International Journal of Multimedia and Ubiquitous Engineering
Vol.11, No.2 (2016), pp.371-380
http://dx.doi.org/10.14257/ijmue.2016.11.2.36

Sparsity Based Denoising of PET-CT Images

Zhi Cui* and Xian-Pu Cui

School of Communication and Electronic Engineering, Hunan City University
zhicui@yeah.net

Abstract

In this paper, we propose an improved method for the removal of additive Gussian white noise from PET-CT images. Different from the traditional sparse representation based denoising methods, our method is composed of two distinctively steps such as the preliminary denoise and the detail compensation. By constructing a sparse representation model, denoising is formulated as an optimization problem that can be solved on an over-complete dictionary. The proposed method effectively trains this dictionary by using K-SVD algorithm with atom replace model. Then the preliminary denoised image is reconstructed through improved OMP algorithm with the fidelity factor of SSIM (Structural Similarity). The detail compensation image is obtained by using the difference between the noisy image and the preliminary de-noised image, and the improved OMP algorithm is employed again to get the denoised detail compensation image. Finally, the final denoised image is reconstructed by adding the denoised detail compensation image to the preliminary denoised image. Experiment results have shown that the proposed method is better than some other denoising methods in terms of PSNR and SSIM.

Keywords*: atom substitution; detail compensation; sparse representation; image denoising*

1. Introduction

With the development of image processing methods, medical diagnostic imaging plays an increasingly important part in the clinical diagnosis and treatment. PET-CT is the product of the integration of PET and CT. It organically combines the functional information of focus obtained by PET and the anatomical information of focus obtained by CT, so it possesses the advantages of both PET and CT, and becomes outstanding in the clinical diagnosis [1]. However, on account of the apparatus, the environment, and other factors, noise interference is inevitable in the process of PET-CT image reconstruction, resulting in the deterioration of the image quality and further the accuracy of clinical diagnosis and treatment. Therefore, effectively denoising PET-CT images has always been a hot spot in the study of medical diagnostic imaging [2-4].

In general, according to their characteristics, common denoising methods in medical image processing can be divided into two major categories. The denoising methods of the first kind are based on transformation domain, such as the classic wavelet-threshold-based denoising [5], Contourlet transformation [6], and Curvelet transformation [7]. The mutual presupposition of these methods is that noise in the image is mainly in the high-frequency region while useful information like the content is mainly in the low-frequency region. They distinguish between the content and the noise in the frequency domain by looking for the distribution regularity of the frequency spectrum in the image, in order to denoise the image. However, experimental results suggest that, in the PET-CT image obtained by the current technology, useful information that reflects the content of the image still exists in high-frequency region while there is also some noise in low-frequency region. In consequence, useful information in high-frequency region could be mistakenly erased and some noise in low-frequency region could still exist after denoising by transformation

ISSN: 1975-0080 IJMUE

domain methods. Additionally, since each transformation can only represent one single characteristic of the image efficiently, the effect of denoising by transformation domain methods needs further improvements, as for the PET-CT images which have various complicated characteristics [8-10]. The denoising methods of the second kind are based on image domain, such as harmonic analysis and Partial Differential Equation (PDE) [11]. Image domain denoising methods have great local adaptivity and design flexibility, but it does not have a good enough performance in retaining the structural characteristics of the image, such as the edge characteristic, limiting its development in medical image technologies.

In recent years, in image analysis and processing, denoising methods on the basis of sparse representation have attracted more and more attention. The theory of sparse representation is to use the linear combination of the columns with the least number to represent a given non-zero vector in a known full row rank matrix (where rows are more than columns) [12]. This known matrix is called dictionary, and its columns are called atoms. Unlike any transformation domain denoising methods and image domain denoising methods, sparse representation replaces orthogonal bases with redundant bases, and regards the useful information in the image as a specific structure closely connected with the atoms, and vice versa as for the noise. Therefore, the separation of signal and noise can be achieved by making sure whether the data have sparse representations on the dictionary or not.

The image denoising method based on sparse representation mainly includes two parts: dictionary design, and sparse decomposition of image on dictionary. Mallat and Zhang firstly proposed the idea of the decomposition of signals on an over-complete dictionary, and after that, researchers proposed various designs of the dictionary, such as the Gabor dictionary [13], and the multiscale ridgelet dictionary [14]. These designs have promoted the development of the sparse representation theory. However, since they adopted fixed atoms unrelated to the content of the image being processed, the calculation became very difficult and the denoising failed to achieve its best result [15]. In 2006, Elad and Aharon proposed the K-SVD algorithm, which upgrades the over-complete dictionary column by column through learning and training and makes the dictionary more adaptive to the image to be processed [16]. The sparse decomposition is another part of the sparse representation denoising; common sparse decomposition methods include matching pursuits algorithm (MP) [17], and orthogonal matching pursuits algorithm (OMP) [18].

Recently, various approaches based on sparse representation have been proposed to address the problem of image denoising. For example, Zhang and Xie proposed a denoising method based on DCT basis and sparse representation [18], with that method, the image's content could be effectively described from an over-complete dictionary which has obtained by learning dictionary from the noisy image itself, and the denoised image could be obtained by combine with the sparse representation coeffcients which have acquired from the pursuit algorithm. Zhang and Fu proposed a denoising method based on adaptively sparse representation in [19], with that method, the K-SVD algorithm has been used to learn an overcomplete dictionary based on image itself by choosing a reasonable threshold, and the denoised image could be effectively and efficiently restored within the application of sparse representation on learned overcomplete dictionary. In [20], Zhou and Luo proposed a novel method for constructed the over-complete dictionary namely K-LMS algorithm to realize the image representation, and the denoising could be achieved by combine the adaptive image sparse decomposition in overcomplete dictionary and the threshold process. These methods greatly extended the application range of sparse representation, but the dictionary based on the K-SVD algorithm training has structural defects, and the current sparse decomposition algorithms generally take the reconstruction error between the images before and after denoising as the fidelity term, and the fixed threshold as the end condition for iteration. On account of the working environment and the detecting object, the PET-CT image possesses a large number of structural characteristics related to human tissue, so if the algorithm above were still to be used, the threshold would be difficult to set, and the noise brought in during the reconstruction would have great influence on the accuracy of the reconstructed image. Therefore, effective methods to obtain the characteristics of the image and suppress noise are the keys to better accuracy of PET-CT imaging.

In this paper, we propose an improved sparse denoising algorithm based on the atom substitution model and the structural similarity. Firstly, the utilization efficiency of atoms is taken into consideration in training an over-complete dictionary by K-SVD. In other words, low-utilization-rate atoms will be substituted by suitable image blocks, in order to improve the adaptivity and the structure of the dictionary. Secondly, in the process of sparse decomposition, similarity factors will replace the reconstruction error as the new fidelity term so as to reserve the structural characteristics of images.

2. Sparse Representation

Sparse representation extends the traditional orthogonal basis to an over-complete dictionary, utilizing the linear combination of a small number of atoms in the dictionary to represent the image, limiting the image energy to a small number of non-zero coefficients. These non-zero coefficients and the corresponding atoms represent the major characteristics and the inner structure of the image.

2.1. Denoising Model

Research results have shown that, the quality of reconstructed image will be disturbed because of the voltage fluctuations, the electrostatic interference and the bad grounding in the process of PET-CT detection [22]. The interference signal can be thought of Gaussian white noise. Thus, a PET-CT image polluted by noise can be described as follows:

$$\mathbf{F} = \mathbf{U} + \mathbf{V} \tag{1}$$

Where $\mathbf{F}$, $\mathbf{U}$ and $\mathbf{V}$ represents observed image, original image and noise, respectively. The goal of denoising is to remove or reduce the influence of noise in the above model, so that the difference between $\mathbf{F}$ and $\mathbf{U}$ can reach the minimum.

According to the theory of sparse representation, a dictionary $\mathbf{D}$ is defined to establish a sparse representation denoising model for PET-CT image $\mathbf{F}$ with a size of $\sqrt{N} \times \sqrt{N}$. Generally, a PET-CT image as a whole contains a large number of detail features such as edges and mutation, while local small image patch appears simple and has a consistent structure [23]. For this reason, we first establish the denoising model for local image block $\mathbf{f}$ ($\mathbf{f} = \mathbf{u} + \mathbf{v}$) which is comprised by some pixel blocks from the holistically image $\mathbf{F}$ in sequence.

Let $x \in \mathbf{R}^n$ be a $\sqrt{n} \times \sqrt{n}$ ($n \ll N$) image patch and define a redundant dictionary as $\mathbf{D} \in \mathbf{R}^{n \times k}$ （$n < k$）, all the image patch can be represented as follows:

$$\mathbf{f} = \mathbf{D}\boldsymbol{\alpha} \tag{2}$$

Where $\boldsymbol{\alpha} \in \mathbf{R}^k$ is a matrix of the sparse coefficient, it can be obtained by solving the following optimization problem:

$$\hat{\boldsymbol{\alpha}} = \arg\min \|\boldsymbol{\alpha}\|_0 \quad s.t. \quad \|\mathbf{D}\boldsymbol{\alpha} - \mathbf{f}\|_2^2 \le \varepsilon \tag{3}$$

Where $\|\cdot\|_0$ is the l_0 norm, $\|\boldsymbol{\alpha}\|_0$ is the number of the non-zero values, and ε is an extremely small positive number which represents error tolerance. In this paper, we use OMP algorithm to solve equation (2).

Hypothesis that the image $\mathbf{f}$ is obtained by adding Gaussian white noise with standard deviation value σ into the image $\mathbf{u}$, the denoising result of $\mathbf{f}$ is the solution of the following model (4) according to the maximun a posteriori (MAP):

$$\hat{\boldsymbol{\alpha}} = \arg\min \|\boldsymbol{\alpha}\|_0 \quad s.t. \quad \|\boldsymbol{D}\boldsymbol{\alpha} - \mathbf{f}\|_2^2 \le T \tag{4}$$

Where T is the hard threshold whose value is to be determined by ε and σ, and σ is the variations of noise. The denoised image y can be described as $\hat{\mathbf{u}} = \mathbf{D}\hat{\boldsymbol{\alpha}}$.

According to the regularization optimization and transform the constraint to the penalty term, the equation (3) can be changed into:

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_2^2 + r\|\boldsymbol{\alpha}\|_0 \tag{5}$$

Where r is the regularization parameter.

If all the image patch x are in conformity with the provisions of the formula (4), the final denoised model can be described as follows:

$$\{\hat{\boldsymbol{\alpha}}_{ij}, \hat{\mathbf{U}}\} = \arg\min_{\boldsymbol{\alpha}_{ij},\mathbf{U}} \lambda\|\mathbf{U} - \mathbf{F}\|_2^2 + \sum_{i,j} r_{ij}\|\boldsymbol{\alpha}_{ij}\|_0 + \sum_{i,j}\|\mathbf{D}\boldsymbol{\alpha}_{ij} - \mathbf{R}_{ij}\mathbf{U}\|_2^2 \tag{6}$$

Where λ is the Lagrange multiplier, r_{ij} is the regularization parameter, $\boldsymbol{\alpha}_{ij}$ is the sparse coefficient, and $\mathbf{R}_{ij}$ is a $n \times N$ matrix.

From equation (6), it can be found that the dictionary $\mathbf{D}$ is fixed. Assume that $\mathbf{D}$ is not fixed, then equation (6) can be translated into the following model:

$$\{\hat{\boldsymbol{\alpha}}_{ij}, \hat{\mathbf{D}}, \hat{\mathbf{U}}\} = \arg\min_{\mathbf{D},\boldsymbol{\alpha}_{ij},\mathbf{U}} \lambda\|\mathbf{U} - \mathbf{F}\|_2^2 + \sum_{i,j} r_{ij}\|\boldsymbol{\alpha}_{ij}\|_0 + \sum_{i,j}\|\mathbf{D}\boldsymbol{\alpha}_{ij} - \mathbf{R}_{ij}\mathbf{U}\|_2^2 \tag{7}$$

2.2. OMP Algorithm

The principle of the OMP algorithm is to use the least of suitable bases to represent functions, and to orthogonalize the selected vectors. In each step of the signal decomposition, components neither of the selected vectors nor others will be introduced into the residual signal.

Assume that $\mathbf{D}$ is an over-complete dictionary, $\boldsymbol{\alpha}_\Gamma$ is the sparse vector, L is the sparseness value, $\mathbf{D}_\Gamma \subset \mathbf{D}$, we can describe the OMP algorithm as follows:

$$\min_{\boldsymbol{\alpha}\in\mathbf{R}^N} \|\mathbf{F} - \mathbf{D}\boldsymbol{\alpha}\|_2^2 \quad (\|\boldsymbol{\alpha}\|_0 \le L) \tag{8}$$

(1) Initialization, satisfying $\Gamma = \varnothing$.

(2) Iteration

1) Choose the atoms $\mathbf{d}$ and $\boldsymbol{\alpha}$ which make the objective function obtain the minimum value from the complementary set Γ_C of Γ:

$\hat{\mathbf{d}} \leftarrow \arg\min_{\mathbf{d}\in\Gamma_C}\left\{\min_{\boldsymbol{\alpha}}\left\|\mathbf{F} - \mathbf{D}_{\Gamma\cup\{\mathbf{d}\}}\boldsymbol{\alpha}\right\|_2^2\right\}$;

2) Update Γ, add the chosen atom $\mathbf{d}$ in it: $\Gamma \leftarrow \Gamma \cup \{\hat{\mathbf{d}}\}$;

3) Update the residual $\boldsymbol{\chi}$: $\boldsymbol{\chi} \leftarrow \left(\mathbf{I} - \mathbf{D}_\Gamma(\mathbf{D}_\Gamma^T\mathbf{D}_\Gamma)^{-1}\mathbf{D}_\Gamma^T\right)\mathbf{F}$;

4) Update $\boldsymbol{\alpha}_\Gamma$: $\boldsymbol{\alpha}_\Gamma \leftarrow (\mathbf{D}_\Gamma^T\mathbf{D}_\Gamma)^{-1}\mathbf{D}_\Gamma^T\mathbf{F}$.

(3) Repeat step (2) J times, over.

3. The Proposed Method

In this section, we introduce the proposed method. One core of our method is the atom replace model which can improve the dictionary training method. In addition, we take SSIM as a new fidelity term to replace the reconstruction error.

3.1. Structural Similarity

Traditional image quality evaluate is dependent on some valuable characteristics such as pixel gray value. Two of the most commonly used are peak signal to noise ratio (PSNR) and mean square error (MSE). However, these two assessments have not considered the relevant of pixel and the perception characteristics of human visual system, which cause difference with subjective feeling. In 2009, Wang et. al proposed a novel image quality assessment method namely histogram structure similarity (SSIM), which takes concentration as the main structure information instead of structure of objects in image [24]. Experiments in [24] show that SSIM is more suitable to outliers and noise compared with other image quality assessment methods, because the distortion extent of image can be calculated by combining histogram concentration, luminance and contrast.

From the fundamental work in [24], we can describe SSIM as follows:

$$SSIM(a,b) = \frac{2\mu_a\mu_b + c_1}{\mu_a^2 + \mu_b^2 + c_1} \cdot \frac{2\sigma_a\sigma_b + c_2}{\sigma_a^2 + \sigma_b^2 + c_2} \tag{9}$$

Where μ_a, μ_b are the means of the noised image and the ideal image, σ_a, σ_b are the variances, c_1, c_2 are the minimal positive constants related to the values of the pixel. SSIM measures the image quality from brightness, contrast and structure, which is more in line with the characteristics of human visual system. It has the value between 0 and 1, the closer it is to 1, the more similar in structure between the noised and the de-noised images.

3.2. Atom substitution Model

K-SVD algorithm is an effectual dictionary training method. However, when noisy image is used as sample, the trained dictionary itself would inevitably contain redundant atoms [25]. If the redundant atoms are used to reconstruct PET-CT image, it would cause quality degradation.

Consider two situations:

(1) If there exists an atom $\mathbf{d}_k$ whose similarity to the atom $\mathbf{d}_l$ is over 0.99 in Matrix $\mathbf{D} = \mathbf{d}_l\mathbf{d}_k^T$ (where $\mathbf{d}_l$ and $\mathbf{d}_k$ refer to any unidimensional column vectors in the dictionary $\hat{\mathbf{D}} \in \mathbf{R}^{N \times k}$), that is, there exist redundant atoms in the dictionary:

$$\text{if} \quad \mathbf{D}(i,j) > 0.99, \quad \mathbf{d}_l = mormlize(\mathbf{R}_{ij}\mathbf{U}) \tag{10}$$

(2) If the non-zero number of the sparse coefficient $\boldsymbol{\alpha}_l$ corresponding to the atom $\mathbf{d}_l$, $\|\boldsymbol{\alpha}_l\|_0 < T$ (where T refers to the lower limit of the number of sparse coefficients), the atom $\mathbf{d}_l$ is considered to be utilized too little, denoted by:

$$\text{if} \quad \|\boldsymbol{\alpha}_l\|_0 < T, \quad \mathbf{d}_l = normlize(\mathbf{R}_{ij}\mathbf{U}) \tag{11}$$

Under the two circumstances, atoms need to be substituted. Research shows that the L^2 norm of the residual vector in the signal decomposition can represent the degree of approximation of the dictionary $\mathbf{D}$ and the sparse coefficient $\boldsymbol{\alpha}$ to the image. Therefore, we propose an atom substitution model in which the image block $\mathbf{R}_{ij}\hat{\mathbf{U}}$ of the lowest degree of approximation is utilized to substitute the atom $\mathbf{d}_l$ as follows:

$$\mathbf{d}_l = \arg\max_{i,j} \left\| \mathbf{R}_{ij}\hat{\mathbf{U}} - \hat{\mathbf{D}}\boldsymbol{\alpha}_{ij} \right\| \tag{12}$$

3.3. The Proposed Method

Based on the above analysis, we propose an improved PET-CT denoising model as follows:

$$\{\hat{\boldsymbol{\alpha}}_{ij}, \hat{\mathbf{D}}, \hat{\mathbf{U}}\} = \arg\min_{\mathbf{D},\boldsymbol{\alpha}_{ij},\mathbf{U}} \lambda \left\|\mathbf{U} - \mathbf{F}\right\|_2^2 + \sum_{i,j} r_{ij} \left\|\boldsymbol{\alpha}_{ij}\right\|_0 + \sum_{i,j} \left\| 1 - SSIM\,(\mathbf{D}\boldsymbol{\alpha}_{ij} - \mathbf{R}_{ij}\mathbf{U}) \right\|_2^2 \tag{13}$$

Where the first and second items on the right side both are bound terms, and the third one is the similarity factor which replaces the reconstruction error as the new computational fidelity term.

To calculate equation (13), we first define $\mathbf{D}$ is a known over-complete dictionary, satisfying $\mathbf{F} = \mathbf{U}$. Then the optimal solution of each image block can be solved as follows:

$$\hat{\boldsymbol{\alpha}}_{ij} = \arg\min_{\boldsymbol{\alpha}_{ij}} r_{ij} \left\|\boldsymbol{\alpha}_{ij}\right\|_0 + (1 - SSIM\,(\mathbf{D}\boldsymbol{\alpha}_{ij} - \mathbf{R}_{ij}\mathbf{U})) \tag{14}$$

Finally, the denoised result can be represented as follows:

$$\hat{\mathbf{U}} = (\lambda \mathbf{I} + \sum_{i,j} \mathbf{R}_{ij}^T \mathbf{R}_{ij})^{-1} (\lambda \mathbf{F} + \sum_{i,j} \mathbf{R}_{ij}^T \mathbf{D}\hat{\boldsymbol{\alpha}}_{ij}) \tag{15}$$

Where $\mathbf{I}$ is the unit matrix.

In conclusion, the principle of our method can be summarized as follows:

(1) Initialization: We set the over-complete DCT dictionary as the initial dictionary $\mathbf{D}$, satisfying $\mathbf{F} = \mathbf{U}$;

(2) Sparse coding: We incorporated SSIM into OMP algorithm and encoding each image block as described in equation (9);

(3) Dictionary training: We define the error matrix as $\mathbf{E}_k = \mathbf{U} - \sum_{j \neq k} \mathbf{d}_j \boldsymbol{\alpha}_j^T$, and set the atom used for sparse decomposition in the dictionary as $\boldsymbol{\omega}_i = \{i \,|\, 1 \le i \le k\}\ (\boldsymbol{\alpha}_j^T(i) \neq 0)$, then switch the dictionary updating as follows:

$$\min_{\mathbf{d}_j, \boldsymbol{\alpha}_j} (1 - SSIM\,(\mathbf{U}, \sum_{j \neq k} \mathbf{d}_j \boldsymbol{\alpha}_j^T)) \quad s.t. \quad \boldsymbol{\alpha}_j^T \subseteq \boldsymbol{\omega}_i \tag{16}$$

We can solve equation (16) as follows by using SVD and fist-order approximation:

$$\mathbf{E}_k = \mathbf{Q}\boldsymbol{\Delta}\mathbf{V}^T \tag{17}$$

Repeat step (3) J times, then update the dictionary.

(4) Atom substitution: We replace the redundant atoms as described in equation (12), and obtain the sparsity dictionary.

(5) Output the denoised result: We first calculate $\hat{\mathbf{U}}$ on the basis of equation (15), and obtain the preliminary denoised image. Then we compensate the preliminary denoised image by using the difference between the preliminary denoised image and the original image, and output the final denoised image.

4. Experiments

In this section, several experiment results of the proposed method are reported to show the denoising performance and compared with other two methods, including K-SVD in [16] and the method in [20]. These methods are applied to several test images, all of them are 256×256 gray scale images with 8 bits per pixel. In our experiments, PSNR values and visual appearance are both adopted as the objective indices to assess the quality of denoised images, and the chosen parameters of the proposed method are set as follows: sparseness number L is set to 6, patch size n is set to 64 (8×8), threshold T used in

OMP algorithm is calculated by $(1.02\sigma + 0.6)\sqrt{n}$, and iteration number J in the K-SVD dictionary training algorithm is 20.

In the first experiment, we perform tests on three PET-CT images and compared the SSIM results obtained by using the detail compensation or not (namely UNCPS). For the sake of simplicity, we add the white noises with the variations of σ = 10, 20 and 40 in the images. The SSIM results are given in Table 1.

As one can see in Table 1, the proposed method outperforms the UNCPS on all SSIM results, values raise at 0.74%, 0.78% and 0.81%. The more the details in the images increases, the higher the value of SSIM would be in the proposed method. It suggests that after the preliminary denoising step, some details in the images would be decreased toghther with the noise, thereafter would lead to the lack of useful information. We obtain the final denoised images by adding the compensation images to the preliminary denoised image, which can help effectively keep structure characteristics, meanwhile, the useful information in PET-CT images can be better retained.

Table 1. Comparison of SSIM between UNCPS and the Proposed Method

Image	Noise σ	UNCPS	proposed
	10	0.916	**0.925**
Lung-1	20	0.893	**0.901**
	40	0.867	**0.874**
	10	0.909	**0.917**
Lung-2	20	0.886	**0.891**
	40	0.859	**0.865**
	10	0.901	**0.907**
Lung-3	20	0.877	**0.884**
	40	0.841	**0.847**

Figure 1 shows the SSIM result obtained by the proposed method and the UNCPS with different σ. As it is shown that, although the growth rate of SSIM caused by the proposed method decreases when the σ is greater than 50, it is still better than the value obtained by the UNCPS. The result indicates the proposed method is effective and feasible.

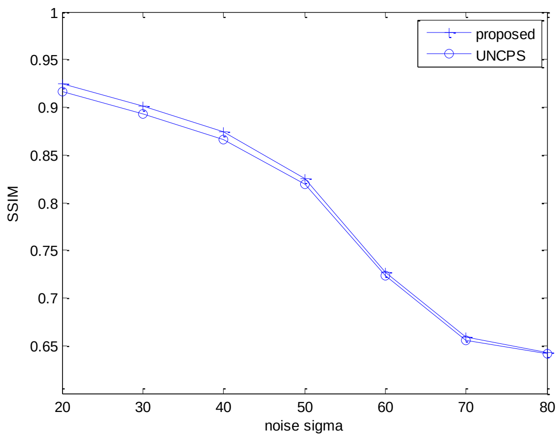

Figure 1. Comparison of SSIM between UNCPS and the Proposed Method

In the second experiment, we compared the SSIM and PSNR results obtained by the proposed method，K-SVD [16] and the method in [20] on five PET-CT images. Table 2 shows the comparison results. As is shown that the tests were performed on the same three images with σ = 10, 20 and 40. It can be found that the results obtained by the proposed method are better than the others for the same image with the same σ. For instance, by adding the white noise with the variance of 20 to the "Image-3", the PSNR result obtained by the proposed method is 24.70 dB, with increased value by 0.49dB and 0.08dB respectively compared with the values in K-SVD and the method in [20], the SSIM result obtained by the proposed method is 0.884, with the increased ratio by 5.23% and 2.07% respectively compared with the values in K-SVD and the method in [20]. It also can be found that the results obtained by the proposed method are better than the others for different images with different σ. For instance, the PSNR result obtained by the proposed method is increased by 0.35 dB and 0.11 dB on average respectively than in K-SVD and the method in [20], and the SSIM result obtained by the proposed method is increased by 5.27%and 2.14% on average respectively than in K-SVD and the method in [20].

Table 2. Performance of the De-Noising Methods by SSIM and PSNR

Image	Noise σ	SSIM			PSNR		
		K-SVD	Method [20]	Proposed	K-SVD	Method [20]	Proposed
Image-1	10	0.878	0.905	**0.925**	28.74	28.85	**28.91**
	20	0.856	0.882	**0.901**	28.13	28.36	**28.47**
	40	0.832	0.855	**0.874**	26.69	26.86	**26.95**
Image-2	10	0.869	0.893	**0.917**	27.48	27.69	**27.83**
	20	0.848	0.873	**0.891**	26.60	26.82	**26.97**
	40	0.822	0.847	**0.865**	24.26	24.58	**24.65**
Image-3	10	0.862	0.889	**0.907**	25.54	25.77	**25.89**
	20	0.840	0.866	**0.884**	24.21	24.62	**24.70**
	40	0.806	0.832	**0.847**	22.48	22.76	**22.94**

We also compared the denoised effects got in the proposed method and the other two for all the five images on a strong noise case with σ = 70. The denoising performance is illustrated in Figure 2. From the aspect of subjective visual effect, we can see that K-SVD produces overly smoothed denoised results where the noise has been suppressed but also edges and other features of the image have been blurred. It has the worst visual quality. Method [20] generates clearer edges and textures than K-SVD. However, it also introduces many disturbing artifacts in both edges and smoothing regions. Our method obtain the best visual quality, where the edges can be better preserved while removing noise without introducing much artifacts.

5. Conclusion

In this paper, we have proposed an adaptive image denoising method for PET-CT images. We established a sparse representation model adapted to solve generalized image restoration problem. We applied this estimator to remove the Guassian white noise in image, and made some improvement by utilizing SSIM and atom replace model. Our experimental results demonstrated that the proposed method can effectively remove noise while keeping sharp edges and clear textures. In addition, our method can achieve a competitive performance in both subjective visual quality and objective PSNR and SSIM value compared with other two denoising algorithms. In feature work, we will consider dictionary training methods.

Acknowledgment

This work is supported by Scientific Research Found of Hunan Provincial Education Department under Grant of 13C122.

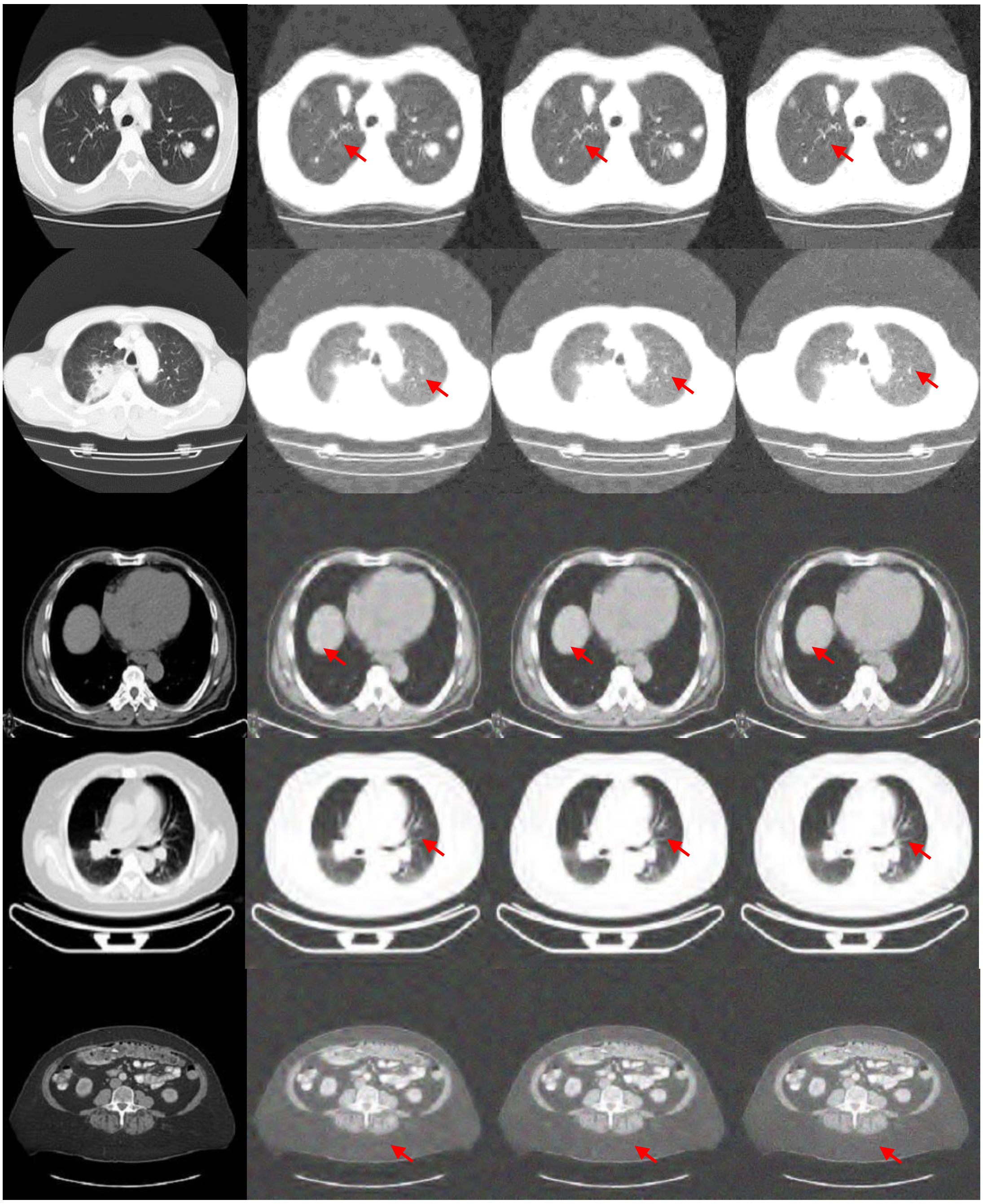

Figure 2. Visual Comparison of the Reconstructed Results on Three PET-CT Images, With Σ=70

The first column: noise-free images. The second column: reconstructed results obtained by K-SVD. The third column: reconstructed results obtained by method [20]. The fourth column: reconstructed results obtained by the proposed method.

References

[1] M. M. Rahman, M. G. Arefin and M. P. K. Kumar, "Computer Engineering and Intelligent Systems", vol. 3, no. 3, **(2012)**.
[2] Z. D. He, M. Abdoli and C. L. Fuentes, "European Journal of Nuclear Medicine and Molecular Imaging", vol. 39, no. 5, **(2012)**.
[3] M. Yin, W. Liu, J. Shui and J. M. Wu, "Mathematical Problems in Engineering", vol. 2012, **(2012)**.
[4] F. X. Yan, S. L. Peng and L. Z. Cheng, "Electronics Letters", vol. 43, no. 18, **(2007)**.

[5] D. L. Donoho, “IEEE Transactions on Information Theory”, vol. 41, no. 3, **(1995)**.
[6] L. A. Cunha, J. P. Zhou and M. N. Do, “IEEE Transactions on Image Processing”, vol. 15, no. 10, **(2005)**.
[7] J. L. Starck, E. J. Candes and D. L. Donoho, “IEEE Transactions on Image Processing”, vol. 11, no. 6, **(2002)**.
[8] S. Rein and M. Reisslein, “Ad Hoc Networks”, vol. 9, no. 4, **(2011)**.
[9] X. Y. Wu, B. L. Guo and L. D. Li, “Journal of Electronics & Info Technology”, vol. 32, no. 2, **(2010)**.
[10] F. Luisier and T. Blu, “IEEE Transactions on Image Processing”, vol. 17, no. 4, **(2008)**.
[11] F. Dai, J. R. Xue and N. N. Zheng, “Journal of Electronics & Info Technology”, vol. 30, no. 3, **(2008)**.
[12] N. N. Yu, T. S. Qiu and F. Y. Ren, “Journal of Mathematical Imaging and Vision”, vol. 45, no. 1, **(2013)**.
[13] K. Dabov, A. Foi and V. Katkovnik, “IEEE Transactions on Image Processing”, vol. 16, no. 8, **(2007)**.
[14] P. Chen and D. Suter, “International Journal of Computer Vision”, vol. 68, no. 1, **(2006)**.
[15] J. Lee, Y. H, Kim and J. H. Nam, “IEEE Transactions on Consumer Electronics”, vol. 54, no. 3, **(2008)**.
[16] M. Aharon, M. Elad and A.M. Bruckstein, “IEEE Transactions on Signal Processing”, vol. 54, no. 11, **(2006)**.
[17] S. G. Mallat and Z. Zhang, “IEEE Transactions on Signal Processing”, vol. 41, no. 12, **(1993)**.
[18] Y. C. Pati, R. Rezallfar, P. S. Kaishnarasad, “IEEE Transactions on Signal Proc”, vol. 41, no. 12, **(1993)**.
[19] F. Zhang and K. Xie, “Editors. A Novel Image Denoising Method Based on DCT Basis and Sparse Representation”, Proceedings of Cross Strait Quad-Regional Radio Science and Wireless Technology Conference, Harbin, China, July 26-30, **(2011)**.
[20] Q. Zhang, Y. Fu, L. C. Li and J. Y. Yang, “Editors. A Millimeter-wave Image Denoising Method Based on Adaptive Sparse Representation”, Proceedings of International Conference on Computational Problem-Solving, Nanjing, China, April 18-21, **(2011)**.
[21] Z. Zhou and L. M. Luo, “Journal of Convergence and Information Technology”, vol. 7, no. 16, **(2012)**.
[22] M. F. Steinkel and E. Bezak, “Australasian Physical and Engineering Sciences in Medicine”, vol. 31, no. 3, **(2008)**.
[23] S. T. Li, H. T. Yin,L.Y. Fang, “Transactions on Geoscience and Remotesensing”, vol. 51, no. 9, **(2013)**.
[24] Z. Wang and A. C. Bovik, “IEEE Signal Processing Magazine”, vol. 26, no. 1, **(2009)**.
[25] L. Y. Fang, S. T. Li, Q. Nie, J. A. Izatt, C. A. Toth and S. Farsiu, “Biomedical Optics Express”, vol. 3, no. 5, **(2012)**.

