

A Novel Multi-Wing Chaotic System and Circuit Simulation

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Abstract

Base on a 3-D chaotic system, a new chaotic system generating three-wing and four-wing chaotic attractors is constructed by introducing a linear term. The phase diagrams, Lyapunov exponent spectrums and bifurcation diagrams of the system are studied by numerical simulation. Furthermore, the system is simulated by circuit, and the three-wing and four-wing chaotic attractors are observed. The circuit simulation results are in agreement with numerical simulation results.

Keywords: chaos; multi-wing chaotic system; circuit simulation

1. Introduction

Since Lorenz found the first chaos model in 1963 [1], people have had a great interest on the chaotic phenomena of the nonlinear systems. Researchers have further understood chaos by constructing and analyzing a large number of chaotic systems [2-5]. In recent years, a class of multi-wing chaotic systems [9-11] with more complicated dynamic behavior attracted much attention. A chaotic system generating three-wing and four-wing chaotic attractors was constructed in Ref. [9]. A chaotic system generating two-wing, three-wing and four-wing chaotic attractors was constructed in Ref. [10]. A chaotic system generating one-wing, two-wing, three-wing and four-wing chaotic attractors was constructed in Ref. [11]. Also, Ref. [12] presented a four-wing and double-wing 3D chaotic system based on sign function. Ref. [13] reported a four-wing hyper-chaotic attractor generated from a 4D memristive system. Ref. [14] presented a 5D hyper-chaotic attractor with four-wing. As the chaotic systems have more complicated dynamic behavior, it is very significant to construct the multi-wing chaotic systems.

In the following section, we present a new chaotic system generating three-wing and four-wing chaotic attractors by introducing a linear term. In the Section 3, the phase diagram of the new system is studied through numerical simulation. In the Section 4, the Lyapunov exponent spectrum and bifurcation diagram of the new system are studied through numerical simulations. In the Section 5, an electronic circuit is designed to implement the system. Finally, a summary and conclusions are drawn in Section 6.

2. Multi-Wing Chaotic System and Its New System

The pseudo four-wing chaotic system [15], which is presented by Liu and Chen, can be described as follows

$$\begin{cases} \dot{x} = ab/(a+b)x - yz, \\ \dot{y} = -ay + xz, \\ \dot{z} = -bz + xy, \end{cases} \quad (1)$$

where, $a = 10$, $b = 4$.

By introducing a linear term in the third equation of system (1), a new system is given by

$$\begin{cases} \dot{x} = ab/(a+b)x - yz, \\ \dot{y} = -ay + xz, \\ \dot{z} = -bz + xy + cy + x, \end{cases} \quad (2)$$

where, $a = 10$, $b = 4$, and c be a constant to be determined.

3. Improved Model Simulation

Let $c = 0.2$, the three Lyapunov exponent values of system (2) are obtained by numerical simulation. And they are equal to 1.2009, -0.0374 and -12.3063, respectively. Obviously, the largest Lyapunov exponent value is greater than zero. The system is in a chaotic state and its phase diagram is shown in Figure 1(a). Let $c = 4$, the three Lyapunov exponent values of system (2) are obtained by numerical simulation. And they are equal to 0.5956, -0.1289 and -11.6096, respectively. Obviously, the largest Lyapunov exponent value is greater than zero. The system is in a chaotic state and its phase diagram is shown in Figure 1(b). From the Figure 1, we can see that the system (2) is able to generate three-wing and four-wing chaotic attractors.

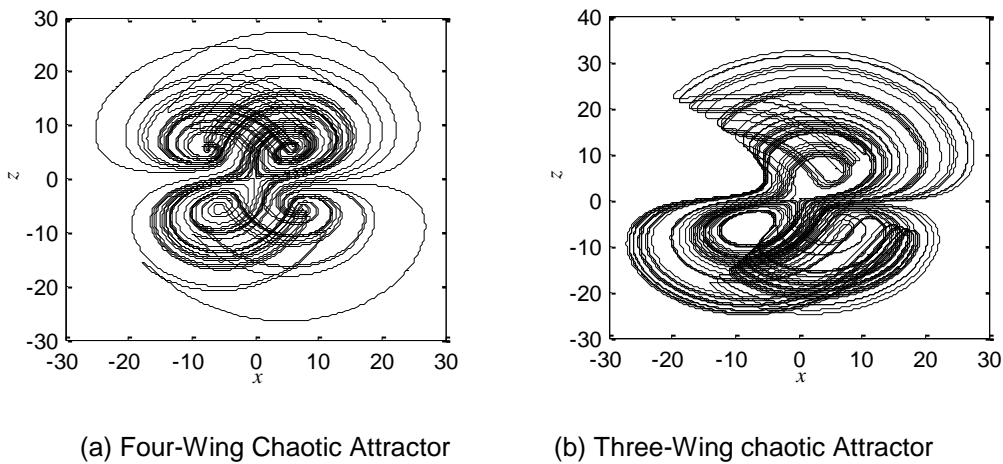


Figure 1. The Phase Diagrams of System (2) in $x - z$ Phase Plane

4. Basic Dynamic Behavior

The Lyapunov exponent spectrum of system (2) and its bifurcation diagram are shown in Figure 2 when $0 < c \leq 7$. From the Figure 2(a), we can see that the largest Lyapunov exponent of system (2) is equal to zero when $c \in [4.5, 4.6] \cup [5.1, 5.3] \cup [5.9, 7]$. The system is in a periodic state and the phase diagram shows in Figure 3. When $c \in (0, 4.5) \cup (4.6, 5.1) \cup (5.3, 5.9)$, the largest Lyapunov exponent of the system is greater than zero. The system is in a chaotic state and the phase diagrams are shown in Figure 1. The bifurcation diagram is in agreement with the Lyapunov exponent spectrum as shown in Figure 2(b).

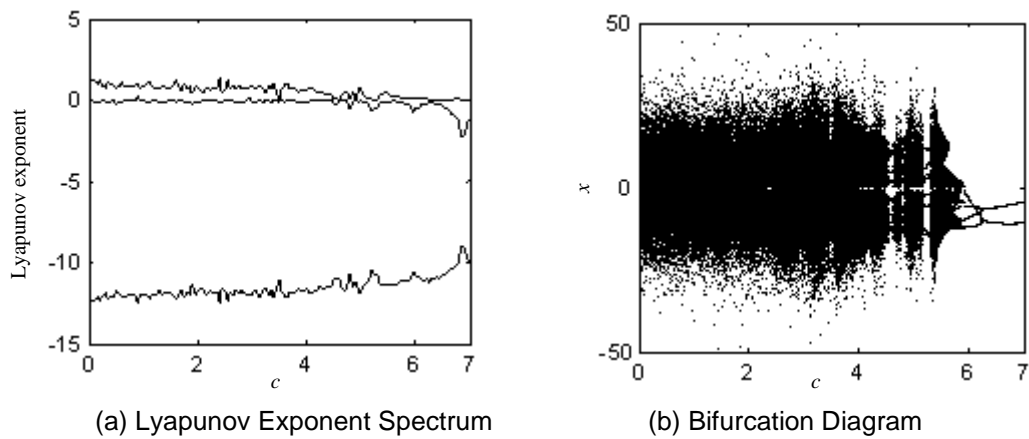


Figure 2. Lyapunov Exponent Spectrum of System (2) and Its Bifurcation Diagram

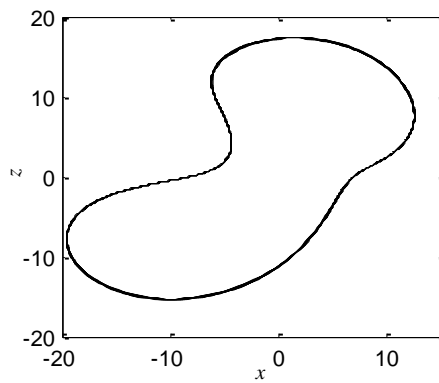


Figure 3. The Phase Diagram When System (2) Is in a Periodic State

5. Circuit Results

From the phase diagrams in Figure 1, it shows that the highest level of signal in a chaotic state is about 40 voltages. In order to ensure the circuit works normally, variable-scale attenuation transform is executed to the system (2) at first. And the system (2) can be changed as below.

$$\begin{cases} \dot{x} = ab/(a+b)x - 10yz, \\ \dot{y} = -ay + 10xz, \\ \dot{z} = -bz + 10xy + cy + x, \end{cases} \quad \square \square \square \quad (3)$$

According to system (3), the circuit was designed as shown in Figure 4.

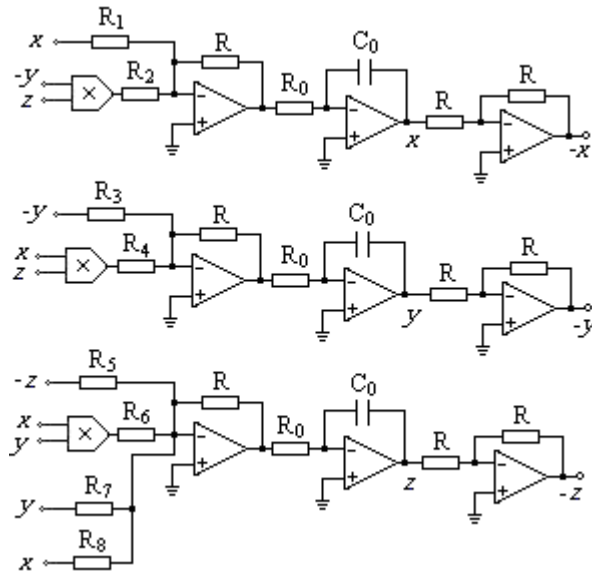


Figure 4. Circuit Diagram of System (3)

In the Figure 4, the gain of the multiplier AD633JN is 0.1. The circuit equation can be obtained as follows

$$\begin{cases} \frac{dx}{dt} = \frac{1}{R_0 C_0} \left(\frac{R}{R_1} x - \frac{R}{10R_2} yz \right), \\ \frac{dy}{dt} = \frac{1}{R_0 C_0} \left(-\frac{R}{R_3} y + \frac{R}{10R_4} xz \right), \\ \frac{dz}{dt} = \frac{1}{R_0 C_0} \left(-\frac{R}{R_5} z + \frac{R}{10R_6} xy + \frac{R}{R_7} y + \frac{R}{R_8} x \right), \end{cases} \quad (4)$$

Let $R = 10\text{k}\Omega$, according (3) and (4), we have $R_1 = 35\text{k}\Omega$, $R_2 = R_4 = R_6 = 1\text{k}\Omega$, $R_3 = 10\text{k}\Omega$, $R_5 = 25\text{k}\Omega$, $R_8 = 100\text{k}\Omega$, $R_7 = 100/c\text{k}\Omega$, $R_0 C_0 = 0.1$. where $R_0 C_0$ is time scale factor. In order to capture the wave easily, we adjust $R_0 C_0 = 10^{-4}$, where $R_0 = 10\text{k}\Omega$, $C_0 = 10\text{nF}$.

Let $c = 0.2$, i.e. $R_7 = 500\text{k}\Omega$. The phase diagram of circuit simulation is shown in Figure 5(a). Let $c = 4$, i.e. $R_7 = 25\text{k}\Omega$. The phase diagram of circuit simulation is shown in Figure 5(b). Let $c = 6.5$, i.e. $R_7 = 15.38\text{k}\Omega$. The phase diagram of circuit simulation is shown in Figure 5(c).

We can see that the circuit simulation results in Figure 5 verify simulation results in Figure 1 and 3.

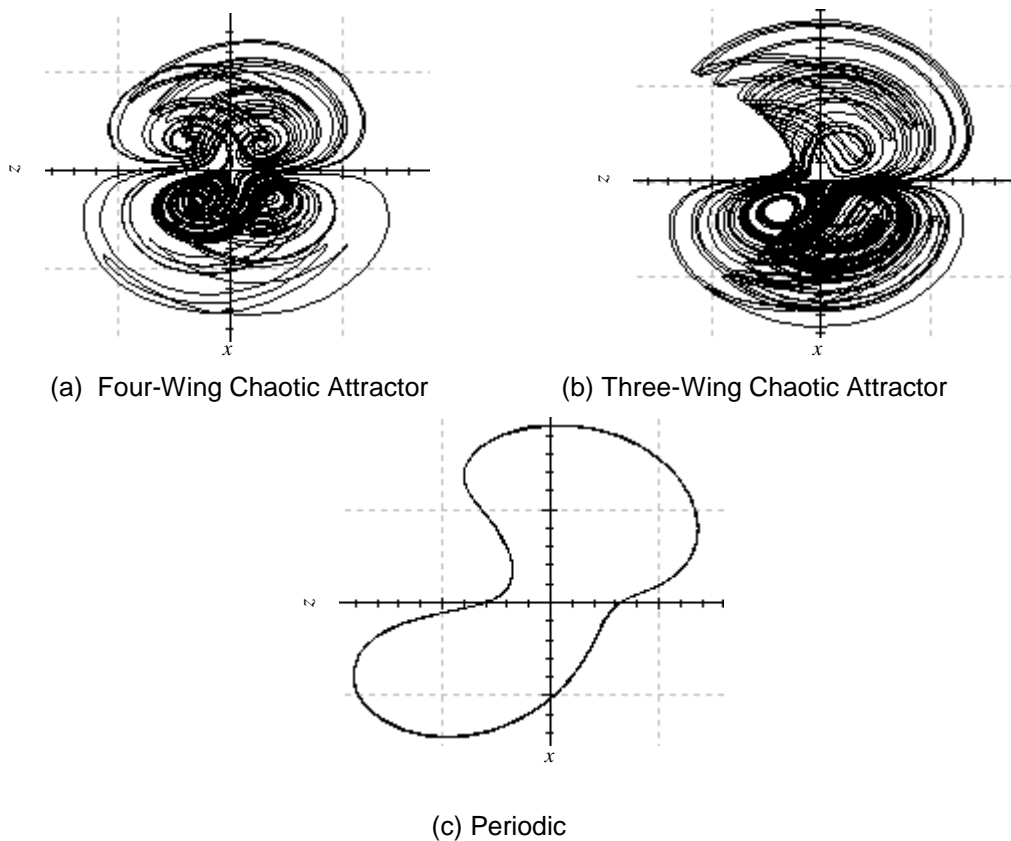


Figure 5. The Phase Diagrams of Circuit Simulation in X – Z Phase Plane

6. Conclusions

In the paper, a chaotic system has been constructed. Though the numerical simulation for the phase diagrams, it shows that the system is able to generate three-wing and four-wing chaotic attractors. The basic dynamic characteristic of the system is analyzed by the Lyapunov exponent spectrums and bifurcation diagrams. At the end, the system is simulated by circuit. The three-wing and four-wing chaotic attractors and the phase diagram of periodic are observed.

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