

## Thermal Field Balance and Efficiency Analysis of Plate Type Electromagnetic Induction Heating

Ping Xue<sup>1</sup>, Hao Sun<sup>2</sup>, Bing Li<sup>3</sup> and Hongmin Wang<sup>4</sup>

*Harbin University of Science and Technology, School of automation, Harbin,  
Heilongjiang Province, China*  
Corresponding Author: Ping Xue  
E-mail: [xueping@hrbust.edu.cn](mailto:xueping@hrbust.edu.cn)

### Abstract

*Owing to flat plate induction heating can't guarantee to heat equally, especially the existence of no heated zone, it haven't been used in these industrial fields like SMD which require high accuracy and easy to control while heating. But some places need its characteristic of contact heating, eco-friendly and high efficiency, for this reason, it is fundamental to research the heating style further as well as the no heating zone. Because induction heating is based on eddy current, firstly, the relationship between eddy current and induction heating is explained and several factors which would influence the eddy current are put forward. Then finite element analysis and calculation of eddy current field is mainly done. At last, the conclusion is obtained by analyzing the results of calculation. Due to the analysis above, after improving the structure of the iron plate, the problem of no heating zone is solved. At the same time, the efficiency evaluation is brought out. It can make the following research more easily.*

**Keywords:** *flat plate induction heating, eddy current, heating balance, finite element analysis, efficiency analysis*

### 1. Introduction

With the development of science and technology, modern industry needs the parts with high reliability and high performance, but heat treatment will bring a great influence to the performance of materials. Because electromagnetic induction heating, a new technology of heat treatment, has advantage at control and efficiency, it has been widely used in many fields such as process and treatment of metal, etc.

Flat plate induction heating also meets some problems. While heating the parts, different conditions need special attention separately. When producing SMD surface mount components, reflow soldering always used has disadvantages at thermal efficiency and utilization of energy. Because of green, energy conservation and easy to control, electromagnetic induction heating becomes the first choice. We need plate type electromagnetic induction heating to heat the welded plate directly. So the coil of this kind is flat. Compared with the former, the later can cause no heated zone in the steel plate. It associates with the distribution of magnetic field induced by the flat coil. So the distribution of eddy current needs to be researched in order to find a solution to no heated zone.

## 2. Finite Element Analysis of 3D Eddy Current Field

### 2.1. The Definition of the Solution Domain

Figure 1 is a typical solving area  $V$  of eddy current problem.  $V_1$  is the vortex region with conductive medium but without the source current.  $V_2$  is a non eddy zone which contains the given source current. The internal interface of  $V_1$  and  $V_2$  is  $S_{12}$ . The outer interface of  $V$  is  $S$ .  $S_H$  and  $S_B$  are the two parts of it. In  $S_H$ , the tangential component of magnetic field intensity is given. In  $S_B$ , the normal component of magnetic induction intensity is given. Assume that the boundary conditions of  $S$  are homogeneous boundary conditions. Then under the condition of knowing the spatial distribution of the source current, The size of geometry and the electromagnetic parameter of the space, eddy current areas ( $J_e$ ) and magnetic field ( $H$  or  $B$ ) along the time and space will be solved [1].

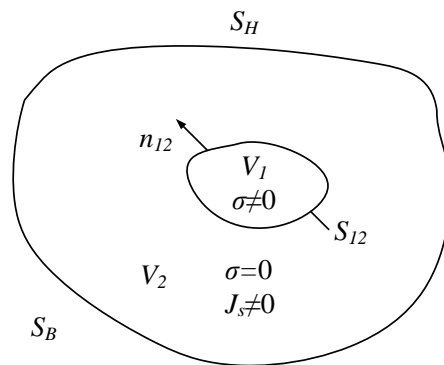


Figure 1. Typical Solution of Eddy Current Problem

### 2.2. The Fixed Solution Problem Presented By Vector Field

Based on Maxwell equations, by using  $E, B, H$  belonged to the vector field, the boundary conditions and the control equations of the eddy current field are expressed in the head region of  $V$ .

$$\begin{aligned} &\text{In } V_1 \\ &\nabla \times \mathbf{H} = \sigma \mathbf{E} \end{aligned} \quad (1)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\begin{aligned} &\text{In } V_2 \\ &\nabla \times \mathbf{H} = \mathbf{J}_s \end{aligned} \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (5)$$

$$\begin{aligned} &\text{In } S_B \\ &\mathbf{B} \cdot \mathbf{n} = 0 \end{aligned} \quad (6)$$

$$\begin{aligned} &\text{In } S_H \\ &\mathbf{H} \times \mathbf{n} = \mathbf{0} \end{aligned} \quad (7)$$

$$\begin{aligned} &\text{In } S_{12} \\ &\mathbf{B}_1 \cdot \mathbf{n}_{12} = \mathbf{B}_2 \cdot \mathbf{n}_{12} \end{aligned} \quad (8)$$

$$\mathbf{H}_1 \times \mathbf{n}_{12} = \mathbf{H}_2 \times \mathbf{n}_{12} \quad (9)$$

In the equations,  $\mathbf{n}_{12}$  represents the unit normal vector of  $S_{12}$  and its direction is from  $V_1$  to  $V_2$ ;  $\mathbf{J}_s$  represents the source current density;  $\mathbf{n}$  represents the unit normal vector of  $S$ .

Also if  $V$  is a single connected region, the unique solution of  $\mathbf{E}$  and  $\mathbf{B}$  can be obtained from (1)~(9) and certain initial conditions.

### 2.3. Vector Potential of the Magnetic and Scalar Potential of the Electric, Mathematical Model Deduced By $\mathbf{A}$ 、 $\phi$ - $\mathbf{A}$ Method

Because the conclusion is deduced by (1)~(9) inconveniently,  $\mathbf{A}$  and  $\phi$  must be used to replace its vector field. That is to say, mathematical model used for analyzing eddy current field must be established by  $\mathbf{A}$ 、 $\phi$ - $\mathbf{A}$  method. This is a method which uses non eddy zone and eddy zone to describe 3D eddy current field [2-3]. The magnetic vector potential  $\mathbf{A}$  is the only unknown function of non eddy current region, the magnetic vector potential  $\mathbf{A}$  and scalar potential  $\phi$  are the unknown functions of eddy current region.

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (10)$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \quad (11)$$

The following formulas can be derived by (1)~(11).

In  $V_1$

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = -\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla \phi \quad (12)$$

In  $V_2$

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}_s \quad (13)$$

In  $S_B$

$$\mathbf{n} \cdot \nabla \times \mathbf{A} = 0 \quad (14)$$

In  $S_H$

$$\frac{1}{\mu} \nabla \times \mathbf{A} \times \mathbf{n} = \mathbf{0} \quad (15)$$

In  $S_{12}$

$$\mathbf{n}_{12} \cdot \nabla \times \mathbf{A}_1 = \mathbf{n}_{12} \cdot \nabla \times \mathbf{A}_2 \quad (16)$$

$$\left( \frac{1}{\mu_1} \nabla \times \mathbf{A}_1 \right) \times \mathbf{n}_{12} = \left( \frac{1}{\mu_2} \nabla \times \mathbf{A}_2 \right) \times \mathbf{n}_{12} \quad (17)$$

Because the magnetic vector potential  $\mathbf{A}$  itself can confirm that  $\mathbf{B}$  don't have the divergence,  $\nabla \cdot \mathbf{B} = 0$  needn't appear obviously here.

The equation above can get the curl of  $\mathbf{A}$  but the divergence of  $\mathbf{A}$  can't be derived. So it has not made the vector potential  $\mathbf{A}$  have unique solution. If we want get the unique solution of  $\mathbf{A}$ , the boundary condition and divergence of  $\mathbf{A}$  must be given out. This requires the introduction of Kulun specification to assure the uniqueness of the  $\mathbf{A}$  [4].

When Kulun specification is adopted

$$\nabla \cdot \mathbf{A} = 0 \quad (18)$$

If the following homogeneous boundary condition is given in the boundary  $S$

In  $S_H$

$$\mathbf{n} \cdot \mathbf{A} = 0 \quad (20)$$

In  $S_B$

$$\mathbf{n} \times \mathbf{A} = \mathbf{0} \quad (19)$$

Through giving out normal and tangential boundary conditions of  $\mathbf{A}$  in boundary  $S$  and determining the curl and divergence of  $\mathbf{A}$ , the uniqueness of  $\mathbf{A}$  can be made sure.

According to (11),  $\nabla \phi$  can be obtained after getting the unique solution  $\mathbf{A}$ . But they are different from each other by an arbitrary constant. If we want to get a unique solution of  $\phi$ , we must find a value of  $\phi$  on a point at the boundary  $S$  or region  $V$  as a reference.

Above all, after merging into Kulun specification, under the given initial conditions, the complete expression of unique solution of eddy current field are as follows:

In  $V_1$

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) - \nabla \left( \frac{1}{\mu} \nabla \cdot \mathbf{A} \right) + \sigma \frac{\partial \mathbf{A}}{\partial t} + \sigma \nabla \phi = \mathbf{0} \quad (21)$$

$$\nabla \cdot \left( -\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla \phi \right) = 0 \quad (22)$$

In  $V_2$

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) - \nabla \left( \frac{1}{\mu} \nabla \cdot \mathbf{A} \right) = \mathbf{J}_s \quad (23)$$

In  $S_B$

$$\mathbf{n} \times \mathbf{A} = \mathbf{0} \quad (24)$$

$$\frac{1}{\mu} \nabla \cdot \mathbf{A} = 0 \quad (25)$$

In  $S_H$

$$\mathbf{n} \cdot \mathbf{A} = 0 \quad (26)$$

$$\left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) \times \mathbf{n} = \mathbf{0} \quad (27)$$

In  $S_{12}$

$$\mathbf{A}_1 = \mathbf{A}_2 \quad (28)$$

$$\frac{1}{\mu_1} \nabla \cdot \mathbf{A}_1 = \frac{1}{\mu_2} \nabla \cdot \mathbf{A}_2 \quad (29)$$

$$\left( \frac{1}{\mu_1} \nabla \times \mathbf{A}_1 \right) \times \mathbf{n}_{12} = \left( \frac{1}{\mu_2} \nabla \times \mathbf{A}_2 \right) \times \mathbf{n}_{12} \quad (30)$$

$$\mathbf{n} \cdot \left( -\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \nabla \phi \right) = 0 \quad (31)$$

### 3. Assumption of the Solution to No Heated Zone and Computer Aided Calculation of Eddy Current Field

#### 3.1. Assumption of the Solution to No Heated Zone

Because there is a strong causal relationship among the vortex electric field, feeling electromotive force and eddy current, the distribution of eddy current can be forecasted and it is similar to the eddy electric field. As a result, the no heated zone must appear and the zone

is the center of the eddy current. Based on the characteristic, we can assume like this: Magnetic field, eddy electric field influence the distribution of eddy current generated by the coil. And they cause the eddy current. But they can't decide the eddy current how to distribute. The distribution must be affected strongly by the shape of the conductor. The distribution depends on the shape of the conductor [5]. Briefly, the magnetic field cause the eddy current, but they are in their own conductor separately. So if changing the center of eddy current and shrinking the separate area of eddy current zone by increasing the quantity of the conductor, the no heated zone may disappear immediately. Then the no heated region will die away.

### 3.2. Computer Aided Calculation

First, the condition of one steel plate and one cake coil will be calculated. Because the whole model is symmetrical, we only research a quarter of it. Three elements--SOLID236 are needed to simulated air, steel plate and coil. Every element should set degree of freedom. In this model, air should set AZ, steel plate should set AZ、VOLT and coil should set AZ、VOLT、EMF. Figure 2 is the sectional view of the physical model [6-7].

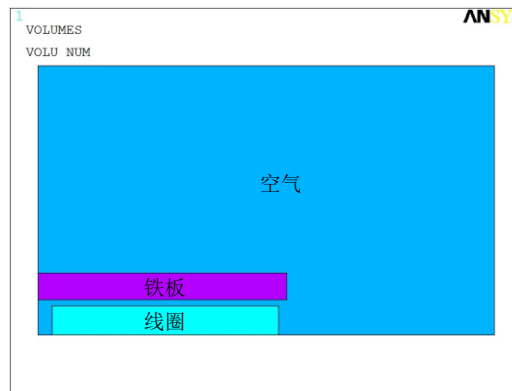


Figure 2. Physical Model of the First Condition

After given material properties and meshed, real constant of coil will be defined and the DOF of current will be coupled. Then the boundary condition of flux parallel is set at the surface of air and the section of steel plate. At last, the solver should be chosen and the solver is harmonic [8-9]. After solving, the distribution of eddy current in the steel plate can be observed as Figure 3.

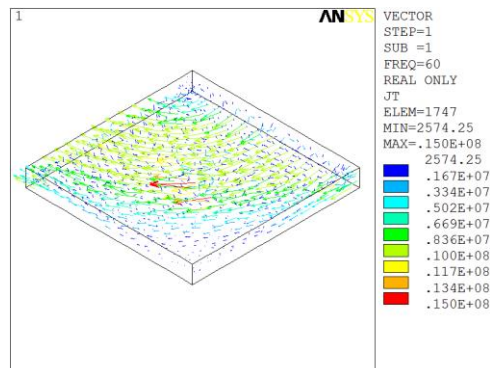


Figure 3. Distribution of Eddy Current of the First Condition

The second situation is that two steel plates are plated symmetrically. Also because of the symmetry, the quarter of it is researched. Physical model is as Figure 4.

The parameters are set as the first condition. After solving, the distribution of eddy current in the steel plate can be observed as Figure 5.

The last situation is four steel plates placed symmetrically. A quarter of the model is as Figure 6.

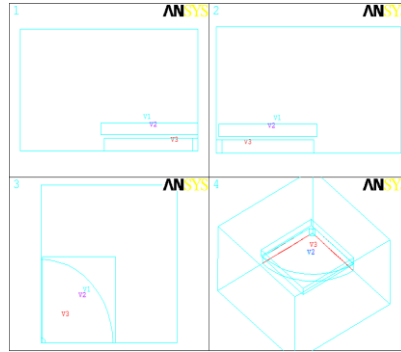


Figure 4. Physical Model of the Second Situation

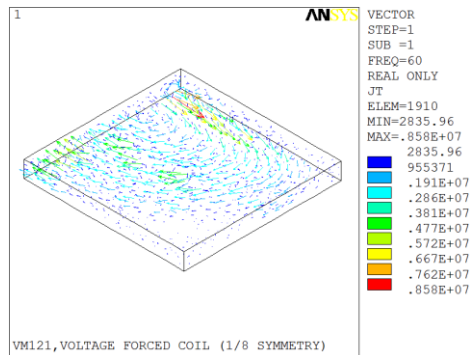


Figure 5. Distribution of Eddy Current of the Second Condition

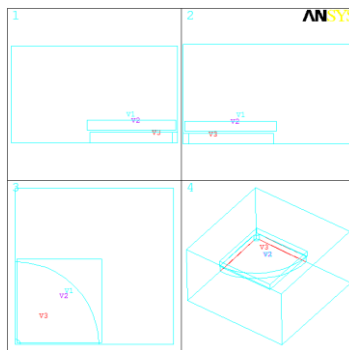
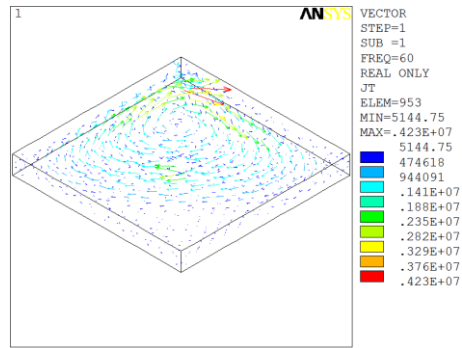


Figure 6. Physical Model of the Third Situation

After the same setting, the result is shown in the Figure 9.



**Figure 7. Distribution of Eddy Current of the Third Condition**

The simulation above tests and verifies the former assumption. If the structure of steel plates is designed like this, the problem of no heated zone can be solved.

### 3.3. Experiments on the Distribution of Eddy Current

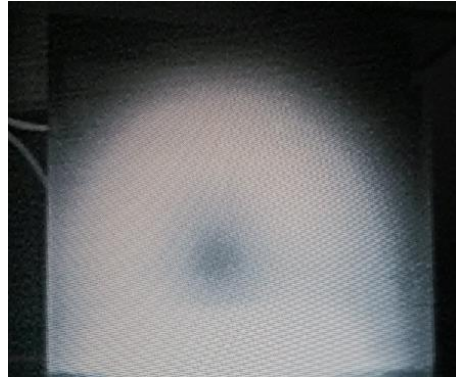
The first condition is that the steel plate is placed above the cake coil. The whole plate is heated all the time. The thermal imaging is as Figure 8.



**Figure 8. Thermal Imaging of One Whole Plate**

The thermal imaging accords with the result of simulation. So the no heated zone exists all along.

The second condition is that two steel plates placed symmetrically above the cake coil. After electrifying ten minutes, the thermal imaging is as Figure 9.



**Figure 9. Thermal Imaging of Two Iron Plates after Heating Ten Minutes**

Conclusion can be gotten from the imaging: The no heated zone will exist in the plate separately.

At last, the third condition in which four steel plates are placed symmetrically is researched. Because of symmetry, only half of it is taken as the object of study.

After 10 sec, the imaging is as Figure 10.



**Figure 10. Thermal Imaging at 10 Sec After Heating**

From the imaging, such conclusion can be derived: Because the heating process is dynamic, on the moment of electrifying, a very fuzzy no heated zone will appear. But it will disappear quickly. So throughout all the heating process, it looks like these four steel plates are heated evenly. Therefore the no heated zone will completely disappear.

#### **4. Analysis of Magnetic Thermal Coupling Field and Evaluation Method of Heating Efficiency**

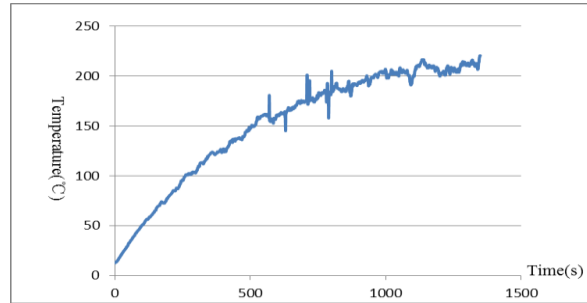
The most basic application requirement of plate type electromagnetic induction heating in industry is heating equilibrium. But the solution will reduce eddy current, as well as the heating efficiency must be reduced [10]. So analyzing the magnetic thermal coupling field and giving evaluation method of heating efficiency are very important.

In this part, the magnetic field and thermal field will be coupled, power consumption can be get throughout the analysis and SMD welding curve. This method can be used to evaluate the heating equilibrium and efficiency [11].



#### 4.1. The Heating Process of Plate Type Induction Heating

After arranging the data gathering from the whole process of electromagnetic heating, the heating curve is shown as Figure 11.



**Figure 11. The Heating Process of Electromagnetic Heating**

From this graph, the whole heating process is that the temperatures changes from rising sharply to rise gently, and then tends to be stable. This statement is related to many factors, such as environmental temperature, heat capacity, heat conduction, power supply *etc.* So the relationship of power supply and stable temperature is the main objective to be researched.

#### 4.2. The Solving Process of the Coupled Field

The coil can generate electromagnetic fields around the iron plate which induced the heat in the plate and temperature of the plate can rise high. Because the change of temperature gradient is very large, iron plate material characteristics changing with temperature must be considered.

The concrete solving process is as follows:

1. Develop attribute relationship.

Develop an attribute relationship for the modeled regions as shown in Table 1: Physics Environment Attributes.

**Table 1. Physics Environment Attributes**

Region	Type	Mat	Real
Billet	1	2	1
Coil	2	3	1
Air	2	1	1
Billet surface	3	2	3

2. Build the Model

Build the model of the entire domain. Assign the attributes to the different regions. (The billet surface will be used to define a surface effect element for thermal radiation. It will be handled differently than the solid regions.)

3. Create Electromagnetic Physics Environment

Create the electromagnetic physics environment by defining element types and material properties as shown below.

**Table 2. Electromagnetic Physics Environment**

Region	Type	Mat	Real
Billet	PLANE13	MURX(T), RSVX(T)	None
Coil	PLANE13	MURX	None
Air	PLANE13	MURX	None
Billet surface	NULL Type (0)	None	None

- (1)Assign appropriate nominal boundary conditions and loads
- (2)Assign appropriate load step and solution options.
- (3)Write the electromagnetic physics environment to a file.

4. Create Thermal Physics Environment

Create the thermal physics environment as follows:

- (1>Delete nominal boundary conditions and reset options.
- (2)Change the element types from electromagnetic to thermal as well as KEYOPT options.

Specify the null element type in the air and coil region (assume the heat transfer analysis only considers the billet).

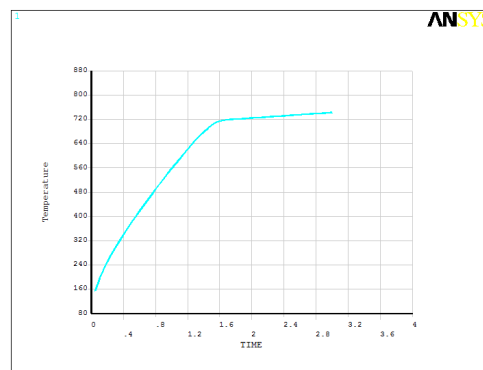
**Table 3. Thermal Physics Environment**

Region	Type	Mat	Real
Billet	PLANE55	KXX(T), ENTH(T)	None
Coil	NULL Type (0)	None	None
Air	NULL Type (0)	None	None
Billet surface	SURF151	EMIS	Stefan-Boltzmann Constant

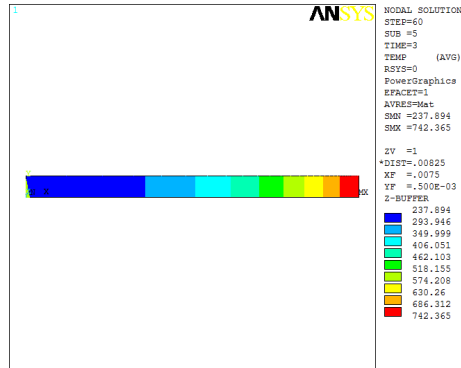
- (3)Define the thermal properties and real constants.
- (4)Assign appropriate nominal boundary conditions and loads.
- (5)Assign appropriate load step options and solution options.
- (6) Write the thermal physics environment to a file.

5. Coupling the magnetic field and the thermal field

In accordance with the above method, the physical model of one whole iron plate was established. After analysis, the time-temperature curve and temperature distribution is shown as below.



**Figure 12. Time-Temperature Curve of Electromagnetic Heating**



**Figure 13. Temperature Distribution of Electromagnetic Induction Heating**

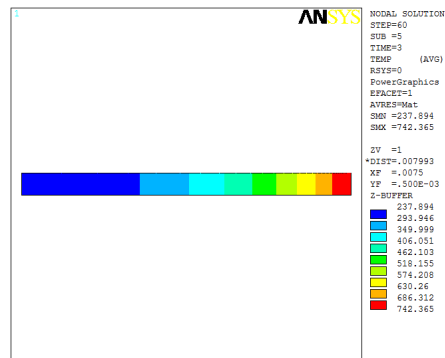
From the Figure 12 and Figure 13, this conclusion can be got: Firstly, the temperature of iron plate is rising gradually, and then tends to be stable. And the temperature decreases from the outside along the radial towards the center of the iron plate, forming a temperature gradient. The no heated zone is generated in the center of iron plate.

#### 4.3. Efficiency Analysis of Induction Heating

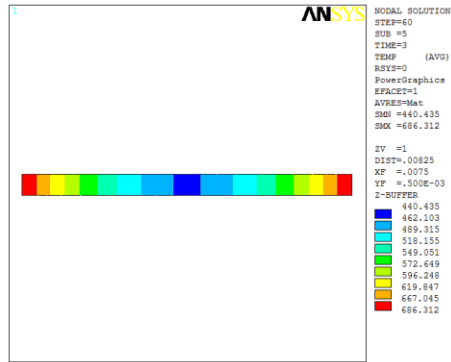
From the analysis above, the solution to heating equilibrium is brought out. But this method must lead to the decline of efficiency. So efficiency analysis becomes very important. The analysis can indicate the feasibility and practicability of this method.

Because the nature of electromagnetic heating is too complex, the research must be simplified. Firstly, the effect of heat conduction is weakened. Secondly, without considering the thermal inertia, temperature of the plate can quickly be stable for a given power.

In this research, the efficiency decline of one iron plate divided into four parts (condition 2) is the researching object compared with the one whole iron plate (condition 1). According to the simulation of ANSYS, the temperature distribution of the two condition while be stable is shown as follows.



**Figure 14. Temperature Distribution of Condition 1**



**Figure 15. Temperature Distribution of Condition 2**

From the temperature distributions, the heating equilibrium of condition 2 increases about 40% compared with condition 1.

Then research the efficiency. The stable temperature of the two conditions for a given power is shown as Table 4 and Table 5.

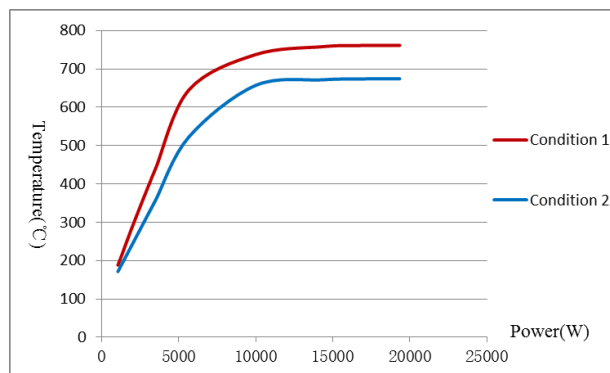
**Table 4. Table Of Given Power and Corresponding Thermometer of the Whole Plate**

Power/W	1050	1340	1630	2200	2780	3500
Temperature/°C	188.191	218.334	250.358	310.671	369.311	440.435
Power /W	5660	9990	14310	16470	19350	
Temperature/°C	645.224	737.445	757.942	760.889	761.44	

**Table 5. Table of given power and corresponding thermometer after dividing**

Power /W	1050	1340	1630	2200	2780	3500
Temperature/°C	171.329	193.427	215.525	258.959	303.155	358.019
Power /W	5660	9990	14310	16470	19350	
Temperature/°C	522. 611	656.937	671.523	673.685	674.258	

The power-temperature curve corresponding with chart data is plotted as Figure 16.



**Figure 16. Power-Temperature Curve of the Two Condition**

Using the trend forecast/regression analysis, power-temperature approximation expression of the two conditions are gotten.

The expression of condition 1:

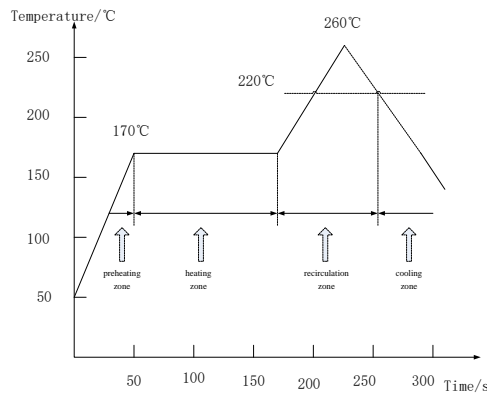
$$y = -2 \times 10^{-16} x^4 + 3 \times 10^{-10} x^3 - 10^{-5} x^2 + 0.1598 x + 20.172 \quad (46)$$

The expression of condition 2:

$$y = 10^{-10} x^3 - 7 \times 10^{-06} x^2 + 0.1179 x + 41.578 \quad (47)$$

Because different industries have their own requirement of heating equilibrium and efficiency, the main research of this passage is the application of SMD welding. Also, the method of evaluating the efficiency can be used in most of the industries.

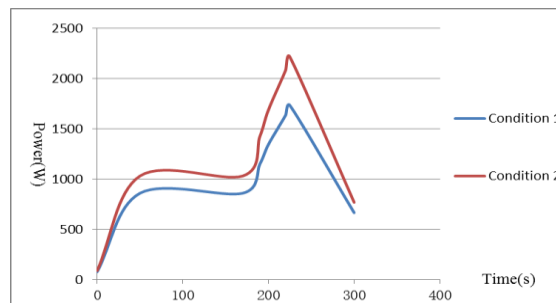
In SMD welding, the change of temperature follows a specific curve. In generally, the curve is divided into four zones: a preheating zone, a heating zone, a recirculation zone and a cooling zone. So the temperature of flat plate induction heating should follow this trend. The SMD welding curve is shown as follows.



**Figure 17. SMD Welding Temperature Diagram**

According to the power- temperature curve of the plate type electromagnetic induction heating and the SMD welding curve, the time-power curve of induction heating in SMD welding can be plotted. The curve corresponding to the two conditions is shown as follows.

From this chart, the energy consumption in the whole heating process is the area surrounded by the curve. So the energy consumption of condition 1 is about 288249.1J, the energy consumption of condition 2 is about 353150.6J. Compared with condition 1, the efficiency of condition 2 drops 22.52%.



**Figure 18. Power-Time Curve of the Two Condition**

Above all, the condition which uses the solution has increased the heating equilibrium by about 40% while the efficiency decreased by 22.52%, and the gain is greater than the loss. So this solution is feasible and it is a universal method in efficiency evaluation.

## 5. Conclusion

According to theory analysis, simulation and experiment, the distribution of eddy current is induced as follows: The magnetic field causes the eddy current, but it can't decide the distribution of eddy current. The shape of conductor which the eddy current is in can influence its distribution largely. The eddy current is in different conductors separately. Based on the conclusion, a solution to no heated zone comes out. Dividing the whole steel plate into several blocks, the eddy current is restricted in its own block. The no heated zone in the center of the plate disappears while the fuzzy no heated zone in every block dies away so quickly that it looks like they are heated equally. Then a method of efficiency evaluation is brought out and it proves the rationality and feasibility of the solution. Although the problem of no heated zone has solved, the temperature of heated zone how to distribute, heat conduction how to affect the temperature equilibrium, how to make the temperature distribute evenly and how to make the temperature increase and decrease equally also need to research. Study in this paper gives a reference to the later research of electromagnetic induction heating and the problem of no heated zone has already proved this heating measure can be used in high accuracy heating field. In order to promote this new heating measure to be used in these heating fields which are high accuracy and easy to control, the matters brought formerly should be researched further.

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