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A Variable Step-Size Least-Mean-Square Adaptive Filtering Algorithm: Design and Application

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Abstract

This paper briefly analyses and discusses the advantages and disadvantages of the commonly used LMS adaptive filtering algorithm, proposes a new nonlinear functional relationship between step factor μ and error signal e (n). And improve methods for variable step-size LMS algorithm appropriately. The algorithm utilizes the correlation value e (n) e (n - D) of output error signal to adjust the step factor μ, solves the inconsistency problem of error performance and convergence time. The algorithm is applied in the simulation of several typical signals processing system, and computer simulation results confirm the theoretical analysis.

Keywords: *Variable Step-Size; adaptive filtering; LMS algorithm; signal processing*

1. Introduction

Filter is the basic unit circuit of the electronic information processing system. The use of technology of filtering to extract useful signal from the mixed signals , at the same time suppress noise or interference signals.The digital filter is refers to the digital signal of input and output, it is the device that through certain operation method to change the input signal frequency components contained ratio or filter out certain frequency components. Adaptive digital filter is to use the filter parameters in the previous time，adjusting the filter parameters of the now moment automatically, to adapt to the unknown or changing with time of signal and noise, to achieve the optimal filter.

Adaptive filter can be divided into linear adaptive filter and nonlinear adaptive filter. Nonlinear adaptive filter such as adaptive filter based on neural network, nonlinear adaptive filter has the stronger ability of signal processing. But due to the high computational complexity of nonlinear adaptive filter, in practical application of linear adaptive filter still use more.

This paper analyzes Least-Mean-Square algorithm (LMS), to adjust the parameters of the linear adaptive filter, to be able to run in the best state of filtering.

2. Principle of Adaptive Filter

Adaptive Filter contains the digital structure of adjustable parameters (or called adaptive processor) and adaptive algorithm of two parts. The principle of adjustable algorithm: Eventually make the error e (n) of the output signal and the desired signal minimum mean square value. The Adaptive Filter is a kind of special Wiener Filter that parameters can automatically adjust itself. In the design of adaptive filter, does not need to know the input signal and noise statistical properties, it can gradually learn or estimate the required

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statistical properties in their work, and according to automatically adjust parameters to achieve the goal of optimal filtering. All in all, the most obvious feature of the adaptive filtering: learning and tracing.

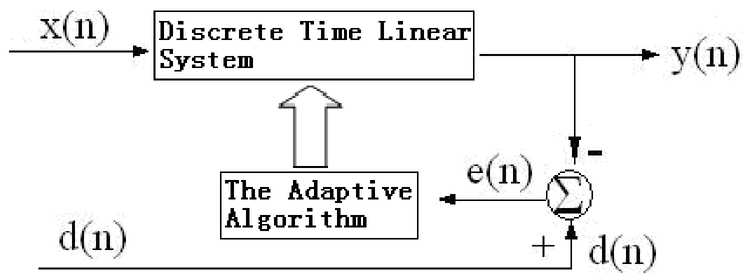

Figure 1. Block Diagram of Adaptive Filter Principle

Figure 1 describes general estimation problem for adaptive filter. Discrete time linear systems as shown could be expressed as a programmable filter; its impulse response is expressed as h (n), or calls the filter parameters. Adaptive filter output signal is expressed as y (n), the desired response signal is expressed as d (n), d(n) and y (n) by subtracting said error signal e (n).H (n) will automatically adjust according to the size of e (n), that the estimates $\hat{y}(n)$ of y (n) is equal to the desired response d (n). Therefore, the impulse response or filter parameters of the adaptive filter are changed along with the change of external environment. After a period of automatic convergence time adjustment and achieve the best filtering requirements. But, the adaptive filter itself has some important adaptive algorithm. This article will discuss the principle and application of LMS for adaptive filtering algorithm.

3. Principle and Improvement for LMS

3.1. LMS Algorithm Principle

LMS (Least-Mean-Squares) algorithm is least mean square error algorithm，is linear adaptive filtering algorithm. Including filtering and adaptive process, the goal is through adjusting coefficient, minimize the mean square value of e(n)=d(n)-y(n).and modify weight coefficient according to the criterion. LMS algorithm is the prominent advantages of small amount of calculation, And no off-line calculation, as long as know the input signal and the reference response. In addition to the basic algorithm, LMS algorithm has many variants. The following LMS algorithm based on the steepest descent method.

The iterative formula is as follows for LMS algorithm based on the steepest descent method.

$$e(n)=d(n)-w(n-1)x(n) \quad (1)$$

$$w(n)=w(n-1)+2\mu(n)e(n)x(n) \quad (2)$$

x (n) as the input of adaptive filter, d(n) for the desired output signal, e(n) for the error, w (n) for weight coefficient and μ(n) for step factor.

LMS algorithm convergence condition is: $0 <\mu< 1/\lambda_{max}$, λ_{max} is the largest eigenvalue of the input signal autocorrelation matrix. Take μ (n) for constant is the most simple parameter selection, that is:

$\mu(n) = \mu \quad (0 < \mu < 1/\lambda max)$

But this fixed step adaptive algorithm for LMS, the requirements is contradictory among convergence rate, tracking rate and steady-state error. Its performance is controlled by step size, and can't be satisfied at the same time. Or, this method will cause the contradiction between convergence rate and stability. The big learning rate parameter to improve the convergent rate of filters, but would reduce the steady state performance. On the other hand, in order to improve the steady-state performance and use small learning rate parameter, tracking speed and convergence rate will slow down.

3.2. The Variable Step-Size LMS Adaptive Filtering Algorithm

Therefore, in order to give consideration to the steady state performance and convergence rate, commonly using different step size parameter in different iterative process. This is the variable step-size algorithm for LMS adaptive filtering. Commonly the basic ideas for the variable step-size algorithm of LMS are as follows: In the initial stage of convergence, step size should be bigger, this makes the algorithm has faster convergence speed. Then with the deepening of convergent gradually reduce the step size to reduce the static error. In the process of research for the variable step-size algorithm of LMS, also proposed to make $\mu(n)$ is proportional to $e(n)$,and proposed to make $\mu(n)$ is proportional to the evaluation of the cross-correlation function for e (n) and x(n), and so on.

Practice shows that these algorithms can give attention to faster convergence rate and smaller maladjustment to a certain extent. Can effectively remove the irrelevant noise interference, and have fewer parameters and smaller amount of calculation of the algorithm itself. And easy to use hardware implementation has been widely used in the adaptive digital filter system.

3.3 Improvement for LMS

After introduces the variable step-size adaptive filtering algorithm what is said above. In this paper, a new variable step-size algorithm is proposed, as shown in the following type:

$$W(n+1)=W(n)+2\mu(n)e(n)X(n) \tag{3}$$

$$\mu(n)=\beta(1-\exp(-\alpha e(n)^2)) \tag{4}$$

Among them, the shape of function is controlled by the parameter $\alpha(>0)$,and the value range of function is controlled by the $\beta(>0)$. The algorithm is simple and has the characteristics of the slowly changing after parameter stability.

However, in the low SNR (Signal to Noise Ratio) environment, the convergence speed and tracking speed and steady-state error of the algorithm are not very perfect. Therefore consider appropriate to improve the algorithm. Not directly use $e^2(n)$ of signal error square to regulate the step size, but by extend a certain period of time for e(n),so that decreases to zero for the correlation of the noise signal. Use the correlation value e (n) e (n - D) to regulate the step size(D is positive integer and the time parameter).Due to the correlation of noise signal fell to zero, so the step factor influence for the noise signal is reduced greatly, and lessen sensibility for noise signal in the variable step-size algorithm of LMS. The improved algorithm formula for the variation step size namely:

$$\mu(n)=(1-\exp(-\alpha e(n)e(n-D))) \tag{5}$$

The algorithm use the correlation value e (n) e (n - D) to regulate the step size ,give consideration to the convergence speed and error performance. And reduce sensitivity of LMS algorithm for the weak- autocorrelation of noise. In this paper, the algorithm was applied to the simulation of several typical signals processing system. From theory and practice prove that the algorithm is optimized obviously. The selection of parameters of α and β affect the performance of the LMS algorithm is briefly described as follows.

By μ (n) and e (n) function relation curve as shown in Figure 2. Initial convergence phase |e (n)| is bigger, corresponding μ (n) is bigger also, and the algorithm convergence speed faster. When the algorithm into the steady state, |e (n)| is minimum, and also μ (n) is minimum. Because of μ (n) should be satisfied: $0 < \mu(n) < 1/\lambda_{max}$, and thus $\beta < 1/\lambda_{max}$. On this condition, the algorithm must be convergence. But not any α、β meet $\beta < 1/\lambda_{max}$, could make the μ(n) is larger in the initial stage of convergence , and after the algorithm convergence μ(n) is smaller.

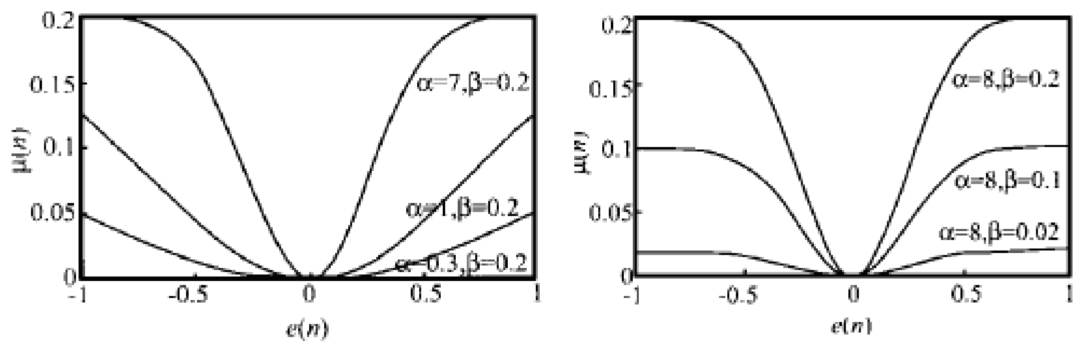

Figure 2. μ (n) and e (n) Relation Curve

The Figure 2 shows: about the same initial error, fixed the $\beta(< 1/\lambda_{max})$ value, the algorithm is convergent. When choosing a larger α value, the convergence speed of the algorithm is faster. By the same token, fixed the α value, when choosing a larger β value, the convergence speed of the algorithm is also faster. But if the value of α、β are too larger, while improving the rate of convergence of the algorithm, but can cause larger steady-state error.

Therefore, the selection of α、 β should follow the following principles: according to the initial error value | e (n)| to choose the α、 β. Make the μ(n) value of initial error | e (n)| as biger as possible. In the practical application, through the experiment to determine the optimal value of α、 β.

4. LMS Algorithm in the Application of Signal Processing

Due to the LMS algorithm has simple structure, small computational complexity, stable performance, and therefore is widely used in adaptive equalization, speech processing, adaptive noise cancellation, system identification and signal processing, *etc.*

4.1 In the Application of Adaptive System Identification

For an actual physical system, people mainly concerned with the input and output characteristics, namely the signal transmission characteristics, and are not required to fully understand its internal structure. System can be one or more input; also can have one or more output. Communication system identification is a very important issue in communication system. The so-called system identification is essentially based on the system input and output signals, to estimate or determine the characteristics of the system and the unit impulse response or transfer function of the system.

System identification and modeling is a very broad concept，there are important significance in the field of control, communication and signal processing. In fact, system identification and modeling not only confined to the traditional engineering field, and can be used to study social systems, economic systems and biological systems. This section only discusses the system identification and modeling problem in the field of communication and signal processing. The filter as communication channel model, and use the adaptive system identification way to identify the communication channel, thus further to the equalization for the communication channel.

If the communication channel as a "black box", and only know "black box" of the input and output. An adaptive filter was used as the “black box” model, and makes the filter with the same input and output as the "black box". Adaptive filter by adjusting its parameters, to match the output of the filter with the output of the "black box". In this way, the filter simulates communication channel for the whole transmission course of signals. Although the structure and parameters of the adaptive filter are different from real communication channel, but they keep highly consistent in the input and output response. Therefore, in this sense, the adaptive filter is the model of an unknown "black box" system. And you can also found that if adaptive filter has the enough degrees of freedom (adjustable parameters), then, the adaptive filter can be arbitrary degree to simulates the "black box".

The following form adaptive filter model of the FIR. Assume that the unknown channel for finite impulse response (FIR) structure, as shown in Figure 3.

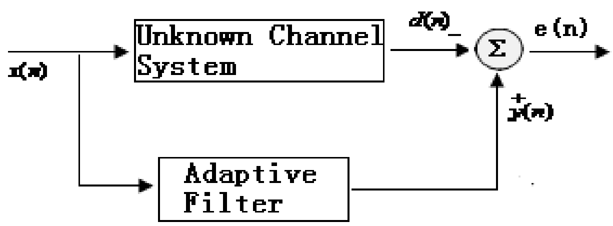

Figure 3.The Principle Diagram of the Adaptive System Identification

In the diagram, using pseudo-random series as the input signal x (n), at the same time was taken to the unknown channel system and adaptive filter. By adjusting the adaptive filter coefficient, minimize the mean square error (MSE) of the error signal e (n). Then the output y (n) of the adaptive filter approximately equal the output d (n) of communication system. We can think so, the two systems have the same similar input and output, should have similar features. As a result, the characteristics of the adaptive filter or the unit impulse response can be used to approximately substitute the unknown system features or the unit pulse response.

Figure 4 illustrates the adaptive FIR filter is able to simulate the unknown system. Original signal after dealing with the two systems, the output effect is very close. In this way, by reference to the parameters of the adaptive FIR filter, get the unknown system function. To reconstructs the same hardware function for the unknown system, it has been widely used in engineering application.

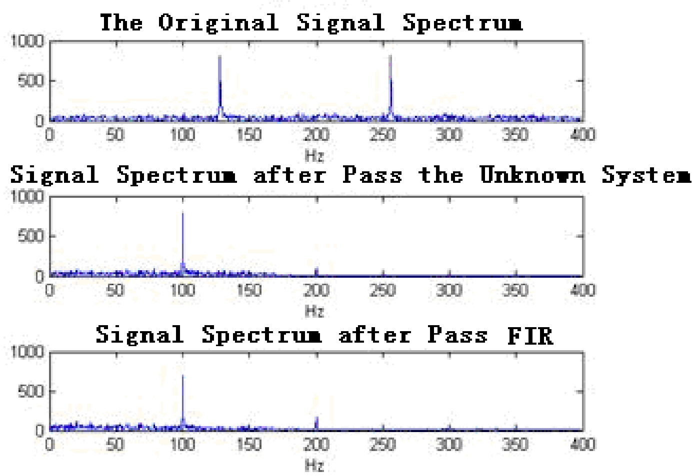

Figure 4. Signal Spectrum of Processing System

4.2 In the Application of the Adaptive Noise Cancellation (ANC) System

Applications in communications, and many other signal processing problems，is often accompanied by interference and noise in received signal, and influence on the reliability of the received signal, lead to higher bit error ratio. Adaptive signal processing is to detect or restore the original signal using the optimal filter in the interference and noise signal. The optimal filter can be fixed, also can be adaptive. Fixed filter depends on the prior knowledge of statistics from signal and noise, and the adaptive filter is not required or little prior knowledge about the noise statistics.

Adaptive Noise Cancellation (ANC) system is a form of adaptive optimal filter. The basic principle of ANC is to get signal cancellation from signal by noise pollution and reference signal; noise may be removed from original signal. The key problem is to ensure that the reference signal and noise have a certain correlation, but uncorrelated to the original signal. In general, minus the noise from receive signals seem to be very unreasonable, very likely lead to noise not only cannot be eliminated, and it will weaken the useful signal. However, adaptive noise cancellation system with adaptive control and adjustment of the system could effectively restore the original signal from the noise.

The basic principle of Adaptive Noise Cancellation (ANC) system is discussed as follows. Assuming that the original input by d (n) = s (n) +$n_{0 \text{ in}}$ ANC system, n_0 is the noise of the offset, and no correlation to the original signal s (n). Reference input with x (n), x (n) =n_1 has correlation to n_0, and no correlation to s (n). The output of the system for z (n) =y (n)-d (n), as shown in Figure 5.

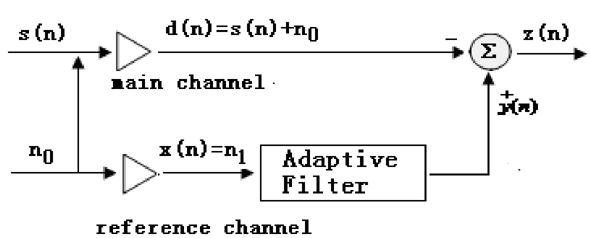

Figure 5. Principle Block Diagram for ANC

The following design a second-order weighted adaptive noise canceller according to the Figure 5. For sine signal with additive white Gaussian noise by filtering processing, a new variable step-size LMS adaptive filtering algorithm is used in the application (As shown in formula (3-5).

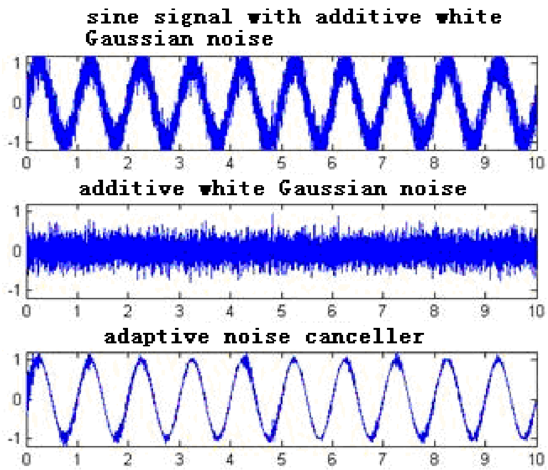

Figure 6. Simulation Results for Adaptive Noise Canceller

As shown in Figure 6, signal source generates a sine waveform, and sine signal superposition of white Gaussian noise into the main channel of adaptive noise canceller. The input of the adaptive filter is a single noise signal (see Figure 5). Through adaptive LMS algorithm to adjust the weight coefficient of the linear combiner, then give rise to signal cancellation for noise in main channel and reference channel. Eventually the output signal is the desired sine signal.

4.3 In the Application of Adaptive Signal Separator

In the adaptive noise cancellation system requires reference signal has correlation to noise. However, in some applications, it is difficult to find a good correlation with the noise of the reference signal. This will make the adaptive noise cancellation system is difficult to run. In fact, if the noise in the broadband signal is cyclical, even if no other reference signal being correlation with noise, also can use adaptive noise cancellation system to eliminate the periodic noise interference.

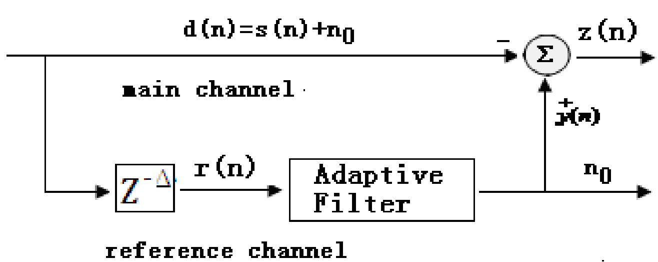

Figure 7. Principle Block Diagram for Adaptive Signal Separator

In the Figure 7, original input signals d (n) is the combination of the periodical noise signal n_0 and broadband signal s (n). Input signal d(n) into the main channel directly, at the same time via a Δ delay circuit into the reference channel. Delay Δ for long enough, make broadband signal of r(n) in the reference channel no or little correlation to broadband signal of d(n). And the periodical noise signal n_0 of d(n) and r(n) by its periodicity, its correlation is cyclical also. After delay Δ, its correlation remains the same. And then treated with adaptive noise cancellation system, the adaptive filter in the reference channel to adjust its weighting coefficients, make the output y(n) on the LMS algorithm is close to the correlation components--the periodical noise signal $n_{0,}$ and the output z(n) is close to the no correlation components—the broadband signal s(n).

The following is an example for adaptive signal separator. Selection of sine signal s = sin (2 * PI * t / 10) for the periodical noise signal $n_{0,}$ the broadband signal s(n) is white Gaussian noise, then to extract s(n) from d(n)=s(n)+ $n_{0.}$

The simulation results as shown in Figure 8. Sine signal superposition of white Gaussian noise into the main channel of adaptive signal separator, set the reference channel delay is 50. Through adaptive LMS algorithm to adjust the weight coefficient of the linear combiner, and separate the broadband signal of white Gaussian noise and the periodical noise signal of sine from input signal d(n).

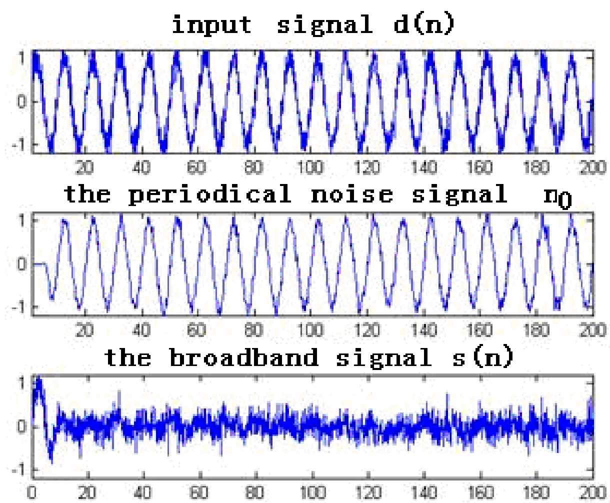

Figure 8. Separation of Broadband Signal with Periodic Noise Signal

5. Simulation Analysis and Conclusion

This paper analysis and discusses the advantages and disadvantages of several main LMS adaptive filtering algorithm. The variation step length LMS algorithm is improved appropriately, by the error correlation value e(n) e (n - D) to adjust step size factor, to overcome the noise sensitivity for weaker autocorrelation, obviously improve the performance of the original algorithm. And this algorithm is applied to the simulation and analysis in application of several typical signals processing, and get well application effect. In fact, the application of adaptive signal processing is far more than these. With the continuous improvement and perfection of the adaptive algorithm, and the rapid development of signal processing chip, it will be more widely used in control system, communication and signal processing, and other fields.

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