) Control Error

Figure 4. The Practical Tracking Circular Path

The simulation time of Figure 3 and Figure 4 is the same as Figure 1 and Figure 2,but there is no peaking phenomenon in Figure 3 and Figure 4. This shows that the peaking phenomenon is eliminated according to theorem 3. However, the state error curve of Figure3 and Figure4 doesn't converge to zero, but converge to a small neighborhood of zero.

In addition, in Figure 3 and Figure 4, other control error exponential converge to zero expect $v - v_r$ in Figure 4. It seems to be inconsistent with the conclusion of Theorem 2. The reason is the transformation from robot coordinate system $(x, y, \theta, v, \omega)$ and three-dimensional chain coordinate system $(x_1, x_2, x_3, u_1, u_2)$. Considering the coordinate transformation (21), Theorem 2 shows that (u_1, u_2) will exponential converge to (u_{1r}, u_{2r}) so $(\omega, v - \omega x_3)$ will exponential converge to $(\omega_r, v_r - \omega_r x_{3r})$. In Figure 3, $\omega_r = 0$,so $v \to v_r$.In Figure 4, ω_r is a nonzero constant, and x_3 can only converge to the neighbor of x_{3r}, v can only converge to v_r as well.

4. Conclusion

For the first order nonholonomic system, paper studies three universal controller based on dynamic feedback, realizes the exponential of convergence, asymptotic stability and the practical stability respectively, which can be used to point stabilization and trajectory tracking at the same time without modifying. Detailed simulations on the three-dimension chained system are carried out, and the results show the effectiveness of the proposed controllers.

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