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Selection of Optimal Decomposition Layer for Thresholding Denoising Using Singular Spectrum Analysis and Wavelet Entropy

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Abstract

To optimize the number of decomposition layers in wavelet threshold denoising for ultrasonic signals, we propose a self-adaptive algorithm of determining the number of decomposition layers based on singular spectrum analysis and wavelet entropy. First the noise-containing signals are decomposed by discrete wavelet transform. The slope of the singular value spectrum for each layer is estimated. Then the wavelet entropy over the signal subinterval is calculated for each layer. Finally the optimal number of decomposition layer is determined by combining the entropy ratio of detail coefficients to original signal and the slope of the singular value spectrum. The performance of the algorithm is evaluated using signal-to-noise ratio (SNR) and the relative error of the peak value (REPV). Experiment shows that the algorithm can self-adaptively determine the optimal number of decomposition layers and filter out the noise contained in the ultrasonic signals. It not only increases the SNR, but also preserves valuable components of the original signal.

Keywords*: Optimal decomposition layer; Wavelet entropy; Singular spectrum analysis; denoising*

1. Introduction

Ultrasonic testing is one of the five major non-destructive testing techniques. The other four techniques are X-ray testing, liquid penetrant testing, magnetic particle testing, eddy current testing and ultrasonic testing. Ultrasonic testing has unique advantages in testing large-thickness objects and planar flaws with low cost, fastness, no toxicity to humans and high sensitivity. It has been widely applied in aeronautics and astronautics, mechanics, material science, marine exploration and diagnosis of diseases [1]. One key aspect that affects the quality of ultrasonic testing is the collection and analysis of ultrasonic echo signals. The echo signals containing the flaw information are not only related to the shape and size of flaw, but also to the performance and location of the ultrasonic transducer, the coupling of the ultrasonic transducer and the medium, and the excitation signals. It is almost impossible to directly extract the flaw information from the echo signals unless through the ultrasonic transducer. The ultrasonic waves are affected by electrical noise of equipments and scattering by the material grains. As a result, the flaw signals received may be submerged in noise, causing false/missed detection [2].

Besides time-domain and frequency-domain analysis, amplitude and phase analysis is also applied to the processing of ultrasonic signals. Self-adaptive filtering and split-spectrum technique are also powerful tools in feature extraction of ultrasonic echo signals. However, these techniques only aim at either the time domain or the frequency domain. In practice, we need to extract the frequency spectrum over a specific period, and time-domain or frequency-domain analysis alone cannot satisfy the requirements [3-4].

Considerable progress has been made in the past decade in non-linear signal processing,

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especially wavelet transform. Wavelet threshold denoising is one of the representative techniques [5-7]. Since the wavelet coefficients of useful information and noise differ on the energy spectrum, the signals and the noise can be separated by using proper wavelet base function, optimal number of decomposition layers and threshold. It should be emphasized that the number of decomposition layers is also an important factor that determines the result of wavelet threshold denoising for ultrasonic signals [8-9]. The conventional method is to preset a specific number of decomposition layers. However, the optimal number of decomposition layers varies with SNR. If it is too large, some useful information will be lost; and if it is too small, the noise is not completely filtered out [10-12]. Übeyli proposed an algorithm for calculating the optimal number of decomposition layers based on de-correlation and verification of white noise [13]. Through wavelet transform, this technique decomposes the signals into detail signals and scale signals. With detail signals maintained constant after each decomposition, thresholding is performed for the scale signals. When the termination condition is met, the results of the last decomposition (the n-th decomposition) are discarded. Then the optimal number of decomposition layers is determined as $n-1$. However, this involves the use of finite series composed of a finite number of observed values in the stochastic process which makes the definition of termination condition difficult. Galford *et al*. proposed a method based on autocorrelation function of the noise [14]. By using DWT, the noise-containing signals are decomposed into wavelet coefficients and scale coefficients. It is determined whether the scale coefficients pass the noise test and then the optimal number of decomposition layers is estimated. But this also requires the priori knowledge, *i.e.*, the noise characteristic series, which can be hardly acquired in the real environment of ultrasonic testing. Dhamala *et al*. proposed an algorithm of estimating the optimal number of decomposition layers based on the correlation between wavelet entropy and scale by constructing a correlation function [15]. However, a rough estimation of the maximum number of decomposition layer is needed in advance, and the calculation is complex.

Most techniques require some priori knowledge, or they are unmatched to the actual environment or fail to determine the termination conditions self-adaptively. The real ultrasonic signals usually contain rich information but have low SNR. The above techniques can hardly estimate the optimal number of decomposition layer rapidly and accurately. To this end, we propose the combination of wavelet entropy and singular spectrum analysis by referring to literature [16]. We have made two major modifications. First the wavelet entropy is calculated instead of verification of white noise. The rationale is that the number of decomposition layer is inversely proportional to information entropy; the smaller the entropy, the higher the certainty of the information is [17]. Second singular spectrum analysis is performed. The singular value is calculated for each decomposition layer starting from the first layer, which is an important basis for estimating the optimal number of decomposition layers.

2. Threshold Denoising Model for Ultrasonic Signal

In ultrasonic testing, the ultrasonic signals are interfered by fluctuation of power supply voltage, electrostatic interference, bad grounding as well as the scattering and reflection of the ultrasonic waves from the heterointerface and the coarse grains. These noises usually have flat broad-bands spectrum and are regarded as additive white Gaussian noises. A noise-containing original ultrasonic signal $x(i)$ is expressed as follows:

$$x(i) = y(i) + z(i) \quad (i = 1,2,\cdots N) \tag{1}$$

Where $y(i)$ is the pure ultrasonic signal and $z(i)$ is noise.

The purpose of denoising is to filter out the noise component $z(i)$ and to obtain pure ultrasonic signal $y(i)$ and an accurate estimate $\bar{y}(i)$. The disagreement between the two is represented by a risk function [18]:

$$\varpi_{\bar{y}(i),y(i)} = \sum_{i=1}^{N} (\bar{y}(i) - y(i))^2 \Big/ N \tag{2}$$

Then we can get the following result by performing discrete wavelet transform on formula (1):

$$\varpi_{\bar{y}(i),y(i)} = \sum_{j,k} (\bar{\psi}(j,k) - \psi(j,k))^2 \Big/ N \tag{3}$$

Where $\bar{\psi}(j,k)$ and $\psi(j,k)$ are wavelet coefficients of the pure ultrasonic signal and the estimate of the ultrasonic signal, respectively. After wavelet decomposition, the average power of the wavelet coefficients of noise decreases with the increasing number of decomposition layers. Thus the noise energy is mostly concentrated in many small coefficients and the wavelet coefficients of noise are still noise. In contrast, the signal energy is mostly concentrated in a few large coefficients. As a result, the amplitude of the wavelet coefficients of signal will be larger than that of the wavelet coefficients of noise in the last decomposition layer. Denoising can be done by setting an appropriate threshold that enables wavelet coefficients of 0. Thus, the estimate of the ultrasonic signals after transform through equation (1) is:

$$\bar{y}(i) = \mathrm{thr}(x(i), T) \tag{4}$$

Where $\mathrm{thr}(\ )$ is threshold function and T is threshold.

3. Selection Algorithm of Optimal Decomposition Layer

In this section, we describe the proposed method. One major modification made in this paper is introducing singular spectrum analysis. The signals contaminated by noise will have much more mutation points, which can be measured by singularity. To perform singular spectrum analysis, we can understand the features of the noise-containing signals [19]. In addition, the wavelet entropy over the signal subinterval in each layer is calculated. Finally the optimal number of decomposition layers is determined by combining the entropy ratio of detail coefficient and original signal and the slope of the singular value spectrum.

3.1. Singular Spectrum Analysis for Wavelet Coefficients

Singular spectrum analysis is applied to the prediction and analysis of time series. By singular value decomposition, the trend characteristics, period characteristics and noise characteristics of the signals can be obtained [20].

Suppose the wavelet coefficients in the j-th decomposition layer) constitute a series $\mathbf{x} = \{x_1, x_2, \cdots, x_N\}$ with the length of N. To perform the singular spectrum analysis, the original series $\mathbf{x}$ is transformed into a smoothing matrix. This step, according to Takens's theorem, is also known as embedding [19]. Delaying the series $\mathbf{x}$ by τ creates an $\mathbf{L}$-dimensional embedding, and the smoothing matrix $\mathbf{A}$ is obtained:

$$\mathbf{A} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N-(L-1)\tau} \\ x_{\tau+1} & x_{\tau+2} & \cdots & x_{N-(L-1)\tau+\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x_{(L-1)\tau+1} & x_{(L-1)\tau+2} & \cdots & x_N \end{bmatrix} \tag{5}$$

Where the delay time τ is related to the degree of autocorrelation of the original series $\mathbf{x}$. The lower the degree of autocorrelation of $\mathbf{x}$, the smaller the τ is.

Singular value decomposition is performed for the smoothing matrix $\mathbf{A}$. Literature [19] believes that for any real matrix $\mathbf{A}$ ($\mathbf{A} \in \mathbf{R}^{L \times N}$), there must be orthogonal matrices $\mathbf{U}$ ($\mathbf{U} \in \mathbf{R}^{L \times L}$) and $\mathbf{V}$ ($\mathbf{V} \in \mathbf{R}^{N \cdot N}$) that make $\mathbf{A} = \mathbf{UDV}^T$, where $\mathbf{D}$ is a diagonal matrix, $\mathbf{D} = diag(\lambda_1, \lambda_2, \cdots, \lambda_m)$, $(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m)$. All elements in $\mathbf{D}$ are the eigenvalues of matrix $\mathbf{A}$. They represent m characteristic directions of the energy of the series, *i.e.*, m singular values of series $\mathbf{x}$. The singular value υ is given by:

$$\upsilon_i = \lambda_i \Big/ \frac{1}{m} \sum_{i=1}^{m} \lambda_i \tag{6}$$

Thus, in each layer, the singular values are similar and the singular value spectrum is flat when the noise is predominant in the wavelet coefficients. This is because the wavelet coefficients are noise at low SNR. When the signal is predominant in the wavelet coefficients, there will be large variation between the singular values and the singular value spectrum is tilted. On this basis, the compression of signal energy and the dispersion of noise energy can be evaluated in the wavelet coefficients.

3.2. Entropy and Entropy Ratio

According to information theory, entropy refers to the mean uncertainty of information contained in all signals. By multi-scale wavelet transform, the wavelet coefficients of each scale are converted into series of probability distribution. The entropy calculated from this series reflects the sparsity of wavelet coefficient matrix, in other words, the uncertainty or disorder of the signals.

After discrete wavelet transform of the original ultrasonic signal $x(i)$, the high-frequency detail coefficient at time k in the j-th decomposition layer is $d_{j,k}$ and the low-frequency approximation coefficient is $\alpha_{j,k}$. Thus the energy of detail coefficient in the j-th decomposition layer $(j = 0,1,\cdots,N)$ is given by:

$$E_j = \sum_k \left| d_{j,k} \right|^2 \tag{7}$$

Then total energy of the signal can be calculated as follows:

$$E = \left\| x(i) \right\|^2 = \sum_j \sum_k \left| d_{j,k} \right|^2 = \sum_j E_j \tag{8}$$

The relative wavelet energy is $P_j = E_j \big/ E$. The detail coefficient $d_{j,k}$ is the j-th decomposition layer is equally divided into n subintervals:

$$E_j(k) = \sum_k^{N/n} \left| d_{j,k} \right|^2 \tag{9}$$

Where N is the number of sampling points. $P_{j,k}$ in each subinterval is calculated as the ratio of wavelet energy $E_{j,k}$ in the k-th subinterval to total energy of wavelet coefficient in this layer:

$$P_{j,\mathrm{k}} = E_{j,\mathrm{k}} / E_j \tag{10}$$

$$E_j = \sum_{k=1}^{n} E_{j,\mathrm{k}} \tag{11}$$

Thus the wavelet entropy ω_k in the k-th subinterval is:

$$\omega_k = -\sum_j P_{j,k} \ln(P_{j,k}) \tag{12}$$

The entropy ω_0 of the original signal and the wavelet entropy ω_j of detail coefficient in the j-th layer are calculated. The entropy ratio η is

$$\eta = \omega_j / \omega_0 \tag{13}$$

Higher entropy of high-frequency detail coefficient usually indicates fewer noise components in the wavelet coefficients and higher certainty of the information. Literature [20] believes that when the entropy ratio of detail coefficient and original signal in a layer is 5%, the noise in the wavelet coefficients can be neglected.

3.3. The Proposed Method

From the above analysis, the steps of the proposed algorithm can be described as follows:

(1) Step 1. Discrete wavelet transform is performed for the original ultrasonic signals to obtain high-frequency and low-frequency wavelet coefficients.

(2) Step 2. Using (12) and (13), the maximal number g of decomposition layers with entropy ratio above 5% and the minimal number h of decomposition layers with entropy ratio below 5% are calculated, respectively. g and h satisfy:

$$g = h - 1 \tag{14}$$

(3) Step 3. Singular value analysis is performed for the detail coefficients after wavelet transform of the original signal. Using (6), m singular values are calculated for the g-th and the h-th layer, and the slope of the singular value spectrum K is calculated by $K = \lambda_1 / \lambda_m$.

(4) Step 4. The optimal number j of decomposition layers is calculated using the formula below:

$$j = \begin{cases} g, K_g > K_h \\ h, K_h > K_g \end{cases} \tag{15}$$

4. Experiment and Analysis

To evaluate the performance of the proposed algorithm, simulation experiment is carried out using MATLAB R2010. For singular spectrum analysis, the delay time τ and the dimension m of the embedding are determined with caution since their values have a significant impact on the singular values. Dimension being constant, reducing the delay time will increase the variation rate of the singular value; delay time being constant, increasing the dimension will make the changes of the singular value spectrum too violent and add to the calculation load. By referring to literature [19], the parameters are set as $\tau = 8$ and $m = 8$. Besides SNR, we also adopt $REPV$ as another performance indicator, which is calculated as follows:

$$REPV = \frac{|T_i - T_o|}{T_i} \times 100\% \tag{16}$$

Where T_i is the peak value of the original signal and T_o is the peak value of the denoised signal. $REPV$ is a measure of energy loss after denoising. In ultrasonic testing, the amplitude and shape of ultrasonic wave of flaws are the basis for judging the type and size of the flaws. The higher the SNR and the lower the root mean square error (RMSE) and $REPV$, the better the denoising effect and the less the loss of useful information will be.

Flaw echoes are non-stationary signals modulated by the central frequency of the probe, and they are simulated with the formula below:

$$f(t) = \rho e^{-\mu(t-\tau)} \cos[2\pi f(t-\tau)] \tag{17}$$

Where ρ and μ are modulation factors, f is the central frequency of the probe and τ is echo delay.

The parameters are $\rho = 3.24$, $\mu = 10^{12}$, $\tau = 5 \times 10^{-6}$ and $f = 2 \times 10^{6}$ in formula (17). SNR is set as -5, 0 and 5, respectively. The wavelet base and the threshold are chosen by the method in literature [21]. When SNR is -5, the entropy ratio is 0.0543 and the slope of the singular value spectrum is 1.3346 after decomposition to the 6th layer; the entropy ratio is 0.0491 and the slope of the singular value spectrum is 1.4784 after decomposition to the 7th layer. By the proposed algorithm, the optimal number of decomposition layers is 7. When SNR is 5, the entropy ratio is 0.0513and the slope of the singular value spectrum is 1.7931 after decomposition to the 4th layer; the entropy ratio is 0.04877 and the slope of the singular value spectrum is 1.2337 after decomposition to the 5th layer. Thus the optimal number of decomposition layers is 4.

Under three different SNRs, the output SNR and $REPV$ for layer 1 to 8 are calculated, as shown in Table 1, 2 and 3. The data in bold are the results using the proposed algorithm, and the underlined data are the results using the method in literature [18].

Table 1. Evaluation Result in Different Decomposition Layers ($SNR_{in} = -5$)

Index	L=1	L=2	L=3	L=4	L=5	L=6	L=7	L=8
SNR_{out}	-4.719	-4.223	-3.831	-1.784	-1.132	-0.973	**-0.857**	<u>-0.865</u>
$REPV$	27.94%	24.46%	18.83%	13.54%	9.47%	6.13%	**5.76%**	<u>6.58%</u>

Table 2. Evaluation Result in Different Decomposition Layers ($SNR_{in} = 0$)

Index	L=1	L=2	L=3	L=4	L=5	L=6	L=7	L=8
SNR_{out}	1.927	2.743	2.926	3.305	3.357	3.372	3.378	3.364
$REPV$	28.15%	19.07%	11.71%	5.28%	2.49%	3.52%	3.93%	2.76%

Table 3. Evaluation Result in Different Decomposition Layers ($SNR_{in} = 5$)

Index	L=1	L=2	L=3	L=4	L=5	L=6	L=7	L=8
SNR_{out}	5.424	6.439	9.257	10.308	10.370	10.328	10.311	10.172
$REPV$	24.29%	15.96%	9.12%	4.17%	5.05%	6.91%	7.39%	7.68%

It can be seen from the table that the output SNR is the minimum at input SNR of -5 and 7 wavelet decomposition layers. Using the method in literature [18], when there are 8 decomposition layers, the output SNR increases by 0.93%, while $REPV$ increases by 14.2%. When the input SNR is 0, neither the output SNR calculated by the proposed algorithm nor that by the method in literature [18] is the greatest; the greatest output SNR occurs in the 7th layer. But this does not mean that the proposed algorithm is less good. In the 7th layer, $REPV$ is 3.93%, which is higher by 57.83% than that in the 5th layer. However, the output SNR in the 7th layer is only higher by 0.63% than that in the 5th layer. To improve the accuracy, we combine output SNR and $REPV$ to determine the optimal number of decomposition layers self-adaptively. As a comparison, the output SNR using the method in literature [18] is higher by 0.21% than that using the proposed method, and the $REPV$ is also higher by 10.8%. Although the largest output SNR is obtained using the method in literature [18], which is higher by 0.61% as compared with our algorithm, the $REPV$ is higher by as large as 21.1%. Greater $REPV$ indicates more severe signal distortion and higher probability of false/missed detection. This result has proved the effectiveness and reliability of the proposed algorithm.

The waveforms under the optimal number of decomposition layers estimated by the proposed algorithm and the method in literature [18] with input SNR of -5, 0 and 5 are shown in Figure 2. The visual effect is already the best after reaching the optimal number of decomposition layers, and there is no need for further decomposition.

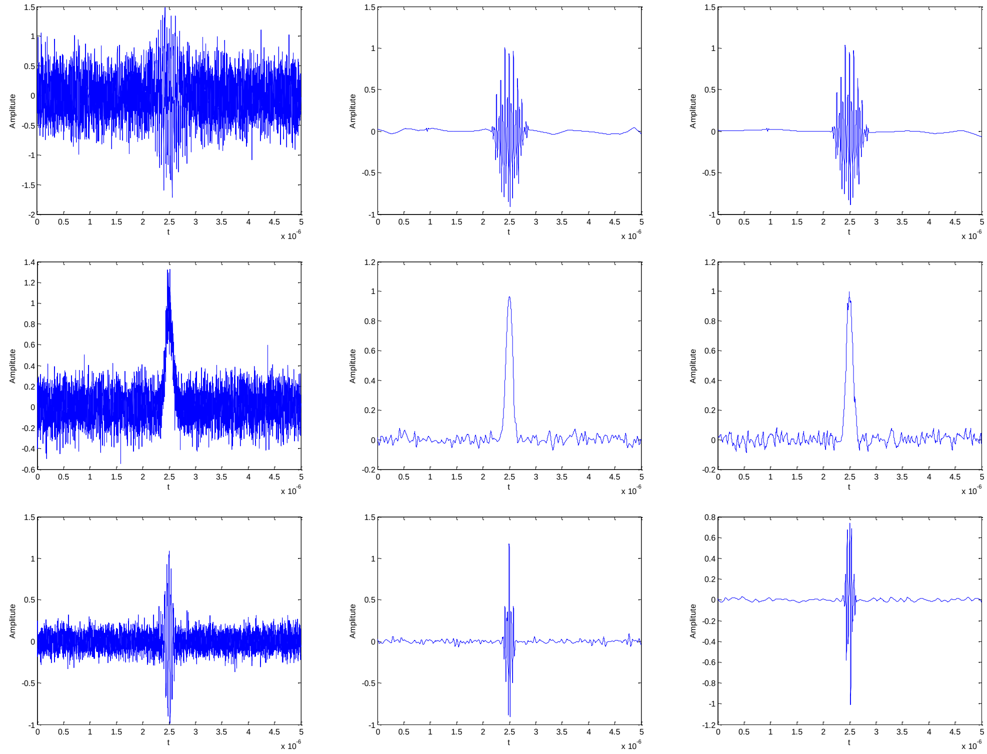

Figure 1. Evaluation Result Obtained by the Proposed Method and the Method in [18]

The first column is the test signal with input SNR of -5, 0, and 5. The second column is the de-noised waveform by the proposed method. The third column is the de-noised waveform by using method in [18].

5. Conclusion

With wavelet threshold denoising, an ideal result can be obtained at a smaller number of decomposition layers for signals with high SNR. But when the SNR is low, the number of decomposition layers should be increased. Obviously, the optimal number of decomposition layers varies with SNR of the original signal. We propose a method to determine the optimal number of decomposition layers by combining the entropy ratio and the slope of the singular value spectrum. Simulation experiment indicates that the optimal number of decomposition layers can be determined self-adaptively for different SNR. The proposed algorithm prevents signal distortion caused by too many decomposition layers, increases the efficiency of ultrasonic testing and reduces the calculation load. The loss of useful information is minimized with higher output SNR and lower $REPV$ using the proposed algorithm. Therefore, the algorithm is effective when the SNR of the original signal is unknown. Designing new threshold function and thresholding rule will be the topic for further research.

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