

Blind Super-Resolution Image Reconstruction Based on Weighted POCS

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Abstract

In the paper a weighted Projection onto to convex sets (POCS) algorithm for image super-resolution reconstruction is proposed, which using POCS scheme to reconstruct image without knowing the degradation model and related parameters. In the proposed algorithm, adaptive block matching is utilized to estimate translational motion among various images firstly, and then APRX algorithm is adopted to estimate the point spread function (PSF) and the ideal high resolution image. Finally based on the traditional POCS algorithm, the paper introduced the prior information about low resolution image definition into POCS to calculate the residual and control the threshold by weighted parameter of image relative definition, and realized the dynamic threshold selection in the whole progress of reconstruction. Results of experiments show that the algorithm in the objective evaluation and visual effects achieves better reconstruction results, which confirm the validity and robustness of the proposed algorithm.

***Keywords:** Blind super-resolution reconstruction; Projection onto convex sets; Motion estimation; Point spread function; Image average gradient*

1. Introduction

In the process of image acquisition, the movement of scene or camera shake will make the original image distorted. In addition, because of the atmospheric disturbances, the difference of optical system, scattered set and the inherent limitations of the image sensor, it is difficult to obtain high resolution images. The super-resolution reconstruction is a technology to estimate one or more non-deformed high resolution images by a set of degraded, deformed and blurred low-resolution images.

So far many kinds of super-resolution image reconstruction methods has been proposed, such as maximum likelihood estimation, maximum a posteriori estimation, POCS method, fast Fourier technologies and so on [1-2]. In the reconstruction, solving the image distortion means to deal with the problem of registration; If not considering the extraction operator and geometry translation, which is just a blind deconvolution image restoration; Assuming fuzzy factor PSF known, and geometric distortion is suppressed to a sub-pixel displacement, which is a traditional problem of super-resolution image reconstruction. In practice, if taking into account these points above, it is the problem of blind super resolution reconstruction. In the most case of practical applications, fuzzy function of the system is unknown or with function type known, blind super resolution reconstruction combine the blur function identification with super-resolution reconstruction technique.

Usually blind super resolution reconstruction methods can be divided into two categories: one is that fuzzy function identification and the super-resolution reconstruction are independent, which means to identify the fuzzy function first and then reconstruct; the other way is blur function identification and the super-resolution rate reconstruction interact simultaneously. The classic blind algorithms are: Nguyen [3]

proposed a fuzzy function identification based on GCV for super-resolution image reconstruction, Wirawan [4] proposed the multichannel blind super resolution reconstruction, Hu [5] proposed a regularization blind super resolution image reconstruction algorithm based on least squares criteria, and Qin [6] proposed a blind super-resolution video reconstruction algorithm based on the error - parametric analysis.

This paper is aim to the blind super-resolution image reconstruction method considering point spread function PSF, in which reconstruct high resolution image without knowing the PSF based on the weighted POCS. The algorithm proposed can simultaneously estimate the unknown fuzzy function in the reconstruction process.

2. Observation Model

Imaging system is a positive non-linear process, thus SR reconstruction is a reverse inversion process, so we first need to create a degradation model to simulate the positive process of the imaging system, which is also the progress to simulate and simplify the nonlinear imaging system by using linear approximation. Degradation model describes the linear relationship between ideal HR and actual LR acquired. The ideal image through a series of degradation processes (including geometric deformation, blurring, down-sampling and noising), result in the sequence of LR. The linear model of imitating the degradation process is shown in Figure 1.

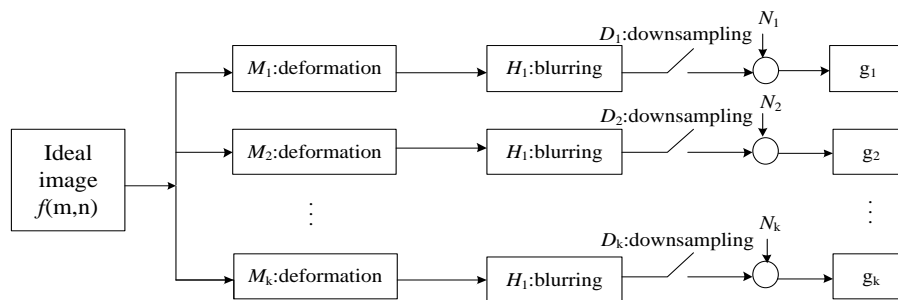


Figure 1. Degradation Model

The model can be expressed as:

$$g_k(x, y) = D_k H_k M_k f(m, n) + N_k = W_k f(m, n) + N_k \quad (1)$$

Where, $f(m, n)$ is the ideal HR, which is the target of image super-resolution reconstruction; $g_k(x, y)$ is the k th LR image; M_k is the geometric deformation matrix; H_k represents the blurring matrix, which is decided by the point spread function (PSF) of the optical system; D_k is the down-sampling matrix of the system, and N_k represents the additive Gauss noise.

The degradation model describes the positive process of achieving LR from the ideal HR, while the super-resolution reconstruction is the process of reconstructing the ideal HR image from the series of LR images. In fact, the reconstruction of HR from these observations may get many different results. Mathematically this is a process of solving the over-determined equation. The diversity and uncertainty of the reconstruction results makes the reconstruction process become a pathological inverse problem, which solutions must be defined according to the prior knowledge. The data acquired by the surveillance camera or camera are the noise images that are deformed, fuzzy and down-sampled, which can be used as a priori knowledge in SR reconstruction. On the other hand, the actual degradation model and the related parameters are not clear, taking into account these factors, it is appropriate to using the projection onto convex sets.

3. Blind Super-Resolution Reconstruction Based on POCS

3.1 Motion Estimation

Motion estimation is a very important step in the SR reconstruction, and the estimation accuracy directly affects the final reconstruction results. Between the close-up images and remote sensing images there are generally only global geometric deformations such as offset, rotation and so on, affine transformation can be used for motion estimation [7]. The motion of different objectives in video images are independent, moving track, direction and speed are different from each other, so the geometrical deformations of video image are non-global, which can be estimated by block matching method [8] and optical flow method [9] *etc.*, where the complexity of block matching is lower.

In this paper, we use adaptive block matching algorithm [10] for motion estimation. Select appropriate search mode according to the motion characteristics of the image, which means faster computation algorithm for small motion block and higher accuracy algorithm for large motion block, so as to select the different search mode adaptively depending on the different motion blocks.

In the search process, due to the evaluation of the similarity of search blocks, the sum absolute error (SAD) is chosen as the criterion, which is defined as follow:

$$SAD(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |f_k(m, n) - f_{k-1}(m+x, n+y)| \quad (2)$$

Where, (x, y) is the displacement vector, f_k and f_{k-1} is respectively the image block of the current frame and the prediction block of the previous frame, $M \times N$ is the size of macro block.

An efficient fast block matching algorithm -DS algorithm is used for searching, the search mode of which are mainly two models [11]: big nine-points search mode (LDSP) and small five -points search mode (SDSP) based on diamond, as shown in Figure 2.

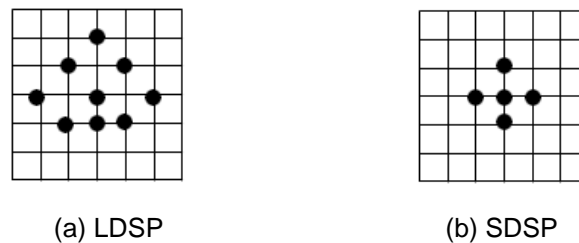


Figure 2. Search Mode

When the search distance to the center is less than two pixels, using LDSP causes excessive searches, but for some large, complex motion block, LDSP search range is too small, which causes inadequate search. For the above problem, we choose the adaptive search mode according to the level of current motion block for motion estimation. Calculation the SAD of the current block and reference block on the same position [9], if the SAD value is less than the pre-set error threshold, the motion vector of the current block is 0, and the search is stopped, thereby greatly reduce the calculation complexity of searching; if the block of the macro block is the first row (column) the motion of the current block is considered low-grade, for small movements, SDSP is directly used for searching, so that the search will not be with more ups and downs; otherwise the motion level of the current block is high, for large movements. In this way conduct a preliminary search using LDSP, then using SDSP for precise positioning. Finally, the motion vectors output.

3.2 PSF Estimation

Accurate PSF estimation is the key to remove the blurring effect. APEX is an excellent PSF estimation algorithm, which is fast and robust, and can be used for the reconstruction of G-class point spread function. Such as the Gauss function, Lorentzians function, as well as their convolution function and so on are all belong to the G-class PSF, so it has a strong representation [12-13].

The optical transfer function (OTF) of G-class point spread function is $H(\varepsilon, \eta)$:

$$H(\varepsilon, \eta) = \int_{\mathbb{R}^2} h(x, y) \exp[-2\pi i(\varepsilon x + \eta y)] dx dy = \exp[-\alpha(\varepsilon^2 + \eta^2)^\beta] \quad \alpha > 0, 0 < \beta \leq 1 \quad (3)$$

For a single frame video or sequence images, when considering only the fuzzy influence of the PSF, the relation of the ideal image and the degraded image is as follow:

$$g(x, y) = h(x, y) * f(x, y) + n(x, y) \quad (4)$$

The Fourier transform is:

$$G(\varepsilon, \eta) = H(\varepsilon, \eta)F(\varepsilon, \eta) + N(\varepsilon, \eta) \quad (5)$$

In the case of small noise, $N(\varepsilon, \eta)$ is ignored, and after normalization, it is considered that there is a region in the frequency domain making the Fourier transform $G_k(\varepsilon, \eta)$ of the image g_k , the Fourier transform $F(\varepsilon, \eta)$ of the image f and PSF satisfies the relation:

$$\log |G(\varepsilon, \eta)| \approx -\alpha(\varepsilon^2 + \eta^2)^\beta + \log |F(\varepsilon, \eta)| \quad (6)$$

$\log |F(\varepsilon, \eta)|$ can be replaced with constant $-A$, make ε, η taking different values for nonlinear least square fitting to $-\alpha(\varepsilon^2 + \eta^2)^\beta - A$, so as to obtain the estimation value of α, β and confirm the OTF model. Then the parameter (α, β) is inserted into equation (7), and with inverse Fourier transform the best estimate of the ideal high resolution image can be obtained.

$$F(\varepsilon, \eta) = \frac{\overline{H}(\varepsilon, \eta)G(\varepsilon, \eta)}{|H(\varepsilon, \eta)|^2 + K^{-2} |1 - H^s(\varepsilon, \eta)|^2} \quad (7)$$

Where, \overline{H} is the conjugation of H , K and s are adjustable parameters, $0.001 \leq s \leq 0.01$.

3.3 POCS algorithm

POCS (projection onto convex sets) algorithm based on the set theory means that high resolution image is contained in a well defined vector space, and the vector space contains some closed convex constraint set. The constraint convex set is defined as the limiting conditions for the feasible solutions of the vector space (e.g. positive definiteness, energy boundedness, observation consistency and smoothness, etc.). By successively projecting the initial value of the high resolution image to the restricted set, the intersection of these sets is obtained as the final estimate of the high resolution image [14].

The residual error limiter set for each pixel of the low resolution image g is defined:

$$C_{m_1, m_2} = \{ \hat{f}(i_1, i_2) : |r^{(g)}(m_1, m_2)| \leq \delta_0 \} \quad (8)$$

Where, $\hat{f}(i_1, i_2)$ represents the estimation of HR image, $r^{(g)}(m_1, m_2)$ is the residual error, δ_0 is the repaired threshold.

The calculation formula of the residual error is as follow:

$$r_i^{(g)}(m_1, m_2) = g(m_1, m_2) - \sum_{i_1=0}^{M_1-1} \sum_{i_2=0}^{M_2-1} f(i_1, i_2) W(m_1, m_2; i_1, i_2) \quad (9)$$

The right side of the above formula represents the difference between the actual pixel value $g(m_1, m_2)$ of the LR image in a point and the pixel value of the high resolution image to simulate the degradation process in the same point, which is denoted as the residual error. If $f(i_1, i_2)$ is the real HR image, the local statistics value of $r_i^{(g)}(m_1, m_2)$ is equal to the statistics value of the noise on the local area of the pixel. The threshold δ_0 can be obtained from the statistical data of the noise changed over time and space. W is the composite

matrix, M is obtained by motion estimation, H is determined by PSF estimation, and D is determined by the resolution enhancement factor Q .

The residual here actually reflects the difference between the reconstructed image and the real image, big residual means the difference between the image estimated and real image pixel value is big, and small residual means the estimation of image pixel value is closer to the true value. Therefore, by calculating the residual value, and then using the residual limit set to determine whether to meet the conditions, so as to modify the image, which purpose is to reduce the difference between the estimated image and real image. In order to make the calculation of the residual value is more accurate, the weighted residual method using the average gradient is put forward.

The average gradient is the physical quantity used to measure the image clarity, which can be used to reflect the tiny details of the images, and the formula is:

$$\overline{G} = \frac{\sum_{i=1}^{M-1} \sum_{j=1}^{N-1} \sqrt{[f(i, j) - f(i+1, j)]^2 + [f(i, j) - f(i, j+1)]^2} / 2}{(M-1) \times (N-1)} \quad (10)$$

Where, $f(i, j)$ refers to the pixel value of the image in (i, j) . M, N respectively is the total number of lines and columns of the image. The average gradient of n LR images in the same scene is $\overline{G}_1, \overline{G}_2, \dots, \overline{G}_n$. The n LR images' different clarity expressed on the average gradient are different, if the fixed residual repair threshold of the POCS reconstruction algorithm is used for image super-resolution reconstruction, there is no good use in the subtle differences of the image clarity, which is not favorable for the reconstruction, so we take different residual calculations due to the different clarity of the low resolution images. The formula (9) is modified as:

$$r_i^{(g)}(m_1, m_2) = k_i \times g(m_1, m_2) - \sum_{i_1=0}^{M_1-1} \sum_{i_2=0}^{M_2-1} f(i_1, i_2) W(m_1, m_2; i_1, i_2) \quad (11)$$

Where, $k_i = \overline{G}_i / \sum_{i=1}^n (\overline{G}_i / n)$, $i = 1, 2, \dots, n$

Take the value k_i as the weight coefficient to measure the relative clarity of the LR image [15], mainly on account of the reliability of the different clarity of the LR image pixel, and at the time of calculating the residual error, the reliability will be in the form of additional weighted in $g(m_1, m_2)$. For better clarity the weight of the LR is big, for poor clarity is small, thus the residual values can be calculated more accurately.

And in the traditional POCS algorithm δ_0 is usually a constant, the fixed selection mode for reconstruction is adverse. Therefore the selection of the threshold should be modified according to some prior knowledge of the image. It is hoped that the threshold can be changed due to the different position of the pixel to be repaired.

The initial HR image in the POCS reconstruction is divided into m blocks, and the weight coefficient of each block is calculated, that is:

$$k_p = \overline{G}_p / \sum_{p=1}^m (\overline{G}_p / m), \quad p = 1, 2, \dots, m \quad (12)$$

Where, \overline{G}_p represents the mean gradient of the p th image block in the HR image, k_p reflects the relative clarity of image block, that is how much of the image block contains details of. The higher clarity, the richer detail the image block has, and k_p is big, the corresponding threshold is smaller, the repair of the region is more meticulous; conversely for lower clarity k_p is big, the corresponding threshold is bigger, the repair of the region is rougher. Using the weighted coefficient to modify the threshold of the residual error:

$$C_{m_1, m_2} = \{ \hat{f}(i_1, i_2) : |r^{(g)}(m_1, m_2)| \leq \delta_0 / k_p \} \quad (13)$$

The projection on the image is defined as:

$$P_r[f(i_1, i_2)] = \begin{cases} f(i_1, i_2) & -\delta_0 / k_b \leq r_i^{(g_i)}(m_1, m_2) \leq \delta_0 / k_b \\ f(i_1, i_2) + \frac{r_i^{(g_i)}(m_1, m_2) - \delta_0 / k_b}{\sum \sum W^2} \mathbf{W}(m_1, m_2; i_1, i_2) & r_i^{(g_i)}(m_1, m_2) > \delta_0 / k_b \\ f(i_1, i_2) + \frac{r_i^{(g_i)}(m_1, m_2) + \delta_0 / k_b}{\sum \sum W^2} \mathbf{W}(m_1, m_2; i_1, i_2) & r_i^{(g_i)}(m_1, m_2) < -\delta_0 / k_b \end{cases} \quad (14)$$

The process of the blind super-resolution reconstruction based on POCS is expressed as follows:

1) Interpolate each frame of the low resolution image g_k for q times and use the first frame as the reference, determine the motion matrix M_k of each amplified image relative to the reference image by the motion estimation method presented in 3.1, and determine the fuzzy matrix H_k using the PSF estimation method described in 3.2;

2) The image obtained by the inverse Fourier transform to the equation (7) is used as the initial estimate of the high resolution image. Divide the image into some blocks, use gradient method for weighted residual to repair threshold. Then define the residual limit set C_{m_1, m_2} according to the formula (13) for each pixel obtained accurately by the motion estimation. And calculate the composite matrix W ;

3) Calculate the residual error according to the formula (11), and the projection operator P_r is used to project through (14), in order to revise the estimation value $f(i_1, i_2)$ of the high resolution image.

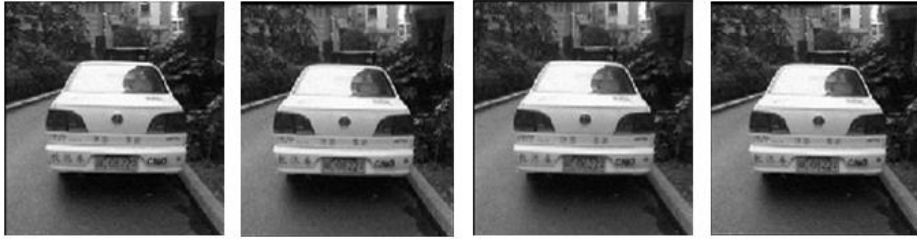
4) If $|\frac{f_i(i_1, i_2) - f_{i-1}(i_1, i_2)}{f_{i-1}(i_1, i_2)}| \leq \varepsilon$, $\hat{f}(i_1, i_2) = f_i(i_1, i_2)$, the iteration is over, otherwise return to step 2).

4. Simulation and Analysis

In order to verify the feasibility and stability, the algorithm is implemented in MATLAB 7.0. And the results of experiment are analyzed by various subjective and objective evaluation indexes.

Experiment 1 Four frames images at different positions and different angles are performed for reconstruction experiments. The four frames images are shown in Figure 3 (a) with the size of 230*230 pixels. First for each frame, the bilinear interpolation method is adopted to expand 2 times in the horizontal direction and the vertical direction. Then the fourth frame image as the reference frame which the displacement matrix of relative to the other frames is determined by using the adaptive block matching algorithm. Blind deconvolution method for the reference frame is used for determining the fuzzy matrix H and the best estimate of high resolution image. Finally, the average gradient of each LR image is calculated to obtain the weighted coefficient of the residual. And the initial HR is divided into blocks with the size of 23*23. Thus 460*460 image is divided into 20 blocks. Then the weighted coefficient is obtained as the average gradient of each block calculated.

In order to verify the effectiveness of the algorithm, we compare the algorithm with the bilinear interpolation, the literature and the traditional POCS algorithm. Figure 3(b) is a reference frame after the bilinear interpolation. Figure 3(c) is the super resolution reconstruction result based on GCV fuzzy identification. Figure 3(d) is the result of the reconstruction using the traditional POCS algorithm after 25 iterations. The reconstruction result of the improved POCS algorithm after 25 iterations is shown in Figure 3(e).



(a) Original LR Images (4 frames)



(b) Bilinear Interpolation



(c) Based on GCV Fuzzy Identification



(d) POCS



(e) Improved POCS

Figure 3. Comparison of Blind Super-Resolution Reconstruction Methods



(a) Based on GCV Fuzzy Identification



(b) Improved POCS

Figure 4. Local Amplification Comparison

As shown from Figure 3, the improved algorithm and image reconstruction algorithm based on GCV fuzzy identification take on better clarity obviously than bilinear interpolation and the traditional POCS reconstruction image. Image detail information are also more prominent. However, the whole effect after using the gradient method for the weighted residual threshold repaired doesn't appear to be much different from the reconstructed image based on the GCV fuzzy identification. But after local amplification, it can be clearly seen from Figure 4 that the text in the image is clearer. So the improved algorithm has better effect.

Experiment 2 Take a high resolution image with the size of 512*512. 4 groups of low resolution images are generated by the high resolution image through the degradation model. Firstly, the high resolution image is translated and rotated. Then the image is convolved respectively with Gauss PSF with variance of 0.8, 1.0, 1.2, 1.5 to simulate blurring in the imaging model. Down-sampling factor is 2. Finally, the Gauss white noise of 30dB is added. Thus 4 groups of low resolution images are obtained with different PSF with different variances, which were used to reconstruct at last, and the normalized mean square error (NMSE) of the PSF in the imaging system is estimated.

From Table 1, we can see that the algorithm can accurately estimate the point spread function of the imaging system, and is less affected by the blur degree of the image.

Table 1. NMSE of the PSF Estimation

Name	Variance			
	0.8	1.0	1.2	1.5
NMSE	0.0126	0.0094	0.0068	0.0101

Experiment 3 the simulation conditions are same with experiment 2. In order to evaluate the reconstruction results objectively, peak signal to noise ratio (PSNR), information entropy, standard deviation and spatial frequency (SF) were calculated respectively. Data of experiments are shown in Table 2, which indicates the indicators obtained from the improved POCS algorithm proposed in this paper have been improved in different degrees.

Table 2. Objective Evaluations

Algorithm	PSNR	information entropy	standard deviation	spatial frequency
Bilinear interpolation	18.8	7.201	51.211	20.143
GCV fuzzy identification	19.6	7.346	54.072	36.671
POCS	19.1	7.352	53.440	22.016
Improved POCS	19.8	7.439	55.354	40.332

5. Summary

In this paper, a new method of blind super-resolution reconstruction based on weighted POCS is proposed in this paper, in which the PSF is unknown. Adaptive block matching algorithm is used to estimate the geometric distortion of different frames of the images. Using APEX algorithm to determine the ambiguity function of the image. And obtain high quality reconstructed image initial estimate for reconstruction. In addition take into account that the traditional POCS algorithm restoring the threshold has fixed selection,

without considering prior information of the image. POCS reconstruction algorithm based on weighted residual method is expressed and weighted threshold is selected for different image blocks, which improves the accuracy of the residual calculation and the selection of the threshold, so as to more accurate revise. Research results show that this algorithm can significantly improve the quality and the degree of clarity of the image, evaluation indexes objectively also prove that the algorithm can improve the access to the actual resolution of the LR image, which verifies the feasibility and effectiveness of the proposed algorithm.

Acknowledgements

This work was supported by projects of Sichuan Provincial Department of Education (13ZB0138) and projects of Artificial Intelligence Key Laboratory of Sichuan Province (2013RYY02).

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