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Multi-Sensor Information Fusion Predictive Control Algorithm

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Abstract

The multi-sensor information fusion predictive control algorithm for discrete-time linear time-invariant stochastic control system is presented in this paper. This algorithm combines the fusion steady-state Kalman filter with the predictive control. It avoids the complex Diophantine equation and it can obviously reduce the computational burden. The algorithm can deal with the multi-sensor discrete-time linear time-invariant stochastic controllable system based on the linear minimum variance optimal information fusion criterion. The fusion method includes the centralized fusion, matrices weighted and the covariance intersection fusion. Under the linear minimum variance optimal information fusion criterion, the calculation formula of optimal weighting coefficients have be given in order to realize matrices weighted. To avoid the calculation of cross-covariance matrices, another distributed fusion filter is also presented by using the covariance intersection fusion algorithm, which can reduce the computational burden. And the relationship between the accuracy and the computation complexities among the three fusion algorithm are analyzed. Compared with the single sensor case, the accuracy of the fused filter is greatly improved. A simulation example of the target tracking controllable system with two sensors shows its effectiveness and correctness.

***Keywords**: Predictive Control, Information Fusion, Centralized fusion, Matrices weighted, Covariance intersection fusion*

1. Introduction

In this paper, steady-state Kalman filter is adopted. Kalman filter is a time varying recursive filter. The optimal Kalman filter is required to compute the gain matrix at every moment, which brings a large computational burden. From the viewpoint of engineering application, the advantage of using steady-state Kalman filter is to avoid computing the gain on-line. The steady-state gain can be calculated in a single time off-line, which simplifies the calculation of the Kalman filter, and reduces the burden of on-line calculation [1-3].

Predictive control is a new type of computer control algorithm that has been successfully developed in recent years. Because of its control strategy, such as prediction model, rolling optimization, and feedback correction, the control effect is good. It is suitable for the control of industrial production process, which is not easy to establish a precise digital model and more complex. At the same time, its theory has received great attention in the industrial field and academia, and has been widely used in the control

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system including the petroleum, chemical, metallurgy, machinery and other industrial department, and it is a promising new class of computer control algorithms [4-12].

The predictive control algorithm based on steady-state Kalman filter in this paper also has the three essential characteristics. Because steady-state Kalman filter algorithm is used, the complex Diophantine functions and the gain are avoided and it can solve the problem of predictive control for time-varying and time-invariant system [11]. For the predictive control algorithm, the accuracy and stability depend on the predictive accuracy of output [13]. For the single sensor system, it can only obtain the partial information. If the sensor has the disturber, the accuracy will get very bad, and even makes the system paralysis. Therefore many advanced and complicated systems require using multisensor to make up the disadvantages of single sensor.

Multisensor information fusion is also known as the multisensor data fusion, and it has become a prevalent field since 1970s, and it has been applied to many fields, such as guidance, robotics, GPS positioning and signal processing [14]. The multisensor measurement fusion based on Kalman filter has two measurement fusion methods. One is State fusion method. The state fusion method is also divided into centralized Kalman filtering and distributed Kalman filtering. Although the centralized Kalman filter can obtain the global optimal fusion state estimation in theory, it has the disadvantage of large computation burden and poor fault tolerance, and the distributed Kalman filtering information fusion can overcome these drawbacks. The other is weighted measurement fusion or distributed measurement fusion, applying weighted method fusing each measurement equation into a measurement equation that its dimension is not too large [15].

In this paper, the multisensor information fusion predictive control algorithm is presented. This algorithm avoids the complex Diophantine equation, but the state predictor is obtained by using steady-state Kalman filter, so it can obviously reduce the computational burden. And the relationship between the accuracy and the computation complexities among the three fusion algorithm are analyzed. Moreover, compared to the single sensor case, using the information fusion algorithm improves the accuracy of the predictive control and the stability of the system. Simulation results verify its effectiveness and correctness.

This paper is organized as follows: Section 2 presents Problem formulation. The local steady-state Kalman filter of the *i*th time-invariant subsystem are presented in Section 3. Section 4 presents the information fusion Kalman filter. Predictive control algorithm base on steady-state Kalman filter is presented in Section 5. A simulation example is given in Section 6. The conclusions are presented in Section 7.

2. Problem Formulation

Consider the multi-sensor linear discrete-time time-invariant stochastic controllable system

$$\boldsymbol{x}(t+1)=\boldsymbol{\Phi}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{\Gamma}\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}_i(t)=\boldsymbol{H}_i\boldsymbol{x}(t)+\boldsymbol{v}_i(t),\quad i=1,\cdots,L \tag{2}$$

where t is the discrete time, the subscript i denotes the ith sensor, L denotes the number of sensor, $\boldsymbol{x}(t)\in\boldsymbol{R}^n$ is the state of the system, $u(t)$ is the input, $\boldsymbol{y}_i(t)\in\boldsymbol{R}^{mi}$ is the measurement of the ith sensor subsystem, $\boldsymbol{v}_i(t)\in\boldsymbol{R}^{mi}$ is the measurement noise of the ith sensor subsystem, $\boldsymbol{w}(t)\in\boldsymbol{R}^r$ is the input noise, $\boldsymbol{\Phi}$, $\boldsymbol{B}$, $\boldsymbol{\Gamma}$, $\boldsymbol{H}_i$ is the suitable dimensional matrix respectively.

Assumption 1 $\boldsymbol{w}(t)\in\boldsymbol{R}^r$ and $\boldsymbol{v}_i(t)\in\boldsymbol{R}^{mi}, i=1,\cdots,L$ are independence white noises with zero mean and covariance are $\boldsymbol{Q}_w$ and $\boldsymbol{Q}_{vi}$ individually

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$$\mathrm{E}\left\{\begin{bmatrix}\boldsymbol{w}(t)\\ \boldsymbol{v}_i(t)\end{bmatrix}\begin{bmatrix}\boldsymbol{w}(k) & \boldsymbol{v}_i(k)\end{bmatrix}\right\}=\begin{bmatrix}\boldsymbol{Q} & 0\\ 0 & \boldsymbol{Q}_{vi}\end{bmatrix}\delta_{tk} \tag{3}$$

Where E is the mathematical expectation, the superscript T denotes the transpose, and δ_{tk} is the Kronecker delta function, $\delta_{tt}=1, \delta_{tk}=0(t\neq k)$.

Assumption 2 The initial state value $\boldsymbol{x}(0)$ is uncorrelated with $\boldsymbol{w}(t)$ and $\boldsymbol{v}_i(t)$, and $\mathrm{E}\boldsymbol{x}(0)=\boldsymbol{x}_0$, $\mathrm{cov}\,\boldsymbol{x}(0)=\boldsymbol{P}_0$.

Assumption 3 $\boldsymbol{u}(t)$ is the known time sequence, or linear function (feedback control) of $(\boldsymbol{y}(t),\ \boldsymbol{y}(t-1),\cdots)$.

Assumption 4 $(\boldsymbol{\Phi},\boldsymbol{H}_i)$ is completely observable pair, and $(\boldsymbol{\Phi},\boldsymbol{\Gamma})$ is completely controllable pair.

Assumption 5 The initial time $t_0=-\infty$.

Our aims are as follows.

1) Based on the measurement $(\boldsymbol{y}(t),\ \boldsymbol{y}(t-1),\cdots,\ \boldsymbol{y}(0))$, using information fusion steady-state Kalman estimation to get the *N*-step-ahead optimal predictive control algorithm.

2) The fusion method includes the centralized fusion, matrices weighted and the covariance intersection fusion. And the accuracy and computational burden among the three fusion algorithm are analyzed.

3. Local Steady-State Kalman Filter

Lemma 1 [16] for system (1) and (2) with the assumption 1-5, the *i*th sensor subsystem has the local optimal steady-state Kalman filter equations:

$$\hat{\boldsymbol{x}}_i(t+1\,|\,t+1)=\boldsymbol{\Psi}_{fi}\hat{\boldsymbol{x}}_i(t\,|\,t)+\boldsymbol{B}\boldsymbol{u}(t+1)+\boldsymbol{K}_{fi}\boldsymbol{y}_i(t+1) \tag{4}$$

$$\boldsymbol{\Psi}_{fi}=[\boldsymbol{I}_n-\boldsymbol{K}_{fi}\boldsymbol{H}_i]\boldsymbol{\Phi} \tag{5}$$

$$\boldsymbol{K}_{fi}=\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{Q}_{\varepsilon i}^{-1} \tag{6}$$

$$\boldsymbol{Q}_{\varepsilon i}=\boldsymbol{H}_i\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}+\boldsymbol{Q}_{vi} \tag{7}$$

$$\boldsymbol{P}_i=[\boldsymbol{I}_n-\boldsymbol{K}_{fi}\boldsymbol{H}_i]\boldsymbol{\Sigma}_i \tag{8}$$

And with the arbitrary initial values are $\hat{\boldsymbol{x}}_i(0\,|\,0)$, further $\sum_i$ satisfies the Riccati equation

$$\boldsymbol{\Sigma}_i=\boldsymbol{\Phi}[\boldsymbol{\Sigma}_i-\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}(\boldsymbol{H}_i\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}+\boldsymbol{Q}_{vi})^{-1}\boldsymbol{H}_i\boldsymbol{\Sigma}_i]\boldsymbol{\Phi}^{\mathrm{T}}+\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{\Gamma}^{\mathrm{T}} \tag{9}$$

Where $\boldsymbol{P}_i$ is steady local filter error covariance matrix.

Lemma 2 [17] The multisensor linear discrete-time time-invariant stochastic controllable system (1) and (2) under the assumption 1-5, the cross covariance $\boldsymbol{P}_{ij}=\lim\boldsymbol{P}_{ij}(t\,|\,t)(t\to\infty)$ between any two local filter satisfies Lyapunov equation:

$$\boldsymbol{P}_{ij}=\boldsymbol{\Psi}_{fi}\boldsymbol{P}_{ij}\boldsymbol{\Psi}_{fj}^{\mathrm{T}}+[\boldsymbol{I}_n-\boldsymbol{K}_{fi}\boldsymbol{H}_i]\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{\Gamma}^{\mathrm{T}}[\boldsymbol{I}_n-\boldsymbol{K}_{fj}\boldsymbol{H}_j]^{\mathrm{T}} \tag{10}$$

4. Fusion Kalman Filter

The system (1) and (2) can be written as

$$\boldsymbol{y}^{(0)}(t)=\boldsymbol{H}^{(0)}\boldsymbol{x}(t)+\boldsymbol{v}^{(0)}(t) \tag{11}$$

$$\boldsymbol{y}^{(0)}(t)=\begin{bmatrix}\boldsymbol{y}_1^{\mathrm{T}}(t) & \cdots & \boldsymbol{y}_L^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}} \tag{12}$$

$$\boldsymbol{H}^{(0)}(t)=\begin{bmatrix}\boldsymbol{H}_1^{\mathrm{T}} & \cdots & \boldsymbol{H}_L^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \tag{13}$$

$$\boldsymbol{v}^{(0)}(t)=\left[\boldsymbol{v}_1^{\mathrm{T}}(t) \quad \cdots \quad \boldsymbol{v}_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{14}$$

$$\boldsymbol{Q}_v^{(0)}=\mathrm{diag}(\boldsymbol{Q}_{v_1},\cdots,\boldsymbol{Q}_{v_L}) \tag{15}$$

Lemma 3 [16] for system (1)-(11) under Assumptions 1-5, the optimal centralized fusion steady-state Kalman filter $\hat{\boldsymbol{x}}^{(c)}(t|t)$ are calculated by

$$\hat{\boldsymbol{x}}^{(c)}(t+1|t+1)=\boldsymbol{\varPsi}_f^{(c)}\hat{\boldsymbol{x}}^{(c)}(t|t)+\boldsymbol{K}_f^{(c)}\boldsymbol{y}^{(0)}(t+1) \tag{16}$$

$$\boldsymbol{\varPsi}_f^{(c)}=[\boldsymbol{I}_n-\boldsymbol{K}_f^{(c)}\boldsymbol{H}^{(0)}]\boldsymbol{\varPhi} \tag{17}$$

$$\boldsymbol{K}_f^{(c)}=\boldsymbol{\varSigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}\boldsymbol{Q}_\varepsilon^{(c)-1} \tag{18}$$

$$\boldsymbol{Q}_\varepsilon^{(c)}=\boldsymbol{H}^{(0)}\boldsymbol{\varSigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}+\boldsymbol{Q}_v^{(0)} \tag{19}$$

$$\boldsymbol{P}^{(c)}=[\boldsymbol{I}_n-\boldsymbol{K}_{fi}^{(c)}\boldsymbol{H}^{(0)}]\boldsymbol{\varSigma}^{(c)} \tag{20}$$

And with the arbitrary initial values are $\hat{\boldsymbol{x}}_i(0|0)$, further $\sum_i$ satisfies the Riccati equation

$$\boldsymbol{\varSigma}^{(c)}=\boldsymbol{\varPhi}[\boldsymbol{\varSigma}^{(c)}-\boldsymbol{\varSigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}(\boldsymbol{H}^{(0)}\boldsymbol{\varSigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}+\boldsymbol{Q}_{vi})^{-1}\boldsymbol{H}^{(0)}\boldsymbol{\varSigma}^{(c)}]\boldsymbol{\varPhi}^{\mathrm{T}}+\boldsymbol{\varGamma}\boldsymbol{Q}\boldsymbol{\varGamma}^{\mathrm{T}} \tag{21}$$

Where $\boldsymbol{P}^{(c)}$ is fused steady-state filter error variance matrix.

Lemma 4 [17] The system (1) and (2), under the assumption 1 to 5, the optimal fused steady-state Kalman filter $\hat{\boldsymbol{x}}_0(t|t)$ weighted by matrices is given as [11]:

$$\hat{\boldsymbol{x}}_0(t|t)=\sum_{i=1}^{M}\boldsymbol{M}_i\hat{\boldsymbol{x}}_i(t|t) \tag{22}$$

Under the linear minimum variance optimal information fusion criterion which minimize the performance index, the optimal weighting coefficients $\boldsymbol{M}_i, i=1,2,\cdots,L$ are given as follows

$$\left[\boldsymbol{M}_1,\cdots,\boldsymbol{M}_l\right]=(e^{\mathrm{T}}\boldsymbol{P}^{-1}e)^{-1}\boldsymbol{e}^{\boldsymbol{T}}\boldsymbol{P}^{-1} \tag{23}$$

Where

$$\boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \cdots & \boldsymbol{P}_{1M}\\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} & \cdots & \boldsymbol{P}_{2M}\\ \cdots & \cdots & \cdots & \cdots\\ \boldsymbol{P}_{M1} & \boldsymbol{P}_{M2} & \cdots & \boldsymbol{P}_{MM}\end{bmatrix},\ \boldsymbol{e}=\begin{bmatrix}\boldsymbol{I}_m\\ \boldsymbol{I}_m\\ \cdots\\ \boldsymbol{I}_m\end{bmatrix},$$

$$\boldsymbol{P}_{ii}=\boldsymbol{P}_i(i=1,2,\cdots,M) \tag{24}$$

Where $\boldsymbol{P}_i(t)$ and $\boldsymbol{P}_{ij}(t)$ are computed by Lemma 1 and 2.

The optimal fused variance matrix is given as

$$\boldsymbol{P}_0=(e^{\mathrm{T}}\boldsymbol{P}^{-1}e)^{-1} \tag{25}$$

And

$$\mathrm{tr}\,\boldsymbol{P}_0\le\mathrm{tr}\,\boldsymbol{P}_j,\ \ j=1,2,\cdots,M \tag{26}$$

Lemma 5 [18, 19] For the system (1) and (2), under the same conditions, when the variance of P_1 and P_2 are known, but the cross covariance P_{12} is unknown, using the covariance intersection (CI) fusion method, this paper proposes a suboptimal fusion steady-state Kalman filter is as follows:

$$\hat{\boldsymbol{x}}_{CI}(t|t)=\sum_{i=1}^{L}\beta_i(t)\hat{\boldsymbol{x}}_i(t|t) \tag{27}$$

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Fusion weight is calculated as follows

$$\beta_i(t)=\lambda_i(t)(\sum_{i=1}^{L}\lambda_i(t)P_i^{-1}(t\,|\,t))^{-1}P_i^{-1}(t\,|\,t) \tag{28}$$

Where

$$\lambda_i(t)=\frac{\mathrm{tr}(P_i^{-1}(t\,|\,t))}{\sum_{i=1}^{L}\mathrm{tr}(P_i^{-1}(t\,|\,t))},\ 0\le\lambda_i(t)\le 1,\ \sum_{i=1}^{L}\lambda_i(t)=1 \tag{29}$$

It is proved in document [16] that the accuracy of above three kinds of weighted fusion estimator from high to low is the centralized fusion, matrices weighted and the covariance intersection fusion. But the computational burden is on the contrary, the centralized fusion estimator has a large computational burden. And covariance intersection fusion avoids solving cross-covariance matrices. So it has the minimal computational burden, and it is suitable for real-time applications.

5. Predictive Control Algorithm Base on Steady-State Kalman Filter1

For system (1) and (2) with the assumption 1-5, selecting the fused state $\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{x}_0(t)$ ($\boldsymbol{e}_i^{\mathrm{T}}=[\underbrace{0\cdots0}_{i-1}1\underbrace{0\cdots0}_{m-i}]$) as the controlled variable, selecting $x_r(t)$ as the reference track at the time t . For the every time t , the N-step ahead control increments $\Delta u(t), \Delta u(t+1),\cdots, \Delta u(t+N_\mu-1)$ need to be obtained to make the states in the future $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0(t+j\,|\,t),\ \ j=1,\cdots,N$ as close as possible to the given reference track $x_r(t+j)$. Where N_μ is the control time domain, N is the optimum time domain.

Theorem1 For the system (1) and (2) under the Assumption 1 to 5, the t-step ahead optimal predictive control increments is obtained:

$$\boldsymbol{\Delta U}(t)=[(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_e(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)+\boldsymbol{\Lambda}_u)^{-1}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_e\times\{\boldsymbol{x}_r-\boldsymbol{e}^{\mathrm{T}}[\boldsymbol{\Phi}_N\hat{\boldsymbol{x}}(t\,|\,t)-\boldsymbol{\Phi}_\mu\boldsymbol{U}(t-1)]\} \tag{30}$$

Where $\boldsymbol{e}^{\mathrm{T}}=\left[\boldsymbol{e}_j^{\mathrm{T}}\ \ \cdots\ \ \boldsymbol{e}_j^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\boldsymbol{Q}_e=\mathrm{diag}(\boldsymbol{Q}_1,\cdots,\boldsymbol{Q}_N)$ and $\boldsymbol{\Lambda}_u=\mathrm{diag}(\boldsymbol{\Lambda}_1,\cdots,\boldsymbol{\Lambda}_{N_\mu})$ are unified called the weighted matrix.

We define the controlled state as

$$\hat{\boldsymbol{X}}_0=\boldsymbol{e}^{\mathrm{T}}\left[\hat{\boldsymbol{x}}_0^{\mathrm{T}}(t+1\,|\,t)\quad \hat{\boldsymbol{x}}_0^{\mathrm{T}}(t+2\,|\,t)\quad\cdots\quad \hat{\boldsymbol{x}}_0^{\mathrm{T}}(t+N\,|\,t)\right]^{\mathrm{T}} \tag{31}$$

And the reference tracking at the time t is defined as

$$\boldsymbol{X}_r=\left[x_r(t+1)\quad x_r(t+2)\quad\cdots\quad x_r(t+N)\right]^{\mathrm{T}} \tag{32}$$

$$\boldsymbol{\Phi}_x(t)=\begin{bmatrix}\boldsymbol{\Phi}(t)\\ \boldsymbol{\Phi}^2(t)\\ \cdots\\ \boldsymbol{\Phi}^N(t)\end{bmatrix},\ \boldsymbol{\Delta U}=\begin{bmatrix}\Delta\boldsymbol{U}(t)\\ \Delta\boldsymbol{U}(t+1)\\ \cdots\\ \Delta\boldsymbol{U}(t+N_\mu-1)\end{bmatrix},\ \boldsymbol{\Phi}_\mu(t)=\begin{bmatrix}\boldsymbol{B}(t)\\ (\boldsymbol{\Phi}(t)+\boldsymbol{I})\boldsymbol{B}(t)\\ \cdots\\ (\boldsymbol{\Phi}^{N-1}(t)+\cdots+\boldsymbol{\Phi}(t)+\boldsymbol{I})\boldsymbol{B}(t)\end{bmatrix},$$

$$\boldsymbol{\Phi}_N(t)=\begin{bmatrix}\boldsymbol{B}(t) & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ (\boldsymbol{\Phi}(t)+\boldsymbol{I})\boldsymbol{C} & \boldsymbol{B}(t) & \cdots & \boldsymbol{0}\\ \cdots & \cdots & \cdots & \cdots\\ (\boldsymbol{\Phi}^{N-1}(t)+\cdots+\boldsymbol{\Phi}(t)+\boldsymbol{I})\boldsymbol{B}(t) & (\boldsymbol{\Phi}^{N-2}(t)+\cdots+\boldsymbol{\Phi}(t)+\boldsymbol{I})\boldsymbol{B}(t) & \cdots & \boldsymbol{B}(t)\end{bmatrix} \tag{33}$$

And the t-step ahead predictive control is computed as

$$\boldsymbol{U}(t)=\boldsymbol{U}(t-1)+\boldsymbol{e}_1\Delta\boldsymbol{U}(t) \tag{34}$$

Where $\boldsymbol{e}_1 = [1\,0 \cdots 0]$, and the filtering $\hat{\boldsymbol{x}}(t \mid t)$ can be obtained through (4)—(9).

Proof: The expanse function equation is defining [20]:

$$J = (\hat{\boldsymbol{A}}_0 - \boldsymbol{A}_r)^{\mathrm{T}} \boldsymbol{Q}_e (\hat{\boldsymbol{A}}_0 - \boldsymbol{A}_r) + \boldsymbol{\varDelta}\bar{\boldsymbol{U}}^{\mathrm{T}} \boldsymbol{\varLambda}_u \boldsymbol{\varDelta}\bar{\boldsymbol{U}} \tag{35}$$

From (1), we have

$$\hat{\boldsymbol{x}}_i(t+1 \mid t) = \boldsymbol{\varPhi}(t)\hat{\boldsymbol{x}}_i(t \mid t) + \boldsymbol{B}(t)\boldsymbol{u}(t) \tag{36}$$

Substituting (36) into (16) or (22) or (27) yields

$$\hat{\boldsymbol{x}}_0(t+1 \mid t) = \boldsymbol{\varPhi}(t)\hat{\boldsymbol{x}}_0(t \mid t) + \boldsymbol{B}(t)\boldsymbol{u}(t) \tag{37}$$

And

$$\hat{\boldsymbol{x}}_0(t+i \mid t) = \boldsymbol{\varPhi}(t)^i \hat{\boldsymbol{x}}_0(t \mid t) + \sum_{m=0}^{i-1} \boldsymbol{\varPhi}(t)^{i-m-1} \boldsymbol{B}(t)\boldsymbol{u}(t+m) \tag{38}$$

Defining

$$\Delta \boldsymbol{U}(t) = \boldsymbol{U}(t) - \boldsymbol{U}(t-1) \tag{39}$$

We have

$$\boldsymbol{U}(t+i-1) = \Delta \boldsymbol{U}(t+i-1) + \Delta \boldsymbol{U}(t+i-2) + \cdots + \Delta \boldsymbol{U}(t) + \boldsymbol{U}(t-1) \tag{40}$$

Putting (40) into (38) obtains

$$\hat{\boldsymbol{x}}(t+i \mid t) = \boldsymbol{\varPhi}^i(t)\hat{\boldsymbol{x}}(t \mid t) + \sum_{n=0}^{j-1} \left[\sum_{m=0}^{j-n-1} \bar{\boldsymbol{\varPhi}}(t)^m \right] \boldsymbol{B}(t)\Delta \boldsymbol{u}(t+n) \tag{41}$$

So that we have

$$\hat{\boldsymbol{X}}_0 = \boldsymbol{e}^{\mathrm{T}} [\boldsymbol{\varPhi}_x \hat{\boldsymbol{x}}_0(t \mid t) + \boldsymbol{\varPhi}_N \Delta \boldsymbol{U} + \boldsymbol{\varPhi}_\mu \boldsymbol{u}(t-1)] \tag{42}$$

Putting (42) into (35) yields

$$J = \{\boldsymbol{e}^{\mathrm{T}} [\boldsymbol{\varPhi}_x(t)\hat{\boldsymbol{x}}(t \mid t) + \boldsymbol{\varPhi}_N(t)\Delta \boldsymbol{U} + \boldsymbol{\varPhi}_\mu(t)\boldsymbol{U}(t-1)] - \boldsymbol{X}_r\}^{\mathrm{T}} \boldsymbol{Q}_e \times \{\boldsymbol{e}^{\mathrm{T}} [\boldsymbol{\varPhi}_x(t)\hat{\boldsymbol{x}}(t \mid t) + \boldsymbol{\varPhi}_N(t)\Delta \boldsymbol{U} + \boldsymbol{\varPhi}_\mu(t)\boldsymbol{U}(t-1)] - \boldsymbol{X}_r\}^{\mathrm{T}} + \boldsymbol{\varDelta U}^{\mathrm{T}} \boldsymbol{\varLambda}_u \boldsymbol{\varDelta U} \tag{43}$$

And letting $\dfrac{\partial J}{\partial \boldsymbol{\varDelta U}} = 0$, we have

$$\boldsymbol{\varPhi}_N^{\mathrm{T}}(t)\boldsymbol{e}\boldsymbol{Q}_e \{\boldsymbol{e}^{\mathrm{T}} [\boldsymbol{\varPhi}_x(t)\boldsymbol{x}(t \mid t) + \boldsymbol{\varPhi}_\mu(t)\boldsymbol{U}(t-1)] - \boldsymbol{A}_r\} + \boldsymbol{\varPhi}_N^{\mathrm{T}}(t)\boldsymbol{e}\boldsymbol{Q}_e \boldsymbol{e}^{\mathrm{T}} \boldsymbol{\varPhi}_N(t)\boldsymbol{\varDelta U} + \boldsymbol{\varLambda}_u \boldsymbol{\varDelta U} = 0 \tag{44}$$

We have the control increments can be computed via (30). From (30) and (39), (34) is obtained.

This completes the proof.

6. Simulation Example

Consider 2-sensor discrete-time linear time-invariant stochastic controllable tracking system (1) and (2), where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ is the state, $y_i(t)$ is the measurement of the ith subsystem, $\boldsymbol{\varPhi}(t) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B}(t) = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{\varGamma}(t) = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $T = 0.7$ is the sampled period, $\boldsymbol{H}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $\boldsymbol{H}_2 = \begin{bmatrix} 0.8 & 0.5 \end{bmatrix}$.

And $\boldsymbol{w}(t)$ and $\boldsymbol{v}_i(t)$ are assumed to be independent Gaussian white noises with zero mean and variances $\boldsymbol{Q} = 0.1$, $\boldsymbol{Q}_{v1} = 0.1$, $\boldsymbol{Q}_{v2} = 0.5$.

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The estimation criterion of the controlled system is defined as the sum of mean square error function (SMSE) of the differences of the state reference track $x_r(t)$ and the controlled state fusion estimator $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0(t|t)$ weighted by scalars [21, 22]

$$\mathrm{SMSE}(k)=\sum_{t=0}^{k}\frac{1}{L}\sum_{j=1}^{L}[\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0^{(j)}(t|t)-x_r(t)]^2 \tag{45}$$

Where $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0^{(j)}(t|t)$ is the *j*th Monte Carlo simulation state estimates at time *t*.

In Monte Carlo simulation for 30 times, $x_1(t)$ is selected as controlled state. Setting the control time domain $N_\mu=3$ and the optimize time domain $N=3$, and the reference track $x_r(t)$ is the 20 units step signal that appears at the time $t=100$, and the error weighted matrix $\boldsymbol{Q}_y=\mathrm{diag}(3,2,1)$, and the controlled weighted matrix $\boldsymbol{R}_\mu=\mathrm{diag}(3,2,1)\times 0.1$.

The simulation results are shown in Figure1-Figure5. Controller output *u*(*t*) is shown in Figure 1. Figure 2-Figure 4 show the comparison curves of the state reference track $x_r(t)$ and the fused steady-state Kalman filter. From Figure 2 and Figure 4, it shows that the fused steady-state Kalman filter can track the state reference track $x_r(t)$ closely, where the straight lines denote the state reference track, and the dashed curves denote the fused steady-state Kalman filter. It indicates that this algorithm has good convergence and attenuation, and the overshoot is small, and the controlled output is stable. The curves of the sum of mean square error (SMSE) for local and fusion steady-state Kalman filters are shown in Figure 5. We can see that that the accuracy of the fused steady-state Kalman filter is higher than single local Kalman filter.

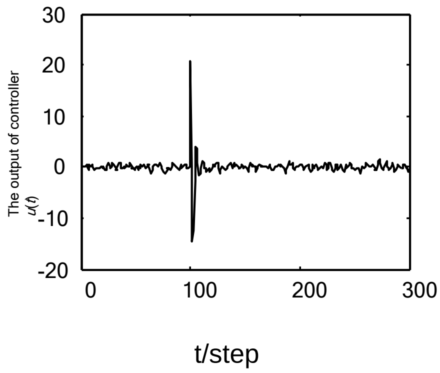

Figure 1. Controller Output *u*(*t*)

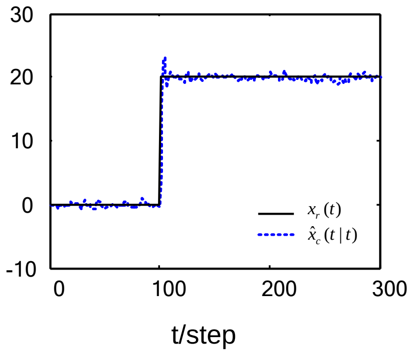

Figure 2. State Reference Track $x_r(t)$ and the Centralized Fused State Estimates $\hat{x}_c(t|t)$

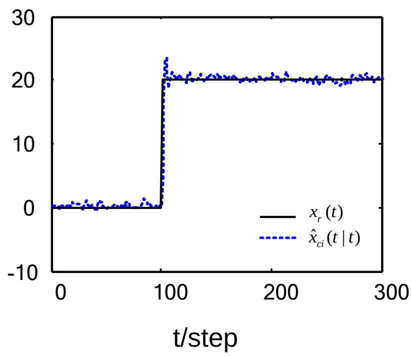

Figure 3. State Reference Track $x_r(t)$ and the Fused State Estimates $\hat{x}_m(t|t)$ Weighted by Matrices

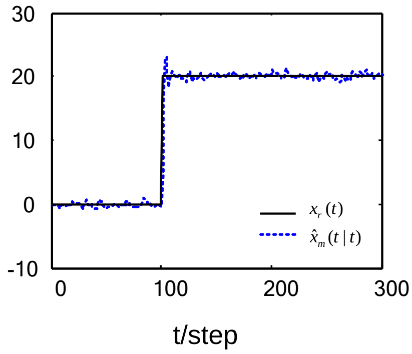

Figure 4. State Reference Track $x_r(t)$ and the Covariance Intersection Fused State Estimates $\hat{x}_{ci}(t|t)$

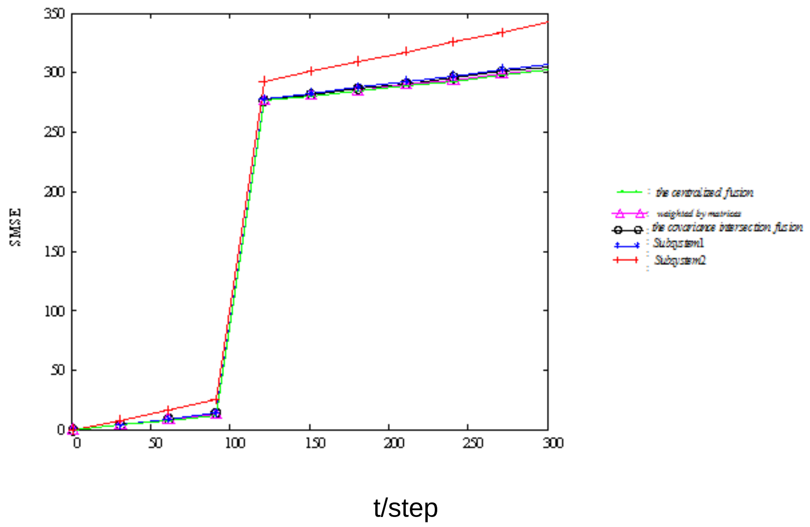

Figure 5. The Monte Carlo Curves of the Sum of Mean Square Error (SMSE)

7. Conclusions

In this paper, multisensor information Fusion Predictive Control for time-invariant systems is presented. The algorithm for time-invariant system combines the fusion steady-state Kalman filter with predictive control firstly. Compared with the classic generalized predictive control, the advantages are as follows:

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1. This algorithm based on steady-state Kalman filter avoids the complex Diophantine equation [23] and computing the gain on-line, so it can obviously reduce the computational burden.

2. Classic generalized predictive control only deals with time-invariant system, or the time-varying system that parameters varies slowly, this is called adaptive generalized predictive control [23]. However steady-state Kalman filter can deal with the time-varying system, so the predictive control system based on steady-state Kalman filter can deal with the linear time-varying and time-invariant system.

3. The stability of the fusion steady-state Kalman filter is making the stability of the system get better, and the ability of anti-jamming is enhanced.

4. Using the information fusion algorithm compared to the single sensor case, the accuracy is improved.

5. The accuracy of above three kinds of weighted fusion estimator from high to low is the centralized fusion, weighted by matrix, and covariance intersection fusion. But the computational burden is on the contrary, the centralized fusion estimator has a large computational burden. And covariance intersection fusion avoids solving cross-covariance matrices and has the minimal computational burden.

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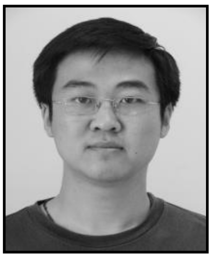

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