# Implementing Constructivist Principles in Early Mathematics Education: Exploring Cognitively Guided Instruction

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# Abstract

The aim of this paper is to review the literature on cognitively guided instruction (CGI) for early mathematics to determine how a CGI classroom embodies constructivist teaching-learning principles. A key philosophy of CGI is that teaching must be based on an understanding of each learner's development of mathematical thinking. CGI also emphasizes contextualization in problem solving, promoting learners' development of mathematical thinking by allowing them to share their thoughts with peers and teachers and solve problems in their own ways. These aspects of CGI correspond to constructivist teaching-learning principles. Therefore, we conclude that CGI may provide appropriate guidelines for developing constructivist teaching and learning practices for mathematics education among young children.

*Keywords:* Constructivism, Teaching and Learning, Mathematics Education for Young Children, CGI (cognitively guided instruction)

## **1. Introduction**

As it has become widely accepted in the field of mathematics education that mathematical knowledge is not directly transferred from teacher to learner, but rather constructed by learners themselves, the constructivist approach to education has enjoyed attention as a new theoretical framework for mathematics education. Within a traditional teaching paradigm, the majority of mathematics lessons involve extensive teacher-directed explanations of material with little or no exploration of mathematical concepts. Traditionalist educators focus on the accurate demonstration of isolated mathematical skills and procedures in their classrooms. In contrast, constructivist educators emphasize the development of deep mathematical understanding and reasoning. They support a decrease in instruction on mathematical procedures while promoting mathematics curricula that are both mathematically rich and contextually situated (NCTM, 1989).

Many early childhood professionals in Korea also advocate constructivist educational approaches and have made efforts to extend constructivist teaching-learning principles to kindergarten contexts. Teachers in the field, however, still experience difficulties in implementing constructivist principles of education in their classrooms. According to Kim (2010), teachers of young children exhibited a mixture of constructivist and traditionalist beliefs in terms of teaching mathematics. Mathematical activities in Korean kindergartens are usually completed during free choice activity time, during which the teacher's role is limited to providing physical manipulatives to the children, with actual teacher-child interactions being rare (Hwang, 2005). It is also often observed that teachers depend on accurate demonstrations and directed explanations in kindergarten mathematics activities (Lee & Lee, 2010).

In this context, we review the literature on cognitively guided instruction (CGI) for early mathematics and examine the possibility of its use as a practical guide for implementing constructivist mathematics teaching in the classroom.

# 2. Background

## 2.1. Mathematics Education for Young Children

Early childhood is a period during which the foundations of children's mathematical abilities are constructed. The mathematical knowledge and skills obtained during this period is important in terms of their effect on future learning achievements (Aunola *et al.*, 2004; Classens & Engel, 2013; Grissmer *et al.*, 2010).

In general, young children tend to make decisions based on intuition rather than logical reasoning. They tend to make errors in problem solving, considering only a single prominent aspect of a task, or generalizing from a single example. However, they are capable of logical reasoning and of adapting their own thinking if they are given appropriate scaffolding from adults or allowed to use concrete manipulatives (Hong, 2010). According to recent research, children are competent mathematical thinkers who are capable of self-developing mathematical understanding from experiences in everyday life (Baroody, 2000; NCTM, 2000). For example, young children can infer simple arithmetic relationships for themselves, understand and use counting principles and strategies, and solve simple addition and subtraction problems (Carpenter Fennema, Franke, Levi, & Empson, 1999; Clements, 2004; Gelman & Gallistel, 1978; Ginsburg, Pappas, & Seo, 2001; Wynn, 1992) In addition, they may possess mathematical abilities for distinguishing, synthesizing, and dividing geometric shapes in space or on a plane (Hong, 2007). In other words, children possess a certain amount of informal knowledge and problem solving abilities applicable to mathematics, and actively participate in acquiring and learning mathematical concepts (Lee, & Bea, 2015).

In highlighting the importance of early childhood mathematical abilities, recent early childhood mathematics education has emphasized the need for a learning environment in which children actively explore and acquire mathematical concepts and procedures (such as problem solving, reasoning, representation, communication, and connecting mathematical ideas) through challenging and meaningful activities related to their experiences and prior knowledge, with a focus on nursing children's affective and cognitive foundations in mathematics (NAEYC & MCTM, 2002). In addition, in order to ensure that teachers provide opportunities for children to connect their real-life experiences with abstract mathematical thinking, they are required to offer instruction based on their understanding of what the children know and what they should learn.

In other words, recent early childhood mathematics education emphasizes children's active participation, the application of mathematical processes, oral interaction, active teacher assistance, and connections between children's experience and informal mathematics knowledge. This reflects a constructivist view, by which knowledge is acquired through an individual's construction and interactions within a social context.

#### 2.2. Constructivist Teaching-Learning Principles

Constructivism is a theory about the construction and acquisition of knowledge that emphasizes an individual's active participation and social interaction. From a constructivist perspective, learning is an interpretative, recursive, and constructive process of building meanings that interacts with the physical and social environment (Fosnot, 1996). Such learning requires a learner-centered classroom in which knowledge and its creation is interactive, different viewpoints are welcomed, and questions from learners are valued (Brooks & Brooks, 1993). Six basic teaching-learning principles for creating such a classroom are summarized below (Duffy & Cunningham, 1996; Fosnot, 1996; Savery & Duffy, 1996).

First, learning is a constructive and active process. It is constructive in the sense that a learner comes to represent knowledge internally, and active in the sense that the learner

develops meaning based on his experience. Thus, each individual creates knowledge for himself by relating existing knowledge to new information. Each learner has his own perceptual framework, and can immerse himself in learning when offered a learning experience that is based on an understanding of his current level of knowledge.

Second, learning is a social and communicative activity. Communication with peers promotes in-depth thinking. According to Vygotsky (1978), joint activities encourage children to use their developing abilities and language and interaction are central to such joint activities. Children internalize collaborative conversations with peers and teachers in acquiring cognitive skills. Children can construct a higher level of meaning through social negotiation with members of the learning community (Steffe & Gale, 1995).

Third, cognitive dissonance engenders learning. Such dissonance is a stimulus for learning and determines the nature and organization of the content to be learnt. When children realize that their constructions do not work in the context of new information, cognitive dissonance occurs. To resolve such dissonance, children either attempt to associate new information with their existing knowledge framework or to alter the existing framework. Disagreement with peers, in particular, leads children to reconsider their existing thoughts through cognitive conflict.

Fourth, reflective abstraction is a primary source of learning. Piaget (1980) suggested that logical and mathematical knowledge is obtained and developed through reflective abstraction. Reflective abstraction is accomplished by activities in which relationships are created between objects and reflection on these activities, which results in qualitative reconstruction that embraces existing knowledge. Reflective abstraction may be promoted when learners are given opportunities to represent their experiences with a range of symbolic systems and discuss the relationships between them.

Fifth, learning occurs when it is related to real world situations. Learners obtain motivation when they participate in tasks that match their own interests and are meaningful. When learning is contextualized in a learner's real life, and related to informal knowledge associated with real-life situations, the learning process and transition are promoted.

Sixth, learning is promoted through tools and symbols. Learning is constructing meanings. The construction of meanings is promoted by the use of tools (physical objects and operations) and symbols (symbolic media and operations, like language, numbers, and maps). External media promote problem solving abilities when logical thinking is required (Venger, 1988). Language, in particular, is a tool that enables abstract and flexible thinking and promotes shared activities. Thinking while speaking allows a learner to understand, clarify, and focus on what he has in mind and thereby assists learning (Bodrova & Leong, 2007).

It is commonly accepted by constructivists that teaching-learning environments corresponding to the above principles are essential in promoting constructive learning.

#### 2.3. Cognitively Guided Instruction

CGI is an approach to mathematics teaching that is based on research on children's mathematical thinking. Its central philosophy is that instruction should be guided by teachers' understanding of children's thinking (Carpenter *et al.*, 2015; Hoosain & Chance, 2004; Jaslow, 2001). Based on an understanding of children's mathematical thinking, CGI teachers support children in applying more effective strategies and more sophisticated mathematical representations through problem solving processes and reflective communication about mathematical thinking (Anthony, Bicknell, & Savell, 2001).

CGI is based on the belief that a teacher's understanding of children's mathematical thinking is a crucial factor in learning with understanding. CGI researchers have found that children are capable of learning with understanding when their teachers truly understand their thinking and provide them with opportunities to build on their own thinking. According to a study by Carpenter *et al* (1988), however, teachers' intuitive

knowledge about students' mathematical thinking is often fragmented and unsystematic. contributing little to decision making regarding instruction. Therefore, a CGI approach focuses directly on helping teachers to gain knowledge about children's mathematical thinking. To this end, CGI provides a detailed framework that can serve as a basis for understanding the development of children's mathematical thinking in whole number arithmetic (e.g., addition, subtraction, multiplication, and division with single-and multidigit numbers). Specifically, the framework may help teachers to understand particular problem types, the strategies invented by children, the relationship between these strategies and problem types, and how children's thinking evolves over time. They can then explore the use of their knowledge of children's thinking in instructional decision making (Beak, 2004). Decisions about what problems to pose, what numbers to use, what questions to ask, who to ask, whose ideas to share, and whose ideas might be connected to a shared strategy can all be supported by knowledge of the development of children's mathematical thinking. Thus, the purpose of CGI is not to provide teachers with a complete framework of children's mathematical thinking and reasoning. Rather, the purpose is to help teachers to construct their own framework about children's thinking, allowing them to decide how best to use that knowledge in the context of their own teaching practice (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

Although CGI does not provide explicit guidelines for instruction, it suggests a range of classroom practices and several characteristics of teaching, based on successful CGI cases, to promote children's mathematical development in ways that respect their thinking (Carpenter *et al.*, 2015).

#### 2.4. Previous Research on CGI

Many studies have shown that CGI leads to positive changes in both teachers' knowledge, beliefs, and practices and children's achievement (Carpenter *et al.*, 2015). For example, Villasenor and Kepner (1993) compared performance in problem solving and counting in a CGI class and a comparison group. Children in the comparison group were taught the problem solving procedures by teacher's demonstrations and spent a long time completing worksheets. In contrast, the children in the CGI class learned the relevant arithmetic through problem solving and were asked to explain their solution processes. Children in the CGI class performed significantly better in solving word problems, as well as in completing number facts, and used advanced strategies more frequently. Similar results were reported by Secada and Brendefur (2000).

Fennema *et al.*'s (1996) longitudinal research also indicated the benefits of CGI classes in an extension of their initial experimental study. The concepts and problem-solving abilities of the children in a CGI class were significantly improved in comparison to the beginning of the research. The students who participated in the CGI class for a longer period of time showed better achievement in the second and third years of the research, implying that the improvement was cumulative. Changes in students' achievement were found to reflect changes in teachers' practice, with a change in a teacher's practice directly followed by significant improvement in concept and problem solving performance among the students.

Kim and Oh (2010) examined the effects of CGI teaching methods on problem solving and mathematical disposition among mathematical underachievers. The results showed that the program was effective in improving their ability to solve problems in various ways and to explain their solutions in spoken or written language and drawings, and also in their development of positive attitudes toward mathematics learning. Based on these results, the researchers concluded that an instructional approach based on a teacher's understanding of an individual child's mathematical knowledge and characteristics may be effective for teaching mathematics underachievers.

Chio and Shin (2006) applied the principles of CGI in mathematics classes in Korean elementary schools, exploring which mathematical concepts students possessed and how

they used informal knowledge and procedures to solve problems. They also attempted to identify difficulties teachers might face when planning CGI-based mathematics teaching. Their study showed that mathematics teaching based on CGI provided opportunities for students to communicate about their own mathematical knowledge. The students appeared to be sure of their thoughts and to learn from the presentations of others. Furthermore, the CGI-based mathematics teaching appeared to make the students think mathematically, trying to understand the meaning of problems and to find various solutions. These researchers concluded that teachers should create an appropriate environment for mathematics teaching based on CGI, and offering appropriate problems and encouraging their students to ask and answer questions.

Despite the above findings, studies of CGI focusing on young children remain somewhat limited. In a study of 70 early childhood subjects, Carpenter *et al.* (1993) investigated young children's ability to solve various word problems and the strategies they used. The results showed that most could solve a wide range of problems involving multiplication and division, and that most used strategies that could be characterized as modeling the actions or relationships within the problems. They concluded that the modeling strategy might provide both a unifying framework for thinking about problem solving in the primary grades and a coherent way of thinking about children's mathematical problem solving.

Warfield (2001) recognized the value of CGI as a tool for analyzing young children's thinking in reporting on a teacher's practices in CGI implementation in an early childhood classroom. The teacher interpreted children's strategies and used information about how children typically process mathematically, allowing her to select appropriate problems and numbers to encourage progression. The teacher also gathered information on the children's thinking by posing word problems, listening to their descriptions of their strategies for solving the problems, and talking to other adults about the children. The teacher used all the information to select problems for later lessons.

Jaslow and Jacobs (2009), in turn, showed that CGI was effective in developing young children's understanding of numbers to 100. In their study, children were engaged in counting activities and solving story problems involving numbers greater than 10 over a period of nine weeks, with teachers providing them with tasks to promote continuous development based on their understanding of children's thinking. The children made significant progress in counting skills and improved in their understanding of place value, exhibiting understanding when counting in 10s and using 10s during problem solving.

Turner and Pattichis (2011) also reported that CGI was successful in developing mathematical problem solving among low-income kindergarten students. Mixed methods were used to examine teaching practices that engaged the students in problem solving and supported their learning. It was observed that although all students showed improvement from pre- to post-assessments, those in the CGI classroom outperformed those in other classroom. The CGI class was distinct from the other classes in spending the most time on problem solving, being exposed to a broader range of problems involving multiplication, division, and multiple steps, and having consistent access to their native language.

While a number of studies on CGI have been conducted in western societies, few have been conducted in Korea. We found only one study on CGI focusing on young children, namely that of Kim and Chung (2014), who explored the changes in mathematical thinking and attitudes toward mathematics among five-year-old preschoolers. They observed that the children's mathematical thinking transited from concrete to abstract thinking, and that they made such extensive progress as to use "records" to assist their own mathematical thinking. In addition, the children began to understand various thinking processes in accordance with different criteria. In terms of attitude toward mathematics, these researchers reported that, the children could only imitate at the beginning of the research, but acquired the ability to express their thinking their own ways and began to understand the joy of cooperation and communication.

# **3.** Constructivist Teaching-Learning Principles and Instruction in the CGI Classroom

Instruction in a CGI classroom can be characterized by a set of six basic classroom practices (Carpenter *et al.*, 2015; Fennema *et al.*, 1996) that follow constructivist principles of teaching and learning. First, children learn important mathematical ideas when they have opportunities to engage in solving a variety of problems and are encouraged to invent their own ways of solving these. CGI assumes that they bring to school a lot of informal mathematical knowledge and problem-solving abilities that can be helpful in understanding mathematics. Therefore, teachers do not demonstrate a strategy for solving a given problem; rather, children learn concepts and skills in a situation in which they solve problems based on their own thinking. They develop mathematical understanding not by means of simple pieces of information but by actively relating their informal knowledge to the problem situation. The teacher drives the children to reconstruct and expand their existing knowledge by selecting challenging problems that are appropriate to an individual child's current understanding and that also cause cognitive conflicts. In addition, the CGI classroom's respect for every child's ideas makes children confident and more involved in their own activities in making sense.

Second, the mathematical problems offered must have a context familiar to the children. Piaget (1980) suggested that emotion (interest and feeling) is the motivation for intellectual development. Word problems in CGI classrooms reflect the interests and context of children's everyday lives, which stimulates their interest and intellectual desire, leads them to engage in the process of mathematical understanding, and allows them to recognize mathematics as useful and valuable. To support children's engagement in and access to the problems, CGI teachers engage children in making sense of the context and the action or situation of the problem to ensure that all children understand the problem.

Third, the teacher ensures that children have manipulatives available to solve problems. In the CGI classroom, children use physical objects, such as counters, cubes, fingers, and drawing paper, to directly model the actions or relationships described in each problem. In CGI, children relate activities to their existing understanding by manipulating physical objects in parallel with their mental operations. Manipulating objects allows them to examine their own thinking and fosters the problem solving process (Bodrova & Leong, 2007). CGI teachers help children to connect their object manipulation with other symbolic tools like talking, writing, and drawing to invoke a second level of abstraction, improving thought recognition and intensifying understanding.

Fourth, CGI teachers elicit children's thinking. The question "Can you tell me how you solved that?" is a central feature of the CGI classroom. The teacher asks questions to find out how the children solve problems, to support them in completing or correcting a strategy, or to extend their mathematical thinking. It is both common and important for children to make their mathematical thinking explicit. Teachers ask ever more specific questions to lead children to share details of their thinking, which encourages them to participate in articulating, explaining, and justifying the details of their strategies. When a child thinks while speaking, language becomes a tool to understanding, clarifying, and focusing his mind (Bodrova & Leong, 2007). Working through explanations allows children to connect and synthesize ideas and make sense of new mathematical ideas.

Fifth, children engage with one another's ideas. Sharing their thinking with their teacher and peers after solving problems is key to children's learning in the CGI classroom. Teachers support this by comparing the ideas of different children, attending to the details of other children's ideas, letting children ask each other questions, discussing whether or not shared strategies are similar, building on or adding to other children's ideas, and even co-constructing a solution with other children. The opportunity for children of different skill and knowledge levels to share ideas by talking or writing allows them to reflect on one another's ideas and strategies, causing cognitive conflicts,

promoting reflective abstraction, and supporting their generation of insights into mathematical relationships and development of more sophisticated strategies. In particular, peer communication supports the internalization of a shared strategy. When children internalize the essential aspects of such regular mathematical conversations, they may be more able to use the strategies reflected in those conversations.

Finally, CGI teachers consider what children know and understand when they make decisions about mathematics instruction. Learners construct meaning by relating new ideas to what they already know and existing knowledge thereby forms a basis for expanding mathematical understanding (Fennema & Romberg, 1999). To help children relate something to be learnt with their existing knowledge, a teacher needs to have a clear understanding of each individual child's mathematical thinking. Therefore, CGI teachers listen to and observe children's strategies and then ask more about what they noticed. Detailed analysis of CGI for the development of children's mathematical thinking can provide teachers with a framework for understanding individual children's thinking and for teaching mathematics in a manner that encourages problem solving, reasoning, reflecting, and communication.

As discussed above, CGI practices building learning communities in which children's construction of mathematical meaning and communication are promoted clearly reflect constructivist teaching-learning principles.

## 4. Conclusion

In this paper, we examined the connections between constructivist teaching-learning principles and CGI classroom practices in order to determine whether CGI might serve as a practical guide for constructivist mathematics teaching for young children. We found that CGI practices clearly reflect constructivist principles and provide a framework for analyzing children's development of mathematical thinking. Furthermore, CGI-related materials, such as literature and videos of CGI practices, would help both researchers and teachers to develop constructivist mathematics teaching-learning processes. Therefore, we recommend the practical application of CGI, not only for meaningful construction of children's mathematical knowledge, but also for positive changes in teachers' practice based on the constructivist paradigm.

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