

## Resource Allocation of Two-relay Networks with Partially Differential Modulation

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### **Abstract**

*Relay networks enhance the system performance by creating cooperative diversity gain and extend communication range by relaying the received signal to other devices. In this paper, the performance of two-relay networks is analyzed by adopting decode-and-forward (DF) relaying protocol. The signaling scheme uses partially differential modulation which applies coherent modulation in source-relays links and differential modulation in relay-destination links. We first analyze the effect of relay location in the system performance. Based on this, we find the optimum relay selection which minimizes the symbol error rate (SER). Then, we explore the effect of energy allocation for the given relay location.*

**Keywords:** *Relay networks, cooperative communications, decode-and-forward, performance, SER*

### **1. Introduction**

In Wireless communications, relay networks have been received much attention for their benefits in communication coverage and system performance. By transmitting signal using relays, communication range is extended and shadowed areas may be vanished. At the destination node, the diversity gain can be achieved by combining the received signals from the several relay nodes, which results in the enhancement of the overall communication performance [1-3]. There two representative relaying protocols in relay networks. One is amplify-and-forward (AF) protocol and the other is decode-and-forward (DF) protocol. The former simply amplifies the received signal from the source node and forwards it to the destination node, the later decodes the received signal and remodulates and forwards it to the destination node. Depending on the usage of channel state information (CSI) at the receiver, the received signal can be demodulated as coherently or noncoherently. Noncoherent schemes are applied in fast fading channels such as vehicular or railway environment, which bypasses channel estimation by sacrificing the performance. Using AF and DF protocols, the performance of relay networks were widely studied both with coherent and noncoherent modulation schemes [3-10]. Resource allocation is one of critical factors to determine the performance of the overall communication systems. The effects of resource allocation in relay networks were analyzed with various conditions by regarding energy and location as the resource, and performance optimization was also investigated depending on the resource allocation [11-15]. However, most of existing works only consider symmetric modulation cases, *i.e.*, both source-relay and relay-destination links adopt coherent modulations or noncoherent modulations where differential modulation is commonly used for noncoherent schemes.

In this paper, we investigate the performance of relay networks by adopting partially differential modulation, *i.e.*, the source-relay and relay-destination links use coherent modulation and differential modulation, respectively. The performance of system is analyzed in the form of symbol error rate (SER) especially focusing on two-relay

scenarios with DF relaying protocol. We first consider the effect of relay location which minimizes SER, then we study the effect of energy allocation for the given relay location.

## 2. System Model

We consider two-relay networks when an arbitrary number of relay exists as shown in in Figure 1. Different from the general relay networks, we only consider two relays among several relay nodes. We denote one source node  $s$ , two relay nodes  $r_k, k = 1, 2$ , and one destination node  $d$ . The DF relaying protocol is used in here, *i.e.*, the relay remodulates the received signal from the source and forwards it to the destination. At the destination node, the received signal is combined to demodulate the received signal from the source node and relay nodes. We consider partial differential modulation scheme, in which the transmission between the source and the relays adopts coherent modulation, and the links between the relays and destination node use differential modulation. The data transmission is carried out time division duplex (TDD) manner. At the first time slot, the source node transmits coherently modulated signal to relay nodes. Then, the relay nodes remodulate the received signal from the source node with differential modulation and transmit to the destination node. The third time slot may be used depending on the usage of direct link between the source and destination node. If the direct link is employed, the source node transmits differently modulated signal to the destination node. Therefore, it is required three and two time slots for one data symbol transmission the transmission with direct link (WD) and without direct link (WOD), respectively. Then  $s-r$  links use coherent modulation, whereas  $r-d$  and  $s-d$  links use differential modulation.

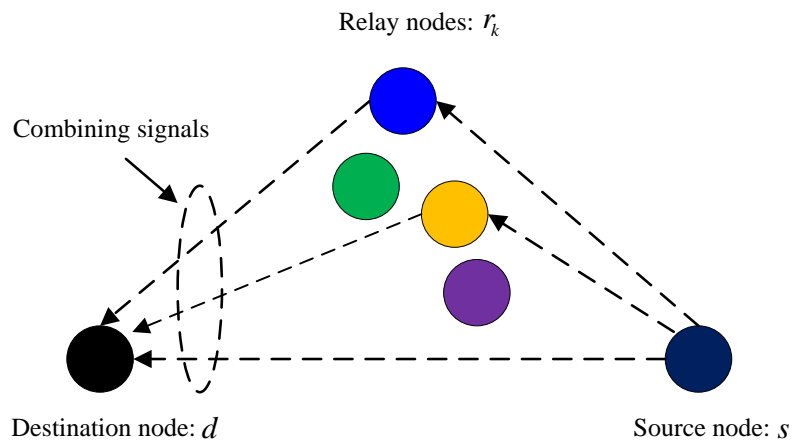


Figure 1. System Model

### 2.1. Channel Model

We let  $n$ th phase-shift keying (PSK) modulated symbol at the source node as  $s_n = e^{j2\pi m_n/M}, m_n \in \{0, 1, \dots, M-1\}$ , then the corresponding  $n$ th differentially modulated signal is  $x_n^s = x_{n-1}^s s_n$  with  $x_0^s = 1$ . In the direct link, *i.e.*,  $s-d$  link, the differentially encoded signal is transmitted. The received  $n$ th signal at the destination node is given by

$$y_n^{s,d} = \sqrt{E_s} h_n^{s,d} x_n^s + z_n^{s,d}. \quad (1)$$

For relay links, *i.e.*,  $s-r$  and  $r-d$  links, the source node broadcasts PSK signal to the relay nodes. The received signal at the relay nodes given by

$$y_n^{s,r_k} = \sqrt{E_s} h_n^{s,r_k} s_n + z_n^{s,r_k}, k = 1, 2. \quad (2)$$

The  $k$ th relay node demodulates the received signal from the source node coherently, and remodulates the signal with differential modulation scheme. Let us  $n$ th demodulated signal at the  $k$ th node as  $\hat{s}_n^{r_k} = e^{j2\pi m_n/M}$ ,  $m_n \in \{0, 1, \dots, M-1\}$ . Notice that this demodulated signal may be erroneous signal depending on the  $s-r$  link status. Then, the remodulated signal at the  $k$ th relay node which is given by  $x_n^{r_k} = x_{n-1}^{r_k} \hat{s}_n^{r_k}$  with  $\hat{s}_0^{r_k} = 1$  is transmitted to the destination node. The received signal at the destination node for each relay is given by

$$y_n^{r_k,d} = \sqrt{E_k} h_n^{r_k,d} x_n^{r_k} + z_n^{r_k,d}, k = 1, 2. \quad (3)$$

In equation (1), (2), and (3),  $E_i, i \in \{s, r_k\}$  represents energy per symbol at the node  $i$ , and fading coefficient  $h_n^{i,j}$  and noise  $z_n^{i,j}$  are zero-mean complex Gaussian distribution with variance  $\sigma_{i,j}^2$  and  $N_{i,j}, \forall i, j \in \{s, r_k, d\}$ , respectively. In this paper, we assume that all fading coefficients are independent. We also assume that all noise components are independent and identically distributed (i.i.d) with  $N_{i,j} = N_0, \forall i, j \in \{s, r_k, d\}$ . Then, we can find instantaneous signal-to-noise ratio (SNR) between the transmitter  $i$  and the receiver  $j$  as

$$\gamma_{i,j} = \frac{|h_n^{i,j}|^2 E_i}{N_0}, i, j \in \{s, r_k, d\}. \quad (4)$$

Then, the average SNR is given by

$$\bar{\gamma}_{i,j} = \frac{\sigma_{i,j}^2 E_i}{N_0}, i, j \in \{s, r_k, d\}. \quad (5)$$

## 2.2. Demodulation and Decision Rule

Since the coherent modulation scheme is applied between the source node and relay node, the demodulator at the  $k$ th relay is given by

$$\hat{s}_n^{r_k} = e^{j2\pi m_n'/M} : m_n' = \arg \max_m \Re \left\{ \left( h_n^{s,r_k} \right)^* e^{j2\pi m_n'/M} y_n^{s,r_k} \right\}, k = 1, 2 \quad (6)$$

where  $\Re(\cdot)$  and  $(\cdot)^*$  represent the real part and conjugate, respectively. At the destination node, the differentially modulated signals are combined for demodulation. The differential modulation signal in (1) and (3) can be represented as the following equation by omitting superscript

$$\begin{aligned} y_n &= h_n x_n + z_n = h_n (x_{n-1} s_n) + z_n \\ &= y_{n-1} s_n + z_{n-1} \end{aligned} \quad (7)$$

where  $z_{n-1} = z_n - z_{n-1} s_n$ . Therefore,  $y_n$  is complex Gaussian with mean  $y_{n-1} s_n$  and variance  $2N_0$ . Then the demodulator at the destination for the system with direct link and without direct link are given by

$$\begin{aligned} \hat{s}_n^d &= e^{j2\pi m_n^d/M} : m_n^d \\ &= \arg \max_m \Re \left\{ \left( y_n^{s,d} \right)^* y_{n-1}^{s,d} e^{j2\pi m_n^d/M} \right\} + \sum_{k=1}^2 \Re \left\{ \left( y_n^{r_k,d} \right)^* y_{n-1}^{r_k,d} e^{j2\pi m_n^d/M} \right\} \end{aligned} \quad (8)$$

and

$$\begin{aligned} \hat{s}_n^d &= e^{j2\pi m_n^d/M} : m_n^d \\ &= \arg \max_m \sum_{k=1}^2 \Re \left\{ \left( y_n^{r_k,d} \right)^* y_{n-1}^{r_k,d} e^{j2\pi m_n^d/M} \right\} \end{aligned} \quad (9)$$

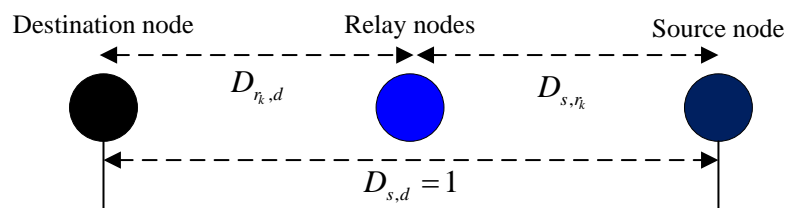
respectively. The only difference between equation (8) and (9) is the existence of direct link.

### 3. Performance Analysis

In this section we analyze the performance of proposed system. We first consider the effect of relay location and then discuss about energy allocation based on the relays' location. We adopt the average symbol error rate (SER) as the performance of the system. By assuming two-relay networks, we analyze the optimum performance depending on the location of two relays. Then, the relay selection which minimizes SER is investigated. We use line topology to analyze the effect of relay locations. As depicted in Figure 1, we assume that the source node is located at (0,0) and the destination node is located (1,0), and two relays  $r_1$  and  $r_2$  can be located an arbitrary location at  $(r_1, r_2), \forall r_k \in (0,1), k=1,2$ . To find optimum location, we let the relationship between the variance of the channel fading coefficient  $\sigma_{h_{i,j}}^2$  and the inter-node distance  $D_{i,j}$  as

$$\sigma_{h_{i,j}}^2 = C \cdot D_{i,j}^{-\alpha}, i, j \in \{s, r_k, d\} \quad (10)$$

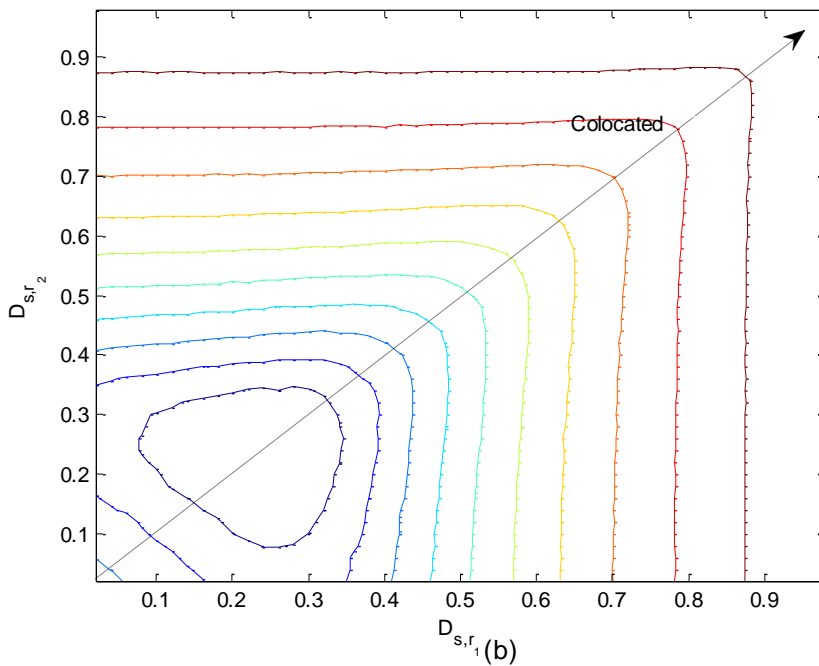
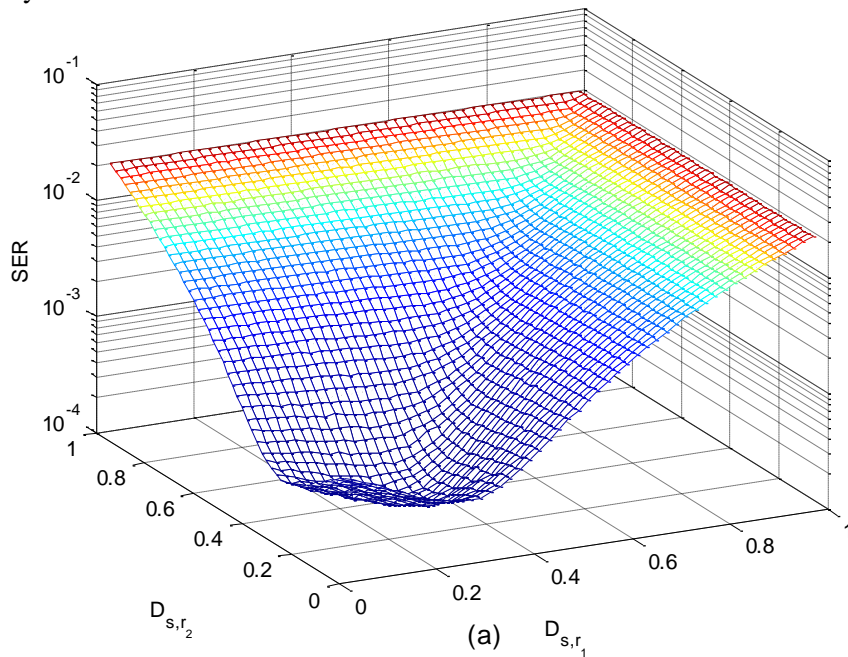
where  $\alpha$  is the path loss exponent of the wireless channel and  $C$  is a constant so we set to 1. Then, our goal is to find the optimum location of two relays which provides minimum SER for the given  $D_{s,d} = 1$ .



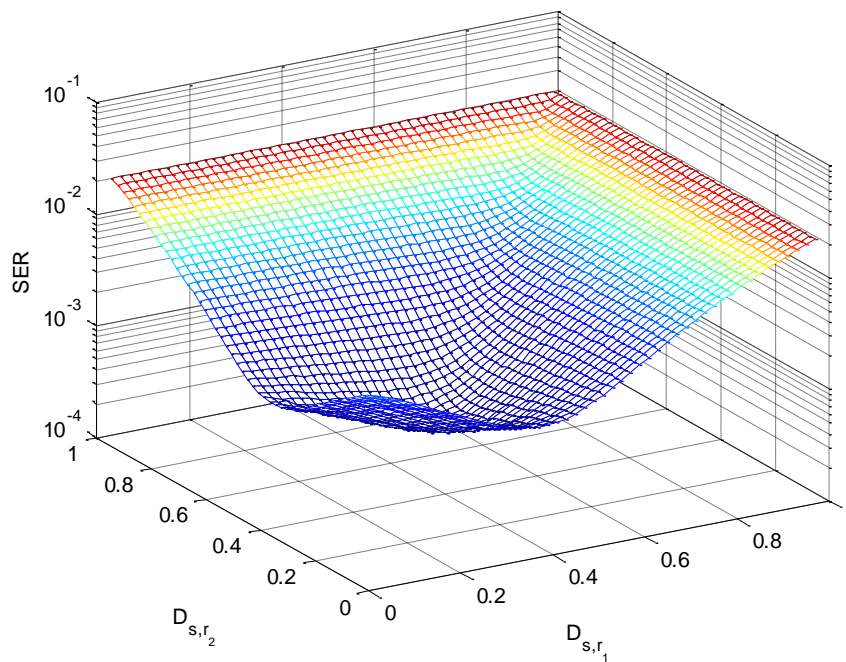
**Figure 2. Network Topology**

For SER simulation, we let  $\alpha=4$  and the transmit SNR at the source  $\rho_s = E_s / N_0$  and relay node  $\rho_{r_k} = E_{r_k} / N_0, k \in \{1,2\}$  are the same. Figure 3 represents SER of the system with direct link. We set  $\rho_s = \rho_{r_k} = 10dB$  and both inter-node distances  $D_{s,r_k}$  and  $D_{r_k,d}$  are changed from 0 to 1 with the constraint of  $D_{s,r_k} + D_{r_k,d} = D_{s,d}$ . Each  $x$  and  $y$  axis represent location of relay node and  $z$  axis shows the corresponding SER for the given location. Figure 3 (a) is SER surface depending on the various locations of relays, and Figure 3(b) represents its

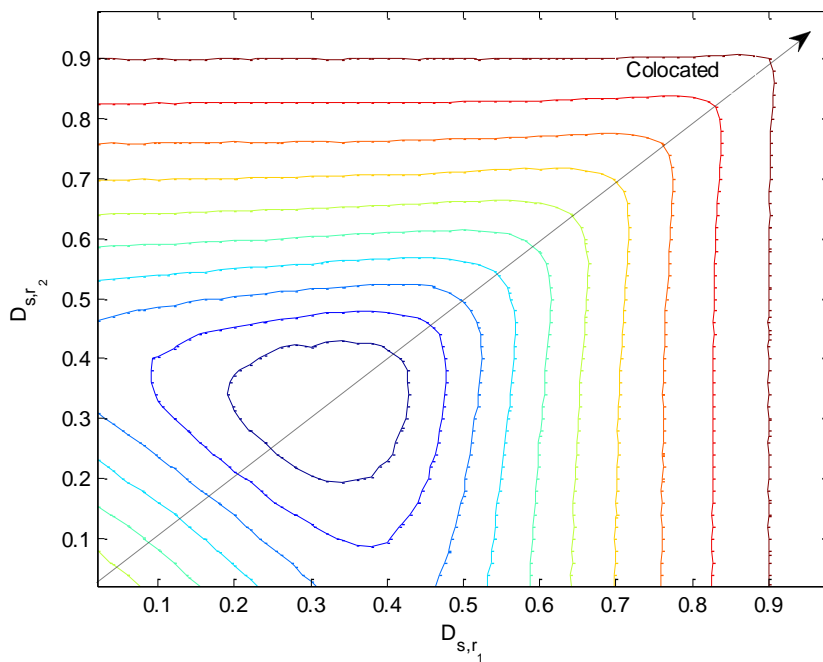
corresponding contour. From the figures, we can find that there exists the optimum point which provides the minimum SER for the given SNR. In general, SER decreases as relays move toward to the source. However, the optimum location of relays is not the closest location of the source. The figures also reveal that the optimum location can be achieved when two relays are colocated. This is due to the fact that a one contaminated link decreases the overall system performance. In two-relay networks, for the given location of relay 1, the optimum SER can be achieved by selecting the same location of relay 2. In practical system, we may enhance the performance of relay networks by choosing the relays which have the same or nearby location.



**Figure 3. SER depending on Various Relay Locations; Systems with Direct Link (a) SER Curve (b) SER Contour**



(a)



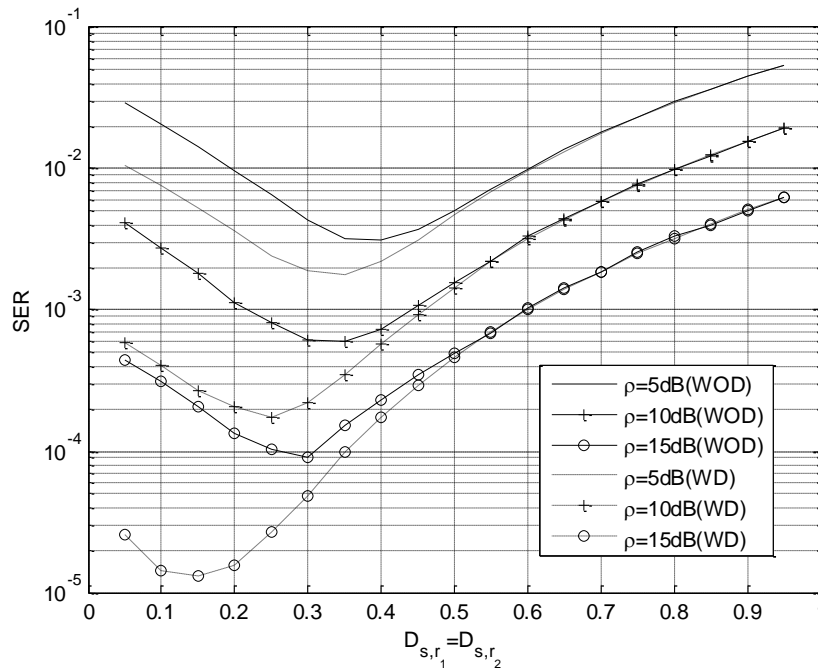
(b)

**Figure 4. SER Depending on Various Relay Locations; Systems without Direct Link (a) SER Curve (b) SER Contour**

Figures 4(a) and 4(b) depict SER surface and its corresponding contour when the direct link does not exist. Similar to the previous case, the minimum SER can be achieved when the relays are located near the source and two relays are colocated. However, the optimum location is different from the system with direct transmission.

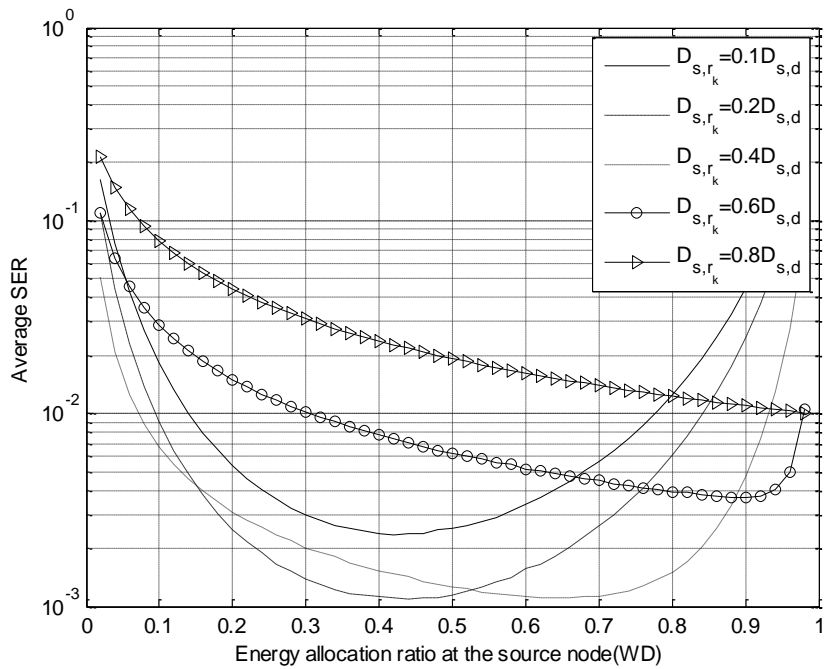
The optimum location is little bit far from the source compared with the system with direct link. This is maybe due to the diversity gain of direct link.

Motivated by above points, we plot SER of the system by locating two relays at the same point in Figure 5. We consider various SNRs by letting  $\rho = \rho_s = \rho_k$  and the scenarios with and without direct link. The figure shows that the optimum point moves toward the source as SNR increases and diversity gain increase. From the figure, we can find that the minimum SER can be achieved if we have information of relays for practical implementation. This finding can be applied for general cases, *i.e.*, the system with an arbitrary number of relays.

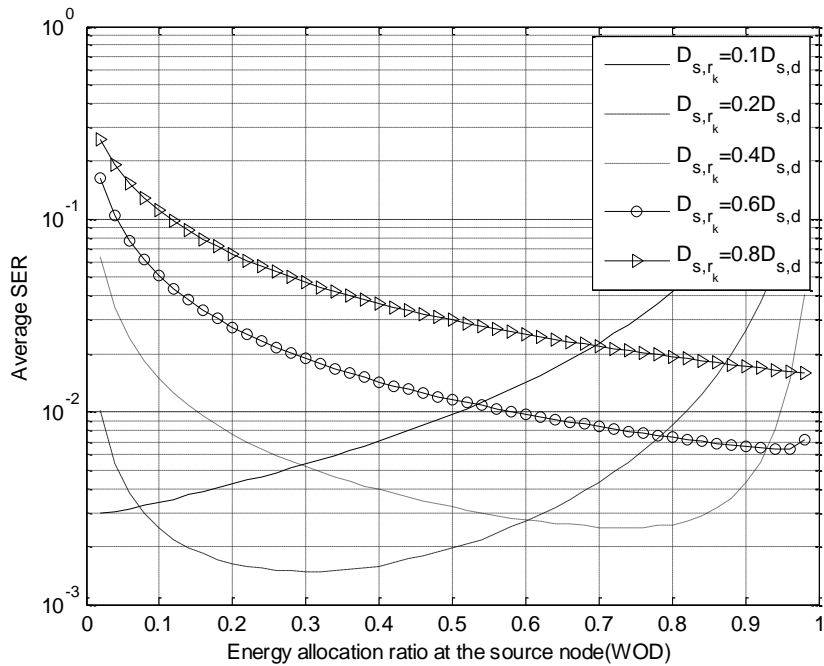


**Figure 5. SER with Collocated Scenarios both the Systems without Direct Link and With Direct Link**

Now, we consider the effect of energy allocation for the given relay locations. In the previous analysis, we conclude that the optimum performance can be achieved by selecting the relay nodes with the same location. We investigate the effect of energy allocation in the system performance by assigning various energies at the source node and relay nodes. We assume collocated relay nodes with the total energy constraint, *i.e.*,  $\rho_T = \rho_s + \rho_k$ . Figs. 6 and 7 represent the performance of two-relay networks depending on the energy allocation with direct link and without direct link, respectively. In the figures,  $x$  axis represents the energy ratio of source node with respect to the total energy. We set the total energy to 10dB. For the system with direct link, we can see that the optimum performance can be achieved by assigning near half of the total energy at the source node by selecting the relay nodes located near the source node. Whereas, for the system without direct link, we can achieve minimum SER by assigning approximated 30% of the total energy at the source node by selecting the relay nodes located near the source node. Notice that we can achieve the optimum performance by selecting the relay nodes which are closely located with the source node but not too close. If we locate the relay node very close to source, we may achieve excellent performance in  $s-r_k$  links.



**Figure 6. SER with Various Energy Allocations with the Same Relay Locations; System with Direct Link**



**Figure 7. SER with Various Energy Allocations with the Same Relay Locations; System without Direct Link**

However,  $r_k - d$  links shows bad condition for signal transmission which results in decrement of the overall system performance. The energy allocation results are slightly different for the system with and without direct links. The performance of the system with direct link is affected by diversity gain which is related to the direct



link, which causes the different result of energy allocation compared with the system without the direct link. Based on location and energy allocation results, we can achieve the global optimum performance which minimizes SER for the given location and energy allocations.

#### 4. Conclusions

In this paper, we consider the optimum relay selection and energy allocation of two-relay networks. DF protocol is adopted by applying partially differential modulation. The simulation results reveal that the optimum performance can be achieved when two relays are located at the same position for both systems with direct link and without direct link. The optimum location moves toward the source when SNR and the number of relays increase. The energy allocation which provides the optimum performance is different depending on the existence of direct link. Our performance analyses confirm that the both location and energy optimization can provide performance enhancement. We may achieve the global minimum SER by applying location and energy allocation iteratively.

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