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Blocking Variable Step Size Forward-Backward Pursuit Algorithm for Image Reconstruction

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Abstract

Compressed sensing is a new signal sampling theory that fully makes use of signal's sparsity or compressibility. The theory shows that, the acquisition of a small amount of the sparse or compressible signal value can be used for exact signal reconstruction. Based on the study and summarization of the existing reconstruction algorithms, this paper proposes a novel blocking variable step size forward-backward pursuit (BVSSFBP). This paper proposed variable step size forward-backward pursuit algorithm by introducing the concept of sparse phase and variable step size to deal with different situations. The algorithm also divides two-dimensional image into blocks, in order to reduce the scale of observation matrix during single processing, reduce the single processing speed and the overall running time. Experimental results show BVSSFBP algorithm can obtain better reconstructed image quality.

Keywords*: compressed sensing; sparse representation; reconstruction algorithm; forward-backward pursuit*

1. Introduction

Signal reconstruction algorithm based on compressed sensing theory is a vital link in the process of perception. The key problem is how to recover the original signal of high dimensional data from the low dimensional ones [1]. Up to now, researchers in this field abroad did rigorous mathematical proof and experiments; there have been many published algorithms for compressed sensing reconstruction. At present, there are many mature signal reconstruction algorithms, including matching pursuit algorithm, direction tracking series algorithm, convex optimization algorithm and statistical optimization algorithm. The matching pursuit algorithm is the most widely used series; the series is the typical greedy iterative algorithm. Representatives are matching pursuit (MP) algorithm, subspace pursuit algorithm (SP), Forward-Backward Matching Pursuit, (FBMP) algorithm, Sparse Adaptive Matching Pursuit(SAMP), *etc.* [2-4].

MP algorithm has more iterative times during the process of operation and the time cost is large. And this kind of algorithm needs to know the sparsity K of signals. In practical application, the sparseness K is often unknown, thus the reconstruction quality is not very accurate [5-7]. In practice, sparse degree of many natural images is unknown, then the application in most reconstruction algorithm can only make a rough estimation of the sparsity. And signal recovery will not be well guaranteed, the performance of the algorithm is greatly reduced. Therefore, in order to further improve the performance of MP algorithm and not to increase the reconfiguration time, this paper presents a novel Block Variable step size Forward-Backward Pursuit, algorithm (BVSSFBP). The algorithm in the premise of the

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signal sparsity is unknown, to control the accuracy of reconstruction through the variable step size, in order to realize the reconstruction of signals.

2. Compressed Sensing and Reconstruction Algorithm

Set *x(n)* is N-dimensional digital signal obtained by traditional sampling, *y(m)* is the observation signals, and it's length is M. If it is a K-sparse (only have K non-zero elements, K≤N) or compressible signal, so it can be well estimated only with a small amount of coefficient under linear transformations [8]. And through the compression perception process, M-dimensional signals can be obtained directly, called the measurement vector. The relationship between them is

$$y = \Phi x \tag{1}$$

In the formula Φ is called sensing matrix or measurement matrix with size of $M \times N$. When $M \geq K \cdot \lg(N)$, $x(n)$ can be accurately reconstructed. Reconstruction algorithm for accuracy verification of the compressed sensing sampling process and later reconstruction of *x(n)* both has important significance. By the following optimization problem to solve, the unknown signal *x* is reconstructed from the measurement vector y.

$$\min \|x\|_0 \quad \text{s.t.} \quad y = \Phi x \tag{2}$$

In practice, allowing some errors exist, therefore the original optimization problem can be transformed into a simpler form of approximate solution, where ε is a tiny constant:

$$\min \|x\|_0 , \quad \text{s.t.} \quad \|y - \Phi x\|_2^2 \leq \varepsilon \tag{3}$$

3. VSFBP Algorithm

VSFBP algorithm improved FBP algorithm by introducing the idea of variable step size of SAMP algorithm [9-10]. The whole operation process is divided into two stages. Firstly, use large step size iteration in the first stage, the approximate solution can be quickly close to the optimal solution. Then use small step iteration in the second stage, the approximate solution can be slowly close to the optimal solution. Finally the better reconstruction performance is obtained [11-12].

Implementation steps of VSFBP algorithm are as follows:

The input parameters: the sensing matrix (atomic library) $\Phi \in R^{M\times N}$, measuring signal $y \in R^M$. Output parameters: the reconstructed signal $\hat{x} \in R^N$ with N dimension of original signal value X. The definition of parameters: forward iteration step length $\alpha = \alpha_1$, backward iteration step $\beta = \beta_1$, the maximum number of iterations is K_{max}, threshold parameter ω is used to change iteration step size and parameter $\varepsilon(\omega > \varepsilon)$ to stop iteration.

Initialization: signal margin $r^{(0)} = y$, index set $\Gamma^{(0)} = \emptyset$, atomic support set $\Phi_{\Gamma^{(0)}} = \emptyset$, iteration times k=0;

1. k=k+1, calculate inner product of the signal margin $r^{(k-1)}$ and all each column vector of the sensing matrix Φ , $g^k = \Phi^T r^{(k-1)}$;

2. Sort the components of $g^{(k)}$ in according to the absolute value, choose deposited prior to the index level T_f index corresponding to the maximum of the α component, put the forward index set T_f into the candidate index set $\tilde{\Gamma}^{(k)}$, $\tilde{\Gamma}^{(k)} = \Gamma^{(k-1)} \cup T_f$, and the corresponding atom set Φ_{T_f} is added to the candidate set $\Phi_{\tilde{\Gamma}^{(k)}}$, $\Phi_{\tilde{\Gamma}^{(k)}} = \Phi_{\Gamma^{(k-1)}} \cup \Phi_{T_f}$;

3. Use the least square method to measuring signal y in candidate atom set $\Phi_{\tilde{\Gamma}^{(k)}}$ spanned by the orthogonal projection space, get the K- raw signal sparse approximation of signal X , $\tilde{X}^{(k)} = \arg\min_{x} \|y - \Phi_{\tilde{\Gamma}^{(k)}} X\|_2$;

4. Sort the approximation signal $\tilde{X}^{(k)}$ in the component according to the absolute value, select the corresponding to the minimum β component index deposit to the index set T_b, delete candidate index set $\Gamma^{(k)} = \tilde{\Gamma}^{(k)} \setminus T_b$ from the backward index set T_b and the corresponding atom set Φ_{T_b} from the candidate set of atoms $\Phi_{\tilde{\Gamma}^{(k)}}$, thus $\Phi_{\Gamma^{(k)}} = \Phi_{\tilde{\Gamma}^{(k)}} \setminus \Phi_{T_b}$.

5. Use the least square method to measuring signal y in candidate atom set $\Phi_{\tilde{\Gamma}^{(k)}}$ spanned by the orthogonal projection space, get the K- raw signal sparse approximation of signal X, $\tilde{X}^{(k)} = \arg\min_{x} \|y - \Phi_{\tilde{\Gamma}^{(k)}} X\|_2$ and update the residual $r^{(k)} = y - \Phi_{\Gamma^{(k)}} X^{(k)}$。

6. If $\|r^{(k)}\|_2 \le \omega \|y\|_2$, then $\alpha = \alpha_2$， $\beta = \beta_2$ and go to 7th step. Otherwise go to 8th step.

7. If $\|r^{(k)}\|_2 \le \varepsilon \|y\|_2$, stop iteration and output approximate solution $\hat{x}$ of the original signal of X, where $\hat{x} = x^{(k)}$, otherwise go to the 8th step.

8. If the number of elements in the index set $\Gamma^{(k)}$ whether meets $|\Gamma^{(k)}| \ge K_{max}$, stop iterative and output approximate solution $\hat{x}$ from the original signal X. Otherwise go to the 1st step to proceed to the next iteration.

In order to verify the performance of the VSFBP algorithm, this paper selects the two-dimensional image Barbara (256×256) for reconstruction at different compression ratio. Nonzero component signal in the process of reconstruction of the location is random, and each nonzero component value is consistent with the random value of Gauss distribution. The measurement matrix is Gauss random matrix. Figure tab1 shows the reconstructed images, where (a) is the original image, (b)-(f) are respectively reconstructed images at M/N = 0.5,0.4,0.3,0.2,0.1.

(a) Original Image (b) M/N=0.5 (c) M/N=0.4

(d) M/N=0.3 (e) M/N=0.2 (f) M/N=0.1

Figure 1. Reconstruction Results by VsFBP at different Sampling Rate

Table 1 gives the reconstruction performance of the VSFBP algorithm for the two-dimensional image signal, according to the different compression ratio. Peak signal to noise ratio (PSNR) of reconstructed image and the running time are given.

Table 1. The Reconstructed Results of VSFBP Algorithm

Sampling rate(M/N)	PSNR(dB)	Time(s)
0.5	37.87	46.61
0.4	26.61	37.93
0.3	15.35	24.26
0.2	8.56	12.42
0.1	3.63	4.17

4. BVSSFBP Algorithm

In the previous work, blocking in compressed sensing can be very good to solve the problem of slow speed of image reconstruction [13]. Here, we still adopt the method of blocking to reconstruct the signal, combining the variable step size in the previous section pursuit algorithm, called as BVSSFBP algorithm.

The algorithm first divides the image into many independent and small signal observations and image reconstruction operation is done for each sub image. Thus the computational complexity is reduced, and at the same time complexity sensing matrix becomes very small. The sampled data are convenient for storage, and the observation data of each sub image have been alone could send go out. The signal receiver can also independently reconstruct image blocks through these data, this ensures the real-time data processing. In order to improve the reconstruction effect, without significant reconstruction time, according to the adjacent phase of reconstruction signal energy difference before and after the tracking algorithm combined with setting an appropriate threshold, using "large step fast approaching, small step gradually approaching" envisaged reconstruction signal to measure the sparse approximate solutions and the optimal approximation of the degree of inter organization dilute. When the signal margin is greater than the threshold, which is the optimal solution of sparse dissociation is far, using large step fast approaching; and when the signal margin is less than this threshold, *i.e.*, sparse dissociation of optimal solution is close, with small step length gradually approximation.

The steps for detailed description of the process of BVSSFBP algorithm are as follows:

1. Image is blocked into size of B × B, the ith block image vector is denoted as x_i, $i = 1, \cdots, n, n = N/B^2$. Use Fourier transform for each block to proceed sparse transform and get the sparse matrix;

2. Design measurement matrixΦ_B. In this paper random Gauss matrix is used to get the observed value y_i;

3. VSSFBP is used to perform the iteration for the vector form of each sub image block and get reconstruction image $\hat{x}_i$ for each sub image block;

4. Finally all the sub image block reconstruction results $\hat{x}_i$ are put together to get the whole image.

5. Experiments and Analysis

This experiment adopts Barbara size of 256 × 256 used as test image, using BVSSFBP to reconstruct. Three experiments are done to test block size, sampling rate and equalization operation’s effect on reconstructed image.

5.1 The Selection of Block Size

Firstly, experiments are done to determine the appropriate image block number of this algorithm, namely the size of B. In Table II, at sampling rate of M/N=0.5, discuss that different B value has influence to the signal reconstruction on Barbara image.

It can be seen that, PSNR gets larger with the increase of B. The size of block becomes larger, the loss of information between blocks gets smaller, and thus the quality of reconstructed image gets better. When B is larger than 16, block effect is obvious and running time is longer. When B is less than 16, reconstructed result is not exact. Thus, BVSSFBP chooses 16 as the size of block to get better reconstructed image.

Table 2. The Reconstructed Results with Different Block Size

Size of B	PSNR(dB)	Time(s)
4×4	14.58	18.68
8×8	17.11	23.82
16×16	22.18	30.98
32×32	36.95	48.32
64×64	38.83	69.23

5.2 The Selection of Sampling Rate

The selection of sampling rate also has effect on BVSSFBP's reconstruction results. If the sampling rate is too low, it will cause PSNR of the reconstructed image is too low, and not up to the requirements of image reconstruction effect; High sampling rate can improve reconstructed image's PSNR, but the reconstruction time required will be increased. For different sampling rate (M/N=0.1, 0.2, 0.3, 0.4, 0.5) reconstruction image analysis is done, as shown in Table III.

Table 3. The Reconstructed Resuits at Different Sampling Rate

Sampling rate(M/N)	PSNR(dB)	Time(s)
0.1	17.75	21.78
0.2	20.08	23.45
0.3	20.25	26.43
0.4	21.38	29.56
0.5	22.18	31.98

From Table III, it can be seen the effect of the selection of sampling rate on the reconstructed image's PSNR and running time. As the sampling rate becomes larger, PSNR becomes larger, and when the sampling frequency is less than 0.2, the lifting range of PSNR is not large, but the reconstruction time required is greatly increased. Therefore, when the sampling rate is 0.2, the reconstructed image not only has good quality but also reconstruction time is relatively short.

5.3 Equalization Operation

Equalization operation is applied based on BVSSFBP, called as EBVSSFBP algorithm. Firstly, row vector of the matrix representation for the image is dealt according to the block sparse adaptive matching pursuit algorithm; signal reconstruction results X_1 can be obtained. Then apply the same method to the transpose of matrix, and get the signal reconstruction results X_2, the reconstruction. Finally, compute the mean of the two results to get the final result. From analysis and comparison of the two signal processing results, it can be known that the first processing destroyed the correlation between columns of matrix, and the second processing destroyed the correlation between rows of matrix. Equalization operation applied to the two reconstruction results can take into account the relationship between the rows and columns of the matrix.

Table IV shows EBVSSFBP algorithm can obtain better PSNR compared to BVSSFBP algorithm (B=4×4). However, this equalization scheme has a mandatory requirement is to be used only when the sub image block is small. The image matrix to represent the original signal is processed for two times, thus increases the processing time. Therefore, in the nature of the signal processing, choose appropriate signal reconstructed method according to the different signal processing requirements, such as processing time or reconstruction accuracy standards.

Table 4. The Reconstructed Resuits with Different Reconstructed Methods (Psnr:Db)

Sampling rate	BVSSFBP	EBVSSFBP
0.1	18.75	18.82
0.2	22.98	23.05
0.3	23.25	23.41
0.4	25.37	25.59
0.5	26.65	26.92

6. Conclusion

This paper mainly studied the signal reconstruction algorithm based on compressed sensing. Based on VSFBP, this paper proposed BVSSFBP algorithm. BVSSFBP divided image into blocks, thus observation matrix became small reducing the complexity of signal calculated and reconfiguration time signal. Meanwhile, BVSSFBP can obtain reconstructed image signal at the premise that the signal sparsity K is unknown and meet the real-time requirements.

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