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Multi-Sensor Information Fusion Predictive Control Algorithm for System with Random Time-Delay Observations

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Abstract

The multi-sensor information fusion predictive control algorithm for discrete-time linear time-invariant stochastic control system with random time-delay observations is presented in this paper. The algorithm applies the fusion steady-state Kalman filter to the predictive control. It avoids the complex Diophantine equation and it can obviously reduce the computational burden. The algorithm can deal with the multi-sensor discrete-time linear time-invariant stochastic controllable system based on the linear minimum variance optimal information fusion criterion. The fusion method includes the centralized fusion, global optimality weighted measurement fusion. And the two fusion method is completely functionally equivalence. Compared with the single sensor case, the accuracy of the fused filter is greatly improved. A simulation example of the target tracking controllable system with two sensors shows its effectiveness and correctness.

***Keywords**: Predictive Control, Information Fusion, Centralized fusion, weighted measurement fusion*

1. Introduction

A system in which a signal is transmitted at a time or several times delay is called time-delay systems. The flow of steam and fluid in the pipe, the electrical signal in the long line of transmission, there is a time-delay. Systems containing such components are time-delay systems. For a specific control system, time delay may be caused by a measurement element or a measurement process, and may be caused by a control element and an actuating element, or caused by them. Strictly speaking, the time-delay in the control system is a common, but the size of the difference. Time-delay system is a system that can not be ignored. Because of the aging of the components, lack of the sensitivity and the delay of the information delivery, there is a time-delay in the system. The study of estimation for time-delay systems is of great significance**Error! Reference source not found.-Error! Reference source not found.**.

In this paper, steady-state Kalman filter is adopted. Steady-state Kalman filtering is not optimal because the gain is not optimal, and is asymptotically optimal. From the viewpoint of engineering application, the advantage of steady state Kalman filter is to avoid the on-line calculation gain, and the gain can be calculated off-line, which can simplify the calculation of Kalman filter and reduce the calculation burden**Error! Reference source not found.-Error! Reference source not found.**.

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Predictive control is a new type of computer control algorithm that has been successfully developed in recent years. Because of its control strategy, such as prediction model, rolling optimization, and feedback correction, the control effect is good. Since last century, predictive control has been widely used in the industrial field, such as the petroleum, chemical, metallurgy, machinery and other industrial department**Error! Reference source not found.**-**Error! Reference source not found.**. Synchronously, its theory has also received great attention in the industrial field and academia[Error! Reference source not found.], especially in the network control system**Error! Reference source not found.**-**Error! Reference source not found.**. It is suitable for the control of industrial production process, which is not easy to establish a precise digital model and more complex.

Since 1970s, with the emergence and development of various kinds of advanced weapon systems, it is urgent to improve the tracking precision and accuracy. In order to solve these problems, with the development of electronic technology and computer application technology, a large number of multi-sensor system with different application background has appeared. Multi-sensor information fusion state estimation is an important branch or field of multi-sensor information fusion. There are two kinds of observation fusion: one is the centralized observation, the observation equation of each sensor is merged into one dimensional observation equation, and then used to realize centralized global optimal Kalman filter with the state equation[Error! Reference source not found.~Error! Reference source not found.]. But its disadvantages are that the computational burden is large. The other is weighted measurement fusion. The method is based on the linear minimum variance criterion to obtain a fusion observation equation of each sensor. The merit of this method is that the dimension of the observation equation is not changed, and the global optimal state estimation can be obtained. The limitation is that each sensor has the same observation.

In this paper, the multi-sensor information fusion predictive control algorithm for system with random time-delay observations is presented. This algorithm avoids the complex Diophantine equation, but the state predictor is obtained by using steady-state Kalman filter, so it can obviously reduce the computational burden. Compared to the single sensor case, using the information fusion algorithm improves the accuracy of the predictive control and the stability of the system. And the two fusion method is completely functionally equivalence. Simulation results verify its effectiveness and correctness.

This paper is organized as follows: Section 2 presents Problem formulation. The local steady-state Kalman filter of the *i*th time-invariant subsystem are presented in Section 3. Section 4 presents the information fusion Kalman filter. Predictive control algorithm base on steady-state Kalman filter is presented in Section 5. A simulation example is given in Section 6. The conclusions are presented in Section 7.

2. Problem Formulation

Consider the multi-sensor linear discrete-time time-invariant stochastic controllable system with random time-delay observations

$$\boldsymbol{x}(t+1)=\boldsymbol{\Phi}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{\Gamma}\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}_i(t)=\boldsymbol{H}_i\boldsymbol{x}(t-\tau_i)+\boldsymbol{v}_i(t),\quad i=1,\cdots,L \tag{2}$$

where t is the discrete time, the subscript i denotes the ith sensor, L denotes the number of sensor, $\boldsymbol{x}(t)\in\boldsymbol{R}^n$ is the state of the system, $u(t)$ is the input, $\boldsymbol{y}_i(t)\in\boldsymbol{R}^{mi}$ is the measurement of the ith sensor subsystem, $\boldsymbol{v}_i(t)\in\boldsymbol{R}^{mi}$ is the measurement noise of the ith sensor subsystem, $\boldsymbol{w}(t)\in\boldsymbol{R}^r$ is the input noise, $\boldsymbol{\Phi}$, $\boldsymbol{B}$, $\boldsymbol{\Gamma}$, $\boldsymbol{H}_i$ is the suitable dimensional matrix respectively, the time-delay $\tau_i>0$.

Assumption 1 $\boldsymbol{w}(t)\in \boldsymbol{R}^r$ and $\boldsymbol{v}_i(t)\in \boldsymbol{R}^{mi}$, $i=1,\cdots,L$ are independence white noises with zero mean and covariance are $\boldsymbol{Q}_w$ and $\boldsymbol{Q}_{vi}$ individually

$$\mathrm{E}\left\{\begin{bmatrix}\boldsymbol{w}(t)\\ \boldsymbol{v}_i(t)\end{bmatrix}\begin{bmatrix}\boldsymbol{w}(k) & \boldsymbol{v}_i(k)\end{bmatrix}\right\}=\begin{bmatrix}\boldsymbol{Q} & 0\\ 0 & \boldsymbol{R}_i\end{bmatrix}\delta_{tk} \tag{3}$$

Where E is the mathematical expectation,, the superscript T denotes the transpose, and δ_{tk} is the Kronecker delta function, $\delta_{tt}=1, \delta_{tk}=0(t\neq k)$.

Assumption 2 The initial state value $\boldsymbol{x}(0)$ is uncorrelated with $\boldsymbol{w}(t)$ and $\boldsymbol{v}_i(t)$, and $\mathrm{E}\boldsymbol{x}(0)=\boldsymbol{x}_0$, $\mathrm{cov}\,\boldsymbol{x}(0)=\boldsymbol{P}_0$.

Assumption 3 $\boldsymbol{u}(t)$ is the known time sequence, or linear function (feedback control) of $(\boldsymbol{y}(t),\ \boldsymbol{y}(t-1),\cdots)$.

Assumption 4 $(\boldsymbol{\Phi},\boldsymbol{H}_i)$ is completely observable pair, and $(\boldsymbol{\Phi},\boldsymbol{\Gamma})$ is completely controllable pair.

Assumption 5 The initial time $t_0=-\infty$.

Our aims are based on the measurement $(\boldsymbol{y}(t),\ \boldsymbol{y}(t-1),\cdots,\ \boldsymbol{y}(0))$, using information fusion steady-state Kalman estimation to get the *N*-step-ahead optimal predictive control algorithm.

Introducing observation and noise transformation

$$\boldsymbol{z}_i(t)=\boldsymbol{y}_i(t+\tau_i),\ \xi_i(t)=\boldsymbol{v}_i(t+\tau_i) \tag{4}$$

So there is no delay observation equation is as follow

$$\boldsymbol{z}_i(t)=\boldsymbol{H}_i\boldsymbol{x}(t)+\xi_i(t),\quad i=1,\cdots,L \tag{5}$$

Based on the measurement $(\boldsymbol{y}(t),\ \boldsymbol{y}(t-1),\cdots,\ \boldsymbol{y}(0))$, for the system (1) and system (2), the problem of local steady-state Kalman filter $\hat{\boldsymbol{x}}_i(t\,|\,t)$ is equivalent to the local steady-state Kalman predictor $\hat{\boldsymbol{x}}_{zi}(t\,|\,t-\tau_i)$ that for the transformed system (1) and (5).

$$\hat{\boldsymbol{x}}_i(t\,|\,t)=\hat{\boldsymbol{x}}_{zi}(t\,|\,t-\tau_i) \tag{6}$$

3. Local Steady-State Kalman Filter

Lemma 1 **Error! Reference source not found.** For system (1) and (5) with the assumption 1-5, the *i*th sensor subsystem has the local optimal steady-state Kalman predictor equations:

$$\hat{\boldsymbol{x}}_{zi}(t+1\,|\,t)=\boldsymbol{\Psi}_{pi}\hat{\boldsymbol{x}}_{zi}(t\,|\,t-1)+\boldsymbol{B}\boldsymbol{u}(t+1)+\boldsymbol{K}_{pi}\boldsymbol{y}_i(t) \tag{7}$$

$$\boldsymbol{\Psi}_{pi}=\boldsymbol{\Phi}-\boldsymbol{K}_{pi}\boldsymbol{H}_i \tag{8}$$

$$\boldsymbol{K}_{pi}=\boldsymbol{\Phi}\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{Q}_{\varepsilon i}^{-1} \tag{9}$$

$$\boldsymbol{Q}_{\varepsilon i}=\boldsymbol{H}_i\,\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}+\boldsymbol{R}_i \tag{10}$$

And with the arbitrary initial values are $\hat{x}_i(0\,|\,0)$, further $\sum_i$ satisfies the Riccati equation

$$\boldsymbol{\Sigma}_i=\boldsymbol{\Phi}[\boldsymbol{\Sigma}_i-\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}(\boldsymbol{H}_i\boldsymbol{\Sigma}_i\boldsymbol{H}_i^{\mathrm{T}}+\boldsymbol{Q}_{vi})^{-1}\boldsymbol{H}_i\boldsymbol{\Sigma}_i]\boldsymbol{\Phi}^{\mathrm{T}}+\boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{\Gamma}^{\mathrm{T}} \tag{11}$$

By type (6), the local steady-state Kalman filter can be computed as

$$\hat{\boldsymbol{x}}_i(t\,|\,t)=\boldsymbol{\Phi}^{\tau_i-1}\hat{\boldsymbol{x}}_{zi}(t\,|\,t-\tau_i) \tag{12}$$

Its steady-state error variance matrix of the filter error matrix is equal to the error variance matrix of the predictor, that is

$$\boldsymbol{P}_i = \boldsymbol{\Phi}^{\tau_i - 1}\boldsymbol{\Sigma}_i \boldsymbol{\Phi}^{(\tau_i - 1)\mathrm{T}} + \sum_{r=0}^{t-2} \boldsymbol{\Phi}^r \boldsymbol{\Gamma} \boldsymbol{Q} \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{\Phi}^{r\mathrm{T}} \tag{13}$$

Where $\tau_i \geq 2$. When $\tau_i = 1$, we have $\boldsymbol{P}_i = \boldsymbol{\Sigma}_i$.

Lemma 2 **Error! Reference source not found.** The multi-sensor linear discrete-time time-invariant stochastic controllable system (1) and (5) under the assumption 1-5, the cross covariance between any two local filter satisfies Lyapunov equation:

$$\boldsymbol{P}_{ij}(k_i, k_j) = \boldsymbol{\Phi}^{-k_i - 1}\boldsymbol{\Sigma}_{ij} \boldsymbol{\Phi}^{(-k_i - 1)\mathrm{T}} + \sum_{j=0}^{-\max(k_i, k_j)-2} \boldsymbol{\Phi}^j \boldsymbol{\Gamma} \boldsymbol{Q} \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{\Phi}^{j\mathrm{T}}, \ k_i, k_j \leq -2 \tag{14}$$

4. Fusion Kalman Filter

The system (1) and (5) can be written as

$$\boldsymbol{z}^{(0)}(t) = \boldsymbol{H}^{(0)}\boldsymbol{x}(t) + \boldsymbol{\xi}^{(0)}(t) \tag{15}$$

$$\boldsymbol{z}^{(0)}(t) = \left[\boldsymbol{z}_1^{\mathrm{T}}(t) \quad \cdots \quad \boldsymbol{z}_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{16}$$

$$\boldsymbol{H}^{(0)}(t) = \left[\boldsymbol{H}_1^{\mathrm{T}} \quad \cdots \quad \boldsymbol{H}_L^{\mathrm{T}}\right]^{\mathrm{T}} \tag{17}$$

$$\boldsymbol{\xi}^{(0)}(t) = \left[\boldsymbol{\xi}_1^{\mathrm{T}}(t) \quad \cdots \quad \boldsymbol{\xi}_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{18}$$

$$\boldsymbol{Q}_{\xi}^{(0)} = \mathrm{diag}(\boldsymbol{Q}_{v_1}, \cdots, \boldsymbol{Q}_{v_L}) \tag{19}$$

Theorem 1 **Error! Reference source not found.-Error! Reference source not found.** For system (1)-(15) under Assumptions 1-5, the optimal centralized fusion steady-state Kalman filter $\hat{\boldsymbol{x}}^{(c)}(t|t)$ are calculated by

$$\hat{\boldsymbol{x}}^{(c)}(t+1 \mid t) = \boldsymbol{\Psi}_p^{(c)}\hat{\boldsymbol{x}}^{(c)}(t \mid t-1) + \boldsymbol{B}^{(c)}\boldsymbol{u}^{(c)}(t+1) + \boldsymbol{K}_p^{(c)}\boldsymbol{y}^{(c)}(t) \tag{20}$$

$$\boldsymbol{\Psi}_p^{(c)} = \boldsymbol{\Phi} - \boldsymbol{K}_p^{(c)}\boldsymbol{H}^{(0)} \tag{21}$$

$$\boldsymbol{K}_{pi}^{(c)} = \boldsymbol{\Phi}\boldsymbol{\Sigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}\boldsymbol{Q}_{\varepsilon}^{(c)-1} \tag{22}$$

$$\boldsymbol{Q}_{\varepsilon}^{(c)} = \boldsymbol{H}^{(0)}\boldsymbol{\Sigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}} + \boldsymbol{Q}_v^{(0)} \tag{23}$$

$$\boldsymbol{P}^{(c)} = [\boldsymbol{I}_n - \boldsymbol{K}_{fi}^{(c)}\boldsymbol{H}^{(0)}]\boldsymbol{\Sigma}^{(c)} \tag{24}$$

And with the arbitrary initial values are $\hat{\boldsymbol{x}}_i(0 \mid 0)$, further $\sum_i$ satisfies the Riccati equation

$$\boldsymbol{\Sigma}^{(c)} = \boldsymbol{\Phi}[\boldsymbol{\Sigma}^{(c)} - \boldsymbol{\Sigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}}(\boldsymbol{H}^{(0)}\boldsymbol{\Sigma}^{(c)}\boldsymbol{H}^{(0)\mathrm{T}} + \boldsymbol{Q}_{vi})^{-1}\boldsymbol{H}^{(0)}\boldsymbol{\Sigma}^{(c)}]\boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{\Gamma}\boldsymbol{Q}\boldsymbol{\Gamma}^{\mathrm{T}} \tag{25}$$

Where $\boldsymbol{P}^{(c)}$ is fused steady-state filter error variance matrix.

The system (5) can be written as

$$\bar{\boldsymbol{z}}^{(\mathrm{o})}(t) = \bar{\boldsymbol{H}}^{(\mathrm{o})}\boldsymbol{x}(t) + \bar{\boldsymbol{\xi}}^{(\mathrm{o})}(t) \tag{26}$$

$$\bar{\boldsymbol{z}}^{(\mathrm{o})}(t) = [\sum_{i=1}^{L}\boldsymbol{Q}_{vi}^{-1}]^{-1}\sum_{i=1}^{L}\boldsymbol{Q}_{vi}^{-1}\bar{\boldsymbol{z}}_i(t) \tag{27}$$

$$\bar{\boldsymbol{\xi}}^{(\mathrm{o})}(t) = [\sum_{i=1}^{L}\boldsymbol{Q}_{vi}^{-1}]^{-1}\sum_{i=1}^{L}\boldsymbol{Q}_{vi}^{-1}\bar{\boldsymbol{\xi}}_i(t) \tag{28}$$

$$\boldsymbol{Q}_{\xi}^{(\mathrm{o})} = [\sum_{i=1}^{L}\boldsymbol{Q}_{vi}^{-1}]^{-1} \tag{29}$$

Theorem 2 **Error! Reference source not found.-Error! Reference source not found.** For the system (1) and (26) under the

Assumption 1-5, global optimality weighted measurement fusion steady-state Kalman filter equations:

$$\hat{\boldsymbol{x}}^{(c)}(t+1|t+1)=\boldsymbol{\Psi}_f\hat{\boldsymbol{x}}^{(c)}(t|t)+\boldsymbol{K}_f\bar{\boldsymbol{z}}^{(\mathrm{o})}(t) \tag{30}$$

$$\boldsymbol{\Psi}_f=[\boldsymbol{I}_n-\boldsymbol{K}_f\bar{\boldsymbol{H}}^{(\mathrm{o})}]\bar{\boldsymbol{\Phi}} \tag{31}$$

Where $\boldsymbol{\Psi}_{fi}$ a is a stable matrix, the filtering gain $\boldsymbol{K}_{fi}$ is as follows:

$$\boldsymbol{K}_{fi}=\begin{bmatrix}\bar{\boldsymbol{H}}^{(\mathrm{o})}\\ \bar{\boldsymbol{H}}^{(\mathrm{o})}\bar{\boldsymbol{\Phi}}\\ \vdots\\ \bar{\boldsymbol{H}}^{(\mathrm{o})}\bar{\boldsymbol{\Phi}}^{\beta-1}\end{bmatrix}^{+}\begin{bmatrix}I_{m_1}-\boldsymbol{Q}_v\\ \boldsymbol{M}_1\\ \vdots\\ \boldsymbol{M}_{\beta_i-1}\end{bmatrix} \tag{32}$$

The pseudo inverse X^+ of the matrix X is defined as $X^+=(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$, β is index of correlation of $(\bar{\boldsymbol{\Phi}},\bar{\boldsymbol{H}}^{(\mathrm{o})})$, the coefficient matrix $\boldsymbol{M}_k$ can be recursively computed as

$$\boldsymbol{M}_k=-\boldsymbol{A}_1\boldsymbol{M}_{k-1}-\cdots-\boldsymbol{A}_{n_a}\boldsymbol{M}_{k-n_a}+\boldsymbol{D}_k \tag{33}$$

Where $\boldsymbol{M}_0=I_{m_i}$, $\boldsymbol{M}_k=0(k<0)$, $\boldsymbol{D}_k=0(k>n_d)$.

Optimality weighted measurement fusion filtering error covariance matrix $\boldsymbol{P}_0$ satisfies the Lyapunov equation

$$\boldsymbol{P}_0=\boldsymbol{\Psi}_f\boldsymbol{P}_0\boldsymbol{\Psi}_f^{\mathrm{T}}+[\boldsymbol{I}_n-\boldsymbol{K}_f\bar{\boldsymbol{H}}^{(\mathrm{o})}]\bar{\boldsymbol{\Gamma}}\boldsymbol{Q}_{\bar{w}}\bar{\boldsymbol{\Gamma}}^{\mathrm{T}}[\boldsymbol{I}_n-\boldsymbol{K}_f\bar{\boldsymbol{H}}^{(\mathrm{o})}]^{\mathrm{T}}+\boldsymbol{K}_f{}_{\xi}^{(\mathrm{o})}\boldsymbol{K}_f^{\mathrm{T}} \tag{34}$$

The multi-sensor linear discrete-time time-invariant stochastic controllable system (1) and (5) under the assumption 1-5, the optimal centralized fusion steady-state Kalman filter from (20) ~ (25) is fully functional equivalence to weighted measurement fusion steady-state Kalman filter (30) ~ (34). That they have the same values of Kalman estimators and corresponding error variance matrix.

5. Predictive Control Algorithm Base on Steady-State Kalman Filter

For system (1) and (4) with the assumption 1-5, select the fused state $\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{x}^{(c)}(t)$ ($\boldsymbol{e}_i^{\mathrm{T}}=[\underbrace{0\cdots 0}_{i-1}\ 1\underbrace{0\cdots 0}_{m-i}]$) as the controlled variable, select $x_r(t)$ as the reference track at time t. For the every time t, control increments $\Delta u(t), \Delta u(t+1), \cdots, \Delta u(t+N_\mu-1)$ need to be obtained to make the states in the future $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}^{(c)}(t+j|t), j=1,\cdots,N$ as close to the given reference track $x_r(t+j)$ as possible. Where N_μ is the control time domain, N is the optimize time domain.

Defining the expanse function **Error! Reference source not found.**:

$$J=(\hat{\boldsymbol{X}}^{(c)}-\boldsymbol{X}_r)^{\mathrm{T}}\boldsymbol{Q}_y(\hat{\boldsymbol{X}}^{(c)}-\boldsymbol{X}_r)+\boldsymbol{\Delta U}^{\mathrm{T}}\boldsymbol{R}_u\boldsymbol{\Delta U} \tag{35}$$

Where $\boldsymbol{X}_r=\begin{bmatrix}x_r(t+1) & x_r(t+2) & \cdots & x_r(t+N)\end{bmatrix}^{\mathrm{T}}$ is the reference track of state, and setting $\boldsymbol{e}^{\mathrm{T}}=$ $\begin{bmatrix}\boldsymbol{e}_i^{\mathrm{T}} & \cdots & \boldsymbol{e}_i^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, $\hat{\boldsymbol{X}}^{(c)}=\boldsymbol{e}^{\mathrm{T}}\begin{bmatrix}\hat{\boldsymbol{x}}^{(c)\mathrm{T}}(t+1|t) & \hat{\boldsymbol{x}}^{(c)\mathrm{T}}(t+2|t) & \cdots & \hat{\boldsymbol{x}}^{(c)\mathrm{T}}(t+N|t)\end{bmatrix}^{\mathrm{T}}$ is called the controlled state, and $\boldsymbol{Q}_y=\mathrm{diag}(q_1,\cdots,q_N)$ is called the error weighted matrix, $\boldsymbol{R}_\mu=\mathrm{diag}(r_1,\cdots,r_{N_\mu})$ is called the

Controlled weighted matrix.

From (1), we have

$$\hat{\boldsymbol{x}}_i(t+1|t) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_i(t|t) + \boldsymbol{B}u(t) \quad (36)$$

Substituting (20) or (30) into (36) yields

$$\hat{\boldsymbol{x}}^{(c)}(t+1|t) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}^{(c)}(t|t) + \boldsymbol{B}u(t) \quad (37)$$

And

$$\hat{\boldsymbol{x}}^{(c)}(t+j|t) = \boldsymbol{\Phi}^j\hat{\boldsymbol{x}}^{(c)}(t|t) + \boldsymbol{\Phi}^{j-1}\boldsymbol{B}u(t) + \cdots + \boldsymbol{B}u(t+j-1) \quad (38)$$

Defining

$$\Delta u(t) = u(t) - u(t-1) \quad (39)$$

Having

$$u(t+j-1) = \Delta u(t+j-1) + \Delta u(t+j-2) + \cdots + \Delta u(t) + u(t-1) \quad (40)$$

Substituting (40) into (38) yields

$$\hat{\boldsymbol{x}}^{(c)}(t+j|t) = \boldsymbol{\Phi}^j\hat{\boldsymbol{x}}^{(c)}(t|t) + (\boldsymbol{\Phi}^{j-1} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B}\Delta u(t) + (\boldsymbol{\Phi}^{j-2} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B}\Delta u(t+1) + \cdots + \boldsymbol{B}\Delta u(t+j-1) + (\boldsymbol{\Phi}^{j-1} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B}u(t-1) \quad (41)$$

so that

$$\hat{\boldsymbol{X}}^{(c)} = \boldsymbol{e}^{\mathrm{T}}[\boldsymbol{\Phi}_x\hat{\boldsymbol{x}}^{(c)}(t|t) + \boldsymbol{\Phi}_N\Delta\boldsymbol{U} + \boldsymbol{\Phi}_\mu\boldsymbol{u}(t-1)] \quad (42)$$

Defining

$$\boldsymbol{\Phi}_x = \begin{bmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Phi}^2 \\ \cdots \\ \boldsymbol{\Phi}^N \end{bmatrix}, \boldsymbol{\Phi}_N = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ (\boldsymbol{\Phi}+\boldsymbol{I})\boldsymbol{B} & \boldsymbol{B} & \cdots & \boldsymbol{0} \\ \cdots & \cdots & \cdots & \cdots \\ (\boldsymbol{\Phi}^{N-1} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B} & (\boldsymbol{\Phi}^{N-2} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B} & \cdots & \boldsymbol{B} \end{bmatrix}, \boldsymbol{\Delta U} = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \cdots \\ \Delta u(t+N_\mu-1) \end{bmatrix},$$

$$\boldsymbol{\Phi}_\mu = \begin{bmatrix} \boldsymbol{B} \\ (\boldsymbol{\Phi}+\boldsymbol{I})\boldsymbol{B} \\ \cdots \\ (\boldsymbol{\Phi}^{N-1} + \cdots + \boldsymbol{\Phi} + \boldsymbol{I})\boldsymbol{B} \end{bmatrix} \quad (43)$$

Substituting (42) into (35), setting $\frac{\partial J}{\partial \boldsymbol{\Delta U}} = 0$, it yields that

$$\boldsymbol{\Delta U}(t) = [(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_y(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N) + \boldsymbol{R}_\mu]^{-1}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\Phi}_N)^{\mathrm{T}}\boldsymbol{Q}_y \times \{\boldsymbol{X}_r - \boldsymbol{e}^{\mathrm{T}}[\boldsymbol{\Phi}_N\hat{\boldsymbol{x}}(t|t) - \boldsymbol{\Phi}_\mu u(t-1)]\} \quad (44)$$

From (39) and (44) , the predictive control is obtained as

$$u(t) = u(t-1) + \boldsymbol{e}_1\Delta\boldsymbol{U}(t) \quad (45)$$

Where $\boldsymbol{e}_1 = [1\,0\cdots 0]$.

6. Simulation Example

Consider 3-sensor the multi-sensor linear discrete-time time-invariant stochastic controllable system with random time-delay observations (1) and (2), where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ is the state, $y_i(t)$ is the measurement of the ith subsystem, $\boldsymbol{\Phi} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{\Gamma} = \begin{bmatrix} \frac{1}{2}T^2 & T \end{bmatrix}^{\mathrm{T}}$, $T = 0.4$ is the sampled period, $\boldsymbol{H}_1 = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}$, $\boldsymbol{H}_2 = \begin{bmatrix} 0.1 & 0.6 \end{bmatrix}$, $\boldsymbol{H}_3 = \begin{bmatrix} 0.1 & 0.8 \end{bmatrix}$.

And $\boldsymbol{w}(t)$ and $\boldsymbol{v}_i(t)$ are assumed to be independent Gaussian white noises with zero mean and variances $\boldsymbol{Q} = 0.1$, $\boldsymbol{Q}_{v1} = 0.1$, $\boldsymbol{Q}_{v2} = 0.3$, $\boldsymbol{Q}_{v3} = 0.5$.

The estimation criterion of the controlled system is defined as the sum of mean square error function (SMSE) of the differences of the state reference track $x_r(t)$ and the controlled state fusion estimator $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0(t\,|\,t)$ weighted by scalars **Error! Reference source not found.-Error! Reference source not found.**

$$\mathrm{SMSE}(k)=\sum_{t=0}^{k}\frac{1}{L}\sum_{j=1}^{L}[\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0^{(j)}(t\,|\,t)-x_r(t)]^2 \tag{46}$$

Where $\boldsymbol{e}_i^{\mathrm{T}}\hat{\boldsymbol{x}}_0^{(j)}(t\,|\,t)$ is the *j*th Monte Carlo simulation state estimates at time *t*.

In Monte Carlo simulation for 30 times, $x_1(t)$ is selected as controlled state. Setting the control time domain $N_\mu=3$ and the optimize time domain $N=3$, and the reference track $x_r(t)$ is the 20 units step signal that appears at the time $t=100$, and the error weighted matrix $\boldsymbol{Q}_y=\mathrm{diag}(3,2,1)$, and the controlled weighted matrix $\boldsymbol{R}_\mu=\mathrm{diag}(3,2,1)\times 0.1$.

The simulation results are shown in Figure1-Figure5. Controller output *u*(*t*) is shown in Figure1 and Figure3. Figure2 and Figure4 show the comparison curves of the state reference track $x_r(t)$ and the fused steady-state Kalman filter. From Figure2 and Figure4, it shows that the fused steady-state Kalman filter can track the state reference track $x_r(t)$ closely, where the straight lines denote the state reference track, and the dashed curves denote the fused steady-state Kalman filter. It indicates that this algorithm has good convergence and attenuation, and the overshoot is small, and the controlled output is stable. The curves of the sum of mean square error (SMSE) for local and fusion steady-state Kalman filters are shown in Figure5. We can see that that the accuracy of the fused steady-state Kalman filter is higher than single local Kalman filter.

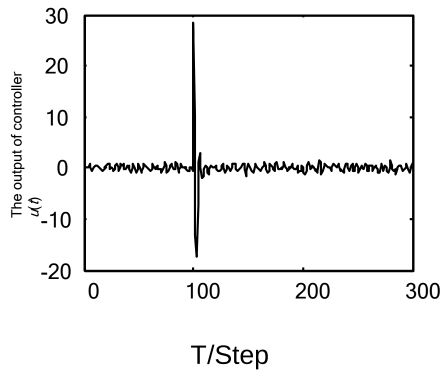


Figure 1. Controller Output *u*(*t*)

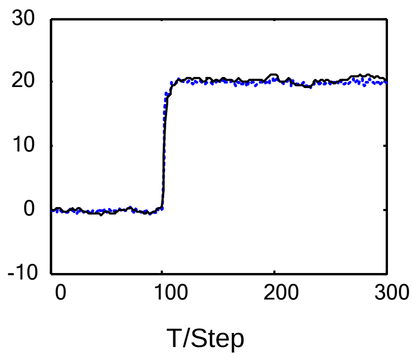


Figure 2. State Reference Track $x_r(t)$ and the Centralized Fusion Steady-State Kalman Filter $\hat{x}^{(c)}(t\,|\,t)$

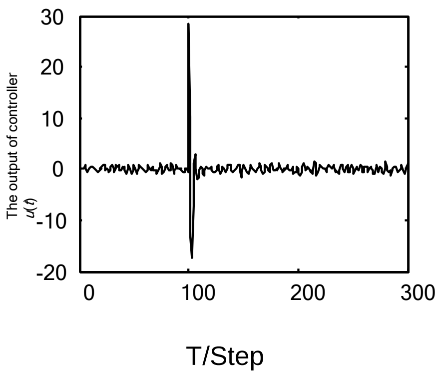


Figure 3. Controller Output *u*(*t*)

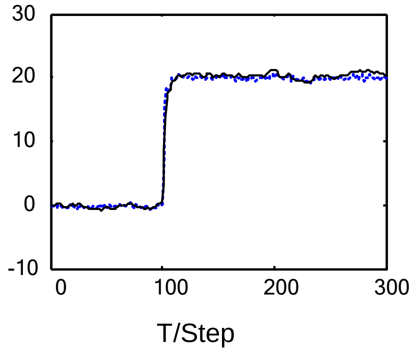


Figure 4. State Reference Track $x_r(t)$ and Weighted Measurement Fusion Steady-State Kalman Filter $\hat{x}^{(c)}(t|t)$

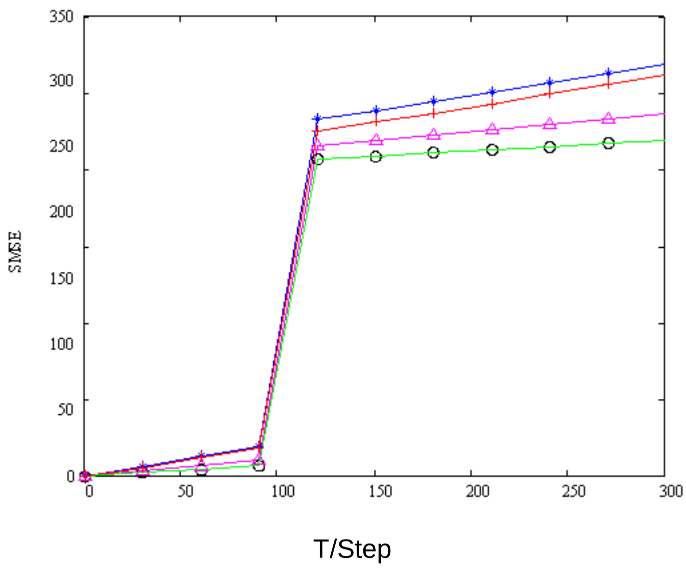
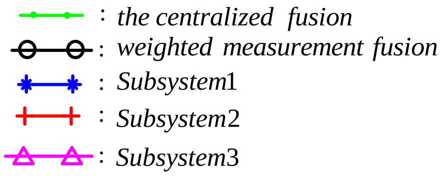


Figure 5. The Monte Carlo Curves of the Sum of Mean Square Error (SMSE)



7. Conclusion

In this paper, multisensor information Fusion Predictive Control for time-invariant systems is presented. The algorithm for time-invariant system combines the fusion steady-state Kalman filter with predictive control firstly. Compared with the classic generalized predictive control, the advantages are as follows:

1. This algorithm based on steady-state Kalman filter avoids the complex Diophantine equation and computing the gain on-line, so it can obviously reduce the computational burden.
2. Classic generalized predictive control only deals with time-invariant system, or the time-varying system that parameters varies slowly, this is called adaptive generalized predictive control **Error! Reference source not found.**. However steady-state Kalman filter can deal with the time-varying system, so the predictive control system based on steady-state Kalman filter can deal with the linear time-varying and time-invariant system.
3. The stability of the fusion steady-state Kalman filter is making the stability of the system get better, and the ability of anti-jamming is enhanced.
4. Using the information fusion algorithm compared to the single sensor case, the accuracy is improved.
5. The optimal centralized fusion steady-state Kalman filter from (20)-(25) is fully functional equivalence to weighted measurement fusion steady-state Kalman filter (30)-(34).

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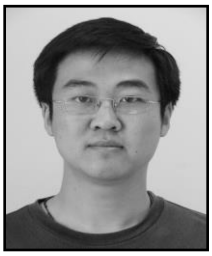

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