

# Cooperative Spectrum Sharing in Cognitive Radio Networks: A Centralized Contracted-Based Approach

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## Abstract

*In this paper, we study the relay-based communication schemes for cooperative spectrum sharing among multiple primary users (PUs) and multiple secondary users (SUs) with incomplete information. Inspired by contract theory, we model the network as a labor market. In this market, PUs and SUs are regarded as employer and employee respectively. Each PU proposes certain contracts to attract the SUs, the contract include relay power and spectrum access time, each SU maximize its utility by selecting the most suitable contract. But PUs and SUs have conflicting objectives in contract content. To tackle this problem, we put forward to a centralized maximizing expectation utility scheme, which convert the optimal contract design into an optimization problem. After that, a approximate method for solving the optimization problem is proposed. Simulation results show that the losses of PUs' total utilities caused by incomplete information and approximate algorithm are relatively small, especially when the number of SUs is very large.*

**Keywords:** cooperative spectrum sharing, Contract theory, multiple primary users

## 1. Introduction

With the explosive development of wireless networks and services, the spectrum becomes more and more crowded and scarce. In addition, low utilization ratio of spectrum caused by static spectrum allocation policy shows that it is urgent to find a new communication mode to improve the utilization ratio of spectrum. Dynamic spectrum sharing (DSS) allows unlicensed users opportunistic spectrum access, is an effective method to overcome the low utilization ratio of spectrum [1]. To solve the incentive problem that PU is lack of motivation in traditional DSS model, researcher put forward to a market-driven spectrum trading mechanism [4], in which PUs sell the spectrum to SUs for monetary reward or resource. In the former spectrum sharing model, PUs provide SUs the spectrum access opportunities so as to obtain monetary rewards, auction theory and bargain theory, as well as market theory [2-7] are widely applied in this spectrum sharing model. When PUs have some temporarily idle spectrum resources, the strategy of monetary reward spectrum sharing is commonly effective. However, when PUs' own demands are high or the primary channels' capacities are low, PUs almost no extra resource for sale. In addition, the monetary reward spectrum sharing requires a trustworthy billing system, which is difficult to design in practice [8]. Therefore, the latter spectrum sharing model is a better choice. Cooperative spectrum sharing is an effective form of resource-exchange spectrum trading, where SUs relay traffics for PUs and get the spectrum resource [9]. Such cooperation can improve PUs' data rate and free up spectrum resource for SUs significantly. Many existing works of cooperative spectrum sharing are based on Stackelberg game formulations with complete information [10], but these work can't extended to the scenario of incomplete information for SU can increase its utility by hiding its private information. As contract theory can elicit the SUs' private information efficiently [11], Paper [12] study cooperative spectrum sharing based contract between

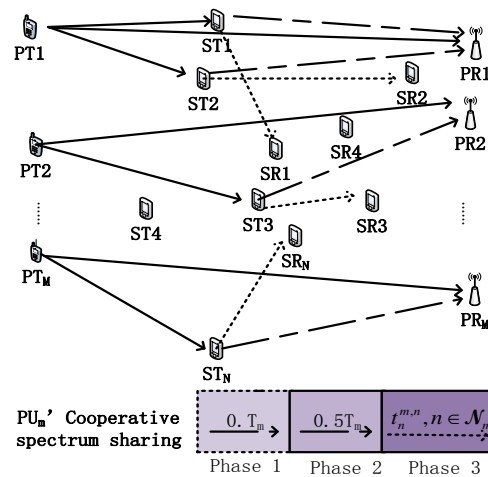
single PUs and multiple SUs under incomplete information. The study of cooperative spectrum sharing between multiple PUs and multiple SUs is discussed in [13], which achieve an optimal match-based scheme with complete information and a stable matching scheme with incomplete information. However, this work may lead to excessive loss which is avoidable.

In this paper, we study the cooperative spectrum sharing between multiple PUs and multiple SUs. Firstly, utility functions of PUs and SUs are proposed, and then we establish a new contract design scheme which maximize the expected PUs' total utility and convert the optimal contract design into an optimization problem under the incomplete information. After that, an approximate method for solving the optimization problem is proposed. Simulation results show that the losses of PUs' total utilities caused by incomplete information and approximate algorithm are relatively small, especially when the number of SUs is very large. Compare with the [13], the scheme in this paper has a better performance.

## 2. System Model and Problem Formulation

### 2.1 System Model

We consider a cognitive radio network with  $M$  PUs' and  $N$  SUs' communication links as shown in Figure1 ( $N > M$ ). All links have a pair of transmitting-receiving antenna, which are defined as  $\{(PT_1, PR_1), (PT_2, PR_2), \dots, (PT_M, PR_M)\}$  and  $\{(ST_1, SR_1), (ST_2, SR_2), \dots, (ST_N, SR_N)\}$  respectively. To protect PU's communication link, SU can relay PU's communication link when PU's channel condition is poor. PU share the spectrum resource with SU, which as the reward of relay service. Taking TDMA for example, the interactions between the PUs and the SUs involve three phases as in Figure1.



**Figure 1. Cooperative Spectrum Sharing Model with M PUs and N SUs**

Phase 1: Every PU's transmitter broadcasts its data to the corresponding receiver and the involved SUs' transmitters.

Phase 2: The involved SUs' transmitter amplify or decode the data received in phase 1 and forward it to the corresponding primary receiver.

Phase 3: The involved SUs can transmit their own data through the dedicated time allocations. SUs using the same spectrum access the spectrum using TDMA and do not interfere with each other.

In above interactions, PU and SUs have conflicting objectives in relay powers and time allocations when exchange their resource. PU wants the involved SUs to offer high power levels when relay its traffic, which will increase the PU's data rate but reduce the SUs' battery levels. The involved SU want to obtain large dedicated transmission time, which will increase the SU's own performance but reduce the PU's average data rate. Therefore, it is necessary to design an effective incentive mechanism to ensure both cooperation parties obtain the maximal benefit of one's own. To analysis the cooperative benefit, the contract theory will be introduced and the problem of both PUs and SUs want to maximize their benefit will be converted into the problem of how to design the optimal contract.

## 2.2. Problem Formulation

To better understand the problem of how to design optimal contract, we imagine PU as the employer and SU as the employee in labor market. PU design contracts and broadcast the contracts, SU chooses one contract that maximizes its benefit and informs the corresponding PU its choice. As SU is passive to select the most suitable contract, the problem of contract design is how PU design contracts to maximize their benefit. In this paper we define the user's benefit as utility. The next job is how PU and SU evaluate the contracts.

### 2.2.1. PU's Utility Model

We assume SU decodes PU's data received in the first phase successfully and forward it to the corresponding PR using the space-time codes assigned by PU from its random code book. To define  $PU_m$ 's utility function, we denote the set of involved SUs as  $\mathcal{N}_m$  and the corresponding contracts are  $\{(p_{mk}, t_{mk}), k \in \mathcal{N}_m\}$ , in which  $p_{mk}$  and  $t_{mk}$  represent the contact relay power and time allocation between  $PU_m$  and  $SU_n$  respectively, we should emphasize that the relay power is measured at PU's receiver. The utility function of  $PU_m$  is:

$$U_m = \frac{1}{1 + \sum_{k \in \mathcal{N}_m} t_{m,k}} \left( \frac{R_m^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{N}_m} p_{m,k}}{n_m} \right) \right) - R_m^{dir} \quad (1)$$

Where  $R_m^{dir}$  is  $PU_m$ 's direct data rate,  $n_m$  is the noise power at the receiver of  $PU_m$ , the first term on the right side of (1) represents  $PU_m$ 's average data rate, therefore, the utility function of  $PU_m$  is the increment of the average data rate during the entire time period by employing SUs.

### 2.2.2. SU's utility model

Assuming  $SU_n$  accepts  $PU_m$ 's contract  $(p_{mn}, t_{mn})$ ,  $SU_n$ 's utility is:

$$u_n^{m,n} = \theta^{m,n} t^{m,n} - p^{m,n}, \text{ where } \theta^{m,n} = \frac{2h_{ST_n, PR_m} (R_n - p_n^{tr})}{C_n} \quad (2)$$

Where  $R_n$ ,  $p_n^{tr}$  represent the data rate and transmission power of  $SU_n$  for its own traffic,  $h_{ST_n, PR_m}$  is the channel coefficient between  $ST_n$  and  $PR_m$ ,  $C_n$  is  $SU_m$ 's sensitivity for unit power consumption,  $\theta^{m,n}$  is the type of  $SU_n$  when it is pitched on by  $PU_m$ . The type reflects all the private information of  $SU_n$ , including its own transmission rate and power, its channel gain over the relay link corresponding to a certain PU, its battery technology. We assume

that a SU can obtain its own type by local measurement of the wireless environment and different SU have different type for a PU.

### 3. Optimal Contracts Design under Incomplete Information

Now we study the contract design under the incomplete information scenario, in which  $PU_m$  only knows the set of SUs' types  $\{\theta_{m,n}, 1 \leq n \leq N\}$  connected to it, and has no knowledge of SU's exact type. Without loss of generality, we assume  $\theta_{m,1} \geq \theta_{m,2} \geq \dots \geq \theta_{m,N}$ . PU design contract for SU's type under incomplete information. As there is multiple PUs in the network, it is possible multiple contracts are designed by different PUs for a SU. SU choose contract according to its utility function and contract designed by PU may be rejected. Therefore, the optimal contract design scheme under incomplete information proposed in paper [12] does not apply to the network with multiple PUs.

#### 3.1 Optimization Problem of Optimal Contracts

The aim of this paper is to design contracts maximize PUs' total utility. The main idea is that each  $PU_m$  select  $n_m$  types from high to low according to  $\{\theta_{m,n}, 1 \leq n \leq N\}$  and design contract for those type, then we can compute PU's expected utility under fixed  $\{n_m, 1 \leq m \leq M\}$ . Through optimize the relay powers and time allocations, we can obtain the maximal expected total utility. The optimal  $\{n_m, 1 \leq m \leq M\}$  can be got by comparing the utility achieved in different  $\{n_m, 1 \leq m \leq M\}$ .

Each PU select  $n_m$  types from high to low according to  $\{\theta_{m,n}, 1 \leq n \leq N\}$  and design contract for those type,  $PU_m$ 's expected utility is:

$$E(U_m(\{(t_{m,1}, p_{m,1}), (t_{m,2}, p_{m,2}), \dots, (t_{m,n_m}, p_{m,n_m})\})) = \sum_{s=0}^{n_m} pr_m(n_m, s) \sum_{\omega \in \Omega(n_m, s)} U_m(\omega) \quad (3)$$

Where  $E(\cdot)$  represents compute expectation operator,  $pr_m(n_m, s)$  is the probability of  $s$  contracts are accepted by the corresponding SUs in  $n_m$  contracts,  $\Omega(n_m, s)$  represents the event set of  $s$  contracts are accepted by the corresponding SUs from  $n_m$  contracts designed by  $PU_m$  and  $\omega$  is the element of  $\Omega(n_m, s)$ .  $U_m(\omega)$  is  $PU_m$ 's utility under the contracts  $\{(t_{m,1}, p_{m,1}), (t_{m,2}, p_{m,2}), \dots, (t_{m,n_m}, p_{m,n_m})\}$  when  $\omega$  happen.

Therefore, PUs' expected total utility is:

$$\sum_{m=1}^M E(U_m(\{(t_{m,1}, p_{m,1}), (t_{m,2}, p_{m,2}), \dots, (t_{m,n_m}, p_{m,n_m})\})) = \sum_{m=1}^M \sum_{s=0}^{n_m} pr_m(n_m, s) \sum_{\omega \in \Omega(n_m, s)} U_m(\omega) \quad (4)$$

According to theory 2 in paper [12], the optimization problem of PU's expected total utility is:

$$\begin{aligned} & \max_{\{n_m, t_{m,1}, \dots, t_{m,n_m}, p_{m,1}, \dots, p_{m,n_m}\}, 1 \leq m \leq M} \sum_{m=1}^M \sum_{s=0}^{n_m} pr_m(n_m, s) \sum_{\omega \in \Omega(n_m, s)} U_m(\omega) \\ & IR: \text{for any } 1 \leq m \leq M \\ & \left\{ \begin{aligned} & t_{m,1} \geq t_{m,2} \geq \dots \geq t_{m,n_m} \geq 0, p_{m,1} \geq p_{m,2} \geq \dots \geq p_{m,n_m} \geq 0 \\ & \theta_{m,n} t_{m,n_m} - p_{m,n_m} \geq 0 \\ & 1 \leq k \leq n_m, p_{m,k+1} + \theta_{m,k+1}(t_{m,k} - t_{m,k+1}) \leq p_{m,k} \leq p_{m,k+1} + \theta_{m,k}(t_{m,k} - t_{m,k+1}) \end{aligned} \right. \quad (5) \end{aligned}$$

### 3.2 The Solution of Optimization Problem of Optimal Contracts

A conceptually straightforward approach to derive the optimal contract is to solve (5) directly. Going through this route, however, is very challenging as (5) is non-convex and involves complicated constraints. In addition, it is difficult to compute  $pr(n_m, s)$ . We proposed an approximate algorithm to solve problem (5). The main idea of the approximate algorithm is reducing the variants by the relationship between relay powers and time allocations, then compute  $pr_m(n_m, s)$  under the condition the numbers of contracts designed by PUs are equal.

According to the theory 3 in [12], we think the optimal relay powers and time allocations have the approximate relationship as theory 3. Thus, (5) can be simplified into (6):

$$\begin{aligned} & \max_{\{n_m, t_{m,1}, t_{m,2}, \dots, t_{m,n_m}\}, 1 \leq m \leq M} \sum_{m=1}^M \sum_{s=0}^{n_m} pr_m(n_m, s) \sum_{\omega \in \Omega(n_m, s)} U_m(\omega) \\ & \text{subject to: } t_{m,1} \geq t_{m,2} \geq \dots \geq t_{m,n_m} \geq 0, 1 \leq m \leq M \end{aligned} \quad (6)$$

Assuming that when multiple PUs design contracts for a SU under incomplete information, the probabilities of each contract is accepted by the SU are equal. If every PU design  $K$  contracts ( $1 \leq K \leq N$ ), the probability of a contract is accepted is:

$$P(K) = \sum_{L=1}^M \frac{1}{L} P_K(L) \quad (7)$$

Where  $P_K(L)$  represents the  $SU^*$  a contract designed for is offered contracts by  $L$  PUs.

**Proposition:**  $M$  PUs and  $N$  SUs, if every PU design contracts for  $K$  SUs, if a PU design contract for  $SU^*$ , the probability of  $L$  PUs offer Contracts for  $SU^*$  is  $P_K(L)$ :

$$P_K(L) = \frac{C_{M-1}^{L-1} (C_{N-1}^{K-1})^{L-1} (C_{N-1}^K)^{M-L}}{(C_N^K)^{M-1}} \quad (8)$$

**Proof.** After a PU design contracts for  $SU^*$ , the number of other PUs' selection schemes is  $(C_N^K)^{M-1}$  and the number of schemes  $L-1$  PUs design contracts for  $SU^*$  is  $C_{M-1}^{L-1} (C_{N-1}^{K-1})^{L-1} (C_{N-1}^K)^{M-L}$ . Where  $C_{M-1}^{L-1}$  represents choosing  $L-1$  PUs from  $M-1$  PUs,  $(C_{N-1}^K)^{M-L}$  is  $M-L$  PUs don't design contract for  $SU^*$ . Therefore, the probability of  $L$  PUs offer Contracts for  $SU^*$  can be expressed as (8).

The probability of a contract is accepted can be written as:

$$P(K) = \sum_{L=1}^M \frac{1}{L} \frac{C_{M-1}^{L-1} (C_{N-1}^{K-1})^{L-1} (C_{N-1}^K)^{M-L}}{(C_N^K)^{M-1}} \quad (9)$$

The probability of  $s$  contracts from  $n_m$  contracts designed by  $PU_m$  are accepted is:

$$pr_m(K, s) = p(K)^s (1 - p(K))^{K-s} \quad (10)$$

Therefore, (6) can be simplified into (11):

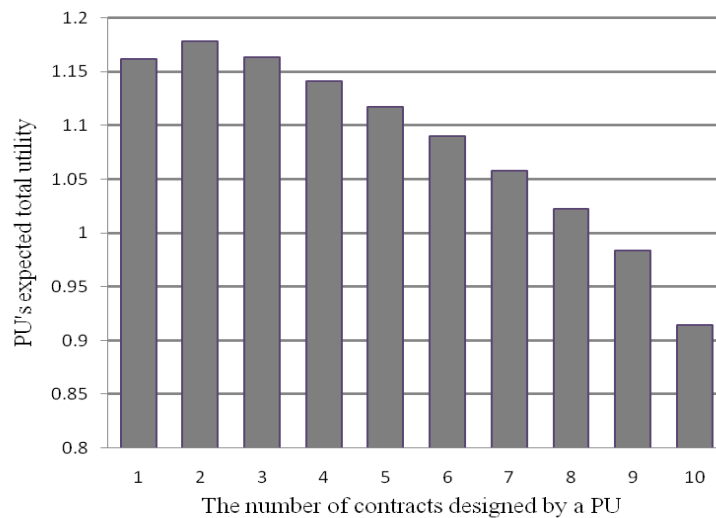
$$\begin{aligned} & \max_{\{K, t_{m,1}, t_{m,2}, \dots, t_{m,K}\}, 1 \leq m \leq M} \sum_{m=1}^M \sum_{s=0}^K p(K)^s (1-p(K))^{K-s} \sum_{\omega \in \Omega(K,s)} U_m(\omega) \\ & \text{subject to } t_{m,1} \geq t_{m,2} \geq \dots \geq t_{m,n_m} \geq 0, 1 \leq m \leq M \end{aligned} \quad (11)$$

As problem (11) isn't convex optimization problem, we can use heuristic algorithm to solve it after (11) has maximal value be proved in theory. Thus, the approximate algorithm works as follows:

- Step1. Let  $K=1:N$ , compute  $P(K)$  according to (9);
- Step2. According to (11), compute the PUs' expected total utility for  $k=1: N$ , respectively;
- Step3. Compare the utility obtained in step 2, pick the maximal utility and select the corresponding  $K$  and contracts.

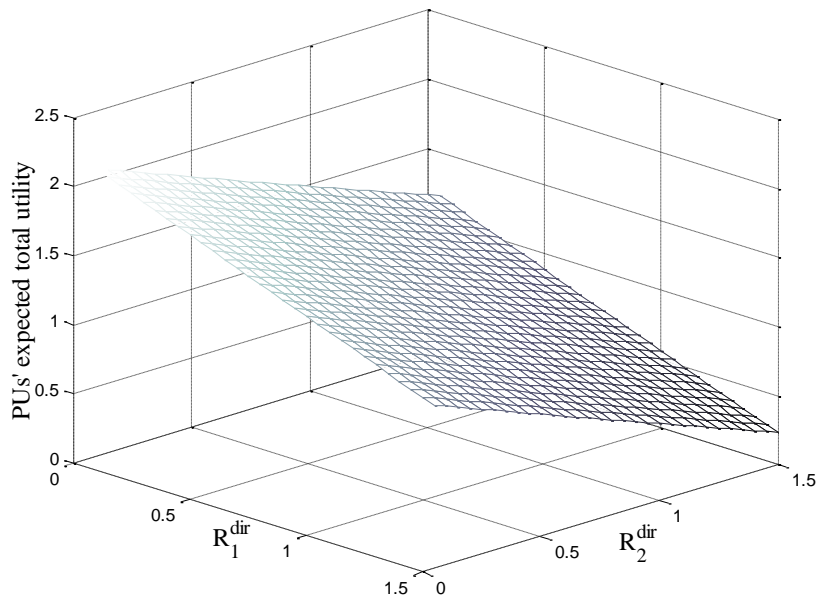
#### 4. Simulation Result and Analysis

In order to verify the correctness and effectiveness of the contract design scheme, we perform the simulation on the matlab in this section.



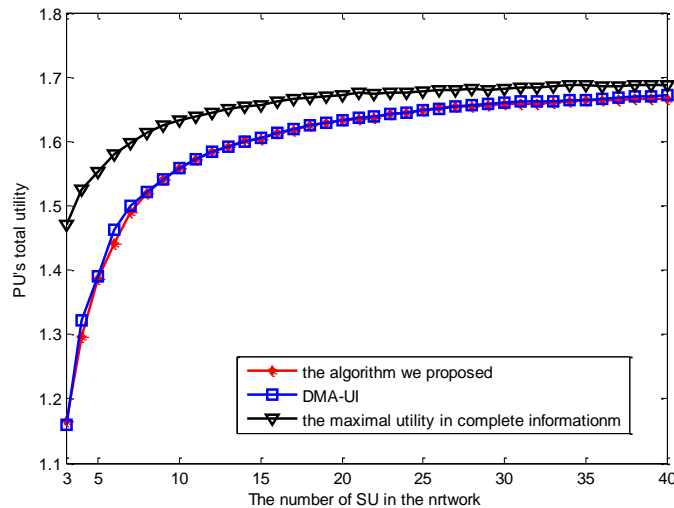
**Figure 2. The Relationship between PU's Expected Total Utility and the Number of Contract Designed by a PU**

Figure2 study how the number of contract designed by a PU affects PU's expected total utility. The main parameter settings of simulation are as follows:  $M=2$ ,  $N=10$ ,  $R_1^{dir} = 0.5$ ,  $R_2^{dir} = 0.5$ , the type of SU connected to PU obey uniform distribution of 5to25. We can conclude that PU's expected total utility achieve the maximal value when all PU design 2 contracts from Figure2. When the number of contracts equal to 1, once the contract is rejected, PU's utility will be damaged seriously and will affect PU's expected total utility. When the number of contracts more than 2, the average quality of SUs will decline with the number of contracts increasing. More than that, we can know a higher type SU can obtain more utility if choose the same contract from formula (2), so when a PU design contracts for multiple SUs, it should be guaranteed that higher type SU can obtain the maximal utility when choose contract designed for its type. PU has to lease their utility to satisfy this constraint and the utility loss is in proportion to the number of the contracts designed by the PU.



**Figure 3. The Relationship Between PU's Expected Total Utility and Pus' Direct Data Rate**

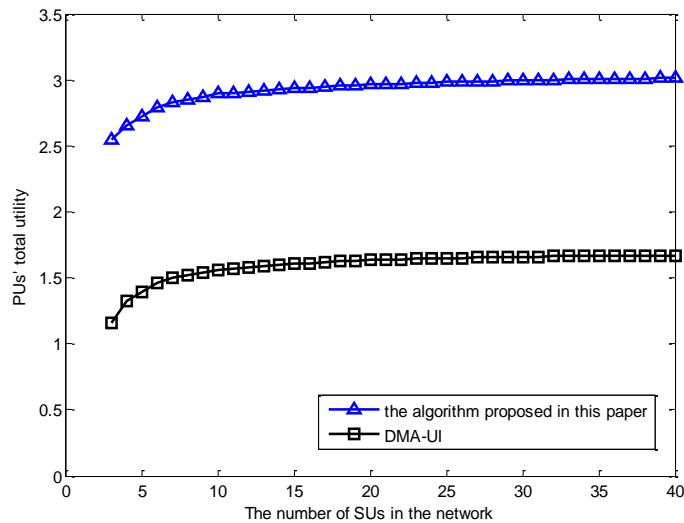
To study the influence of PUs' channel environment to its expected total utility, we perform an experience as Figure3, in which  $M=2, N=10$ , PUs' direct data rate range from 0 to 1. We can conclude that PU's will get a clear utility increment when the environment of PU is poor from Figure3. The reason is when the environment of PU is poor, PU can't use spectrum effectively without employing SU. The relay service can weaken the influence of poor channel environment.



**Figure 4. The Comparison Between Our Algorithm with DMA-UI**

Figure4 compare our algorithm with DMA-UI proposed in paper [13] and study the influence of the number of SUs in the network to PU's expected total utility. The main parameter settings are as follows:  $M=2, N=2, R_1^{dir} = 0.5, R_2^{dir} = 0.7$ , the type of SU connected to PU obey uniform distribution of 5to25. It can be seen from Figure4, there is a little

difference between our algorithm and DMA-UI in PU's total utility, which shows the algorithm we proposed is effective. Besides, a conclusion from Figure4 is PU's expected total utility increase with the number of SUs increasing, finally convergence to the maximal utility achieved in complete information. The reason is the probability of contract is accepted increase with SUs' number and attends to 1. When the probability is equal to 1, the competition between PUs can be ignored, this to say, this situation is equal to complete information.



**Figure 5. The Comparison between Our Algorithm with DMA-UI**

In order to further compare the performance of the algorithm in this paper and the DMA-UI, we consider a simulation scenarios as follows:  $M=2$ ,  $R_1^{dir} = 0.5$ ,  $R_2^{dir} = 0.7$ , the corresponding SU of each PU can be divided into two kinds, one SU included in the first kind, whose type connected to the PU obey uniform distribution of 20 to 40, the reminded SUs' types connected to the PU obey uniform distribution of 5 to 25. Figure5 shows that the algorithm in this paper outperform the DMA-UI, this is because there is a defect in the thought of DMA-UI. The idea of DMA-UI algorithm is each PU designs its contract on the condition that get a fixed utility, then all SUs choose their optimal contract and inform the corresponding PU, PU will raise its utility requirement if more than one SU choose its contract until only one SU choose the contract. When the requirement has increased a very small amount and the number of SU who choose the contract is 1, down from 2, PU won't raise its utility requirement any more, however, as a result of there is a high-performance SU in the network, the SU won't reject the contract even though PU raise its utility requirement, therefore, the DMA-UI algorithm will lead to the loss of PU's utility.

## 5. Conclusion

In this paper, we study the cooperative spectrum sharing between multiple PUs and multiple SUs. Through the mathematical derivation, under incomplete information we obtain an effective optimal contract design scheme and proposed an approximate algorithm to solve the optimization problem derived from the scheme. The simulation result shows us under incomplete information PUs' expected total utilities increase with the number of SUs. When the number of SUs is far larger than the number of PUs, the utility losses caused by incomplete information are negligible. However, SU's information is known by PU as the form of probability distribution in general, the



problem of cooperative spectrum between multiple PUs and multiple SUs worth further studying in future.

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