

## Dynamic Spectrum Access with Spatial Reuse

Ying Liu, Guangqiang Shi and Xiaomin Mu

*School of Information Engineering, Zhengzhou University, Zhengzhou, China  
1069984045@qq.com, 1243611987@qq.com, iexmmu@zzu.edu.cn*

### **Abstract**

*We investigate the problem of achieving utility function optimization for distributed channel selection using game theoretic solutions in opportunistic spectrum access system, where secondary users are spatially located and mutual interference only emerges between neighboring users. In addition, interference graph of spatial position is introduced to measure interference and evaluate the possible opportunities of spectrum access. Based on this we propose an optimization problem of minimizing loss throughput, which is solved by a game theory and an optimal channel selection set is derived. The main work of this paper is to prove that this game is an exact potential game, which has at least one pure strategy Nash equilibrium. Finally simulation results is given to verify the correctness of the theoretical analysis. Especially meanwhile after considering spatial reuse by users' position, the system throughput is significantly increased, thus the spectrum utilization rate is effectively improved.*

**Keywords:** *Dynamic spectrum access, cognitive radio networks, spatial reuse, game model, potential game, Nash equilibrium*

### **1. Introduction**

With the rapid increasing of heterogeneous networks, spectrum resource becomes more scarce which has been a bottleneck to restrict the development of wireless communication, so the dynamic spectrum access technology emerges as a promising technique to improve the utilization rate of spectrum resources in 5G [1-2]. The key challenge of dynamic spectrum access is how to solve the distributed spectrum resources allocation among selfish secondary users. They can transmit in the same frequency band simultaneously without causing any performance degradation if users are located sufficiently far apart, thus improving the spectrum utilization rate [3]. Compared with other optimization approaches about multiple users' resources, from systematic angle, game theory is a powerful tool to predict system's steady state (equilibrium point), reflect the interaction among multiple users, guide users to make a decision in order to improve the system performance [4]. So it is a current active research topic to study the problem of distributed channel selection with game theory for dynamic spectrum access.

A common assumption of most existing references is that secondary users are close-by and interfere with each other when they transmit on the same frequency band simultaneously [5-6]. However, the secondary user's spatial position is arbitrary distributed and user's decision only affects its neighboring users instead of all users in the general wireless access network. Therefore, in recent years, establishing a optimization problem with spatial reuse attracts the attention of specialist, a small amount of research is emerged [7-9]. According to the different object functions design the corresponding optimization frames. From the perspective of maximizing the expected throughput, this optimization framework is to optimize channel selection strategy set when the channel idle probability and the number of secondary users are unknown and the business demand of secondary user is same, is presented in [7]. Although there has been some progress, the problem is not yet solved. Specifically, in general model, spatial position information and

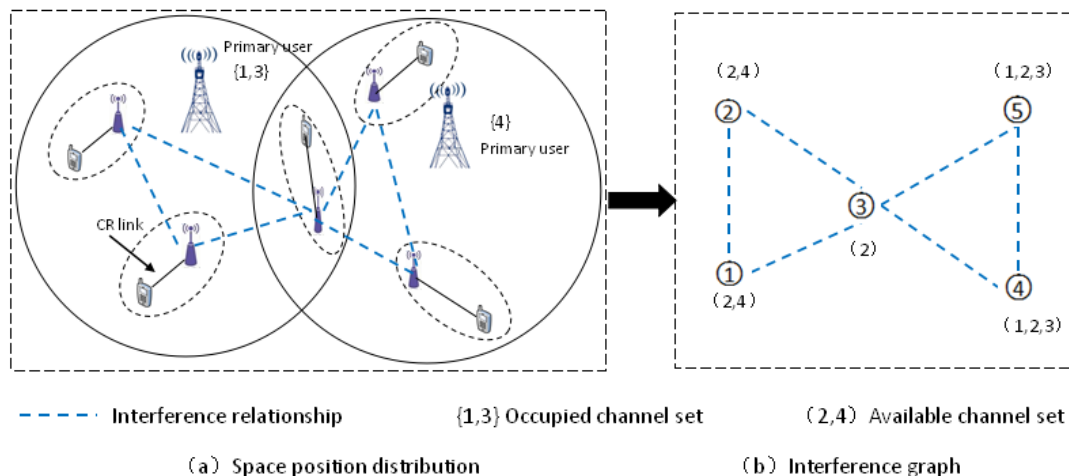
user demand are all affect the certain of optimal channel selection. In view of this, from the perspective of minimizing the system interference, the optimization framework, in which the secondary users make use of the interference graph decided by space position, is to evaluate interference and to optimize channel selection strategy set, is presented in [8]. Then from the perspective of maximizing the expected throughput, this optimization framework is to optimize the joint channel and location selection strategy set, is presented in [9].

User demand is a key factor to affect the system throughput and the selection of optimal channel set, user satisfaction is also related to it. However, the effect of user demand on distributed spectrum access is less understood than many other aspects in the above references, which motivates this study. Thus an optimization goal of minimizing the loss throughput is proposed, taking user demand into account and then studying the problem of distributed channel selection by game model; it is proved that this game would has at least one pure strategy Nash equilibrium. Finally simulation results are given to verify the correctness of the theoretical analysis, after considering spatial reuse, the system throughput is significantly improved, thus the spectrum utilization rate is improved as well compared with the reference [7].

## 2. The Formulation of Distributed Channel Selection and Interference Graph

An interference graph [8] of a corresponding spatial position distribution between users, characterizing the limited range of interference or transmission, can be obtained. Specifically, each node represents a secondary user on the interference graph. Considering available spectrum access opportunities based on users spatial position distribution of the cognitive radio network, where is involving  $N$  secondary users and  $M$  licensed channels which are owned by the primary users and can be opportunistically used by the secondary users,  $N > M > 1$ . Denote the set of the secondary users as  $\mathbf{N} = \{1, 2, \dots, N\}$  and the set of the licensed channels as  $\mathbf{M} = \{1, 2, \dots, M\}$ . For simplicity, we call a cognitive transmitter-receiver pair as a CR link or secondary user. An example of spectrum access with spatial reuse is shown in Figure 1 involving two primary users, four licensed channels (1-4) and five secondary users.

Notice that different primary users occupy different channel and their interference range partially overlap, which leads to heterogeneous spectrum opportunities in cognitive radio network. Then we characterize the heterogeneous spectrum opportunities by the channel available vector  $\mathbf{c}_n$ . Specifically,  $\mathbf{c}_n = \{C_{n1}, C_{n2}, \dots, C_{nM}\}$ , for each  $n \in \mathbf{N}$ , where  $C_{nm} = 1, m \in \mathbf{M}$  indicates that channel  $m$  is available for user  $n$ , while  $C_{nm} = 0$  indicates that it is not available, moreover, it is assumed that the spectrum opportunities vary slowly in time. We assume that the secondary users are located in a spatial domain  $\mathbf{D}$ , *i.e.*, a set of possible spectrum access locations. Denote  $d_n \in \mathbf{D}$  as the location of user  $n$ , and  $\mathbf{a} = (d_1, d_2, \dots, d_N) \in \mathbf{D}^N$  as location profile of all users, each secondary user has a transmission range  $\delta$ . Then given the location profile  $\mathbf{a}$  of all users, we can obtain the interference graph  $\mathbf{G}_a = \{\mathbf{N}, \varepsilon_a\}$  to describe the interference relationship among users. Here the vertex set  $\mathbf{N}$  is the secondary user set, and the edge set  $\varepsilon_a = \{(i, j) : \|d_i, d_j\| \leq \delta, \forall i, j \neq i \in \mathbf{N}\}$  is the interference edges set. If there is an interference edges between two secondary users, they cannot simultaneously transmit on the same channel, so denote  $\mathbf{J}_n = \{j \in \mathbf{N}, (j, n) \in \varepsilon_a\}$  as the set of connected (neighboring) users of user  $n$ .



**Figure 1. An Example of Spectrum Access with Spatial Reuse**

From the interference graph of the corresponding spatial position distribution, we can see that neighboring users can interfere with each other when simultaneously transmitting on the same channel in Figure 1. Then the transmission range among secondary users and the possibility of available spectrum access opportunities with spatial reuse are all closely related to secondary users' spatial location. So it is necessary to study the performance of spectrum sharing based on interference graph with spatial reuse and tapping the spectrum efficiency further.

### 3. The Optimization Problem of Distributed Channel Selection with Game Theory

The access strategy of secondary users and the spectrum utilization rate are two key factors of spectrum sharing mechanism based on spatial reuse. The spectrum utilization rate is usually modeled as loss throughput of secondary users, and through designing the strategy of user access set to max the negative of loss throughput. There is no centralized controller available, which motivates us to model and solve the problem as a game. The previous games, however, are proposed in the premise of the user same demand, so it is possible urgent to set up a potential game in conditions of user different demands.

#### 3.1. The Optimization Model of Spectrum Sharing with Spatial Reuse

Although the current optimization technologies in cognitive radio networks are usually maximizing the throughput, there are other alternative methods that implicitly maximizing the throughput, *e.g.*, the loss throughput minimization. So motivated by this idea, we consider the problem of opportunity spectrum access from the angle of minimizing the loss throughput in cognitive radio network, where secondary users are spatially located and interference only emerges between neighboring users.

It is assumed that all secondary users can perfectly sense all channel, but can transmit on only one channel due to hardware limitation. Efficient distributed mechanism such as CSMA can be applied to data transmission among neighboring and interfering users. Considering user different demand (telephone, SMS *etc.*), access channel probabilities of users is vary from it to it. Let  $a_n$  denote a channel selection of secondary user  $n$ , then the loss throughput, which is equivalent to the normalized contention time, received by secondary user  $n$  is given by:

$$r_n(a_n, a_{J_n}) = \frac{N_c \tau}{T_e} I_{a_n} R_{a_n} \quad (1)$$

Where  $T_e$  is the useful time after channel sensing,  $\tau$  is the length of a mini-contention slot,  $N_c$  is the number of mini-contention slot. It is seen that  $N_c$  is a geometric random variable [10] with the following probability mass function (PMF):

$$\Pr\{N_c = i\} = p_s (1 - p_s)^{i-1}, i \geq 1 \quad (2)$$

Where

$$p_s = p_n \prod_{j \in I_n(a_n, a_{J_n})} (1 - p_j) \quad (3)$$

Is the overall successful channel contention probability in a mini-contention slot,  $p_n$  is access channel probability of user  $n$ ,  $I_{a_n}$  indicates whether the channel  $a_n$  is idle or occupied. It is seen that  $I_{a_n}$  is a Bernoulli random variables with the following PMF:

$$\Pr\{I_{a_n} = x\} = \begin{cases} \theta_{a_n}, & x = 1 \\ 1 - \theta_{a_n}, & x = 0 \end{cases}, \quad (4)$$

Where  $\theta_m$  is the probability of idle channel,  $1 \leq m \leq M$ .

Based on (1)-(4), the expected loss throughput achieved by secondary user  $n$  is given by:

$$\begin{aligned} Q_n(a_n, a_{J_n}) &= E[r_n(a_n, a_{J_n})] \\ &= \frac{\tau R_{a_n} \theta_{a_n}}{T_e p_n \prod_{j \in I_n(a_n, a_{J_n})} (1 - p_j)} \end{aligned} \quad (5)$$

Where  $E[\cdot]$  takes the expectation.

To take the fairness issue into account, the proportional-fair utility function [11] is considered in this paper, *i.e.*,

$$\begin{aligned} D_n(a_n, a_{J_n}) &= 1 - \frac{Q_n(a_n, a_{J_n})}{\sum_{j \in I_n(a_n, a_{J_n})} \rho_j} \\ &= 1 - \frac{\tau R_{a_n} \theta_{a_n}}{p_n T_e \sum_{j \in I_n(a_n, a_{J_n})} \rho_j} \end{aligned} \quad (6)$$

Fair function in other forms will be considered in future research.

Therefore, our goal is to find the optimal channel allocation to minimize loss throughput when each secondary users has different demand, *i.e.*,

$$(P) : \min_{a_n \in \Lambda_n} D_n(a_n, a_{J_n}) \quad (7)$$

Where denote  $\Lambda_n = \{m \in M : C_{nm} = 1\}$  as the available channel set of player  $n$ .

### 3.2. The Optimization Problem Solution with Game Theory

The optimization problem of spectrum sharing based on spatial reuse is modeled as the selection problem of optimal channel set in the paper. As there is no central controller and each secondary user only care about their own utility function maximization, which motivates us to model and solve the problem as a game, then prove that it is a potential game which has at least one pure Nash equilibrium.

As each user is generally selfish and tries to maximize its own utility by choosing a proper channel, the game can be described as follows:

$$G = [\mathbf{N}, \{\mathbf{A}_n\}, \{U_n(a_n, a_{-n})\}_{n \in \mathbf{N}}] \quad (8)$$

Where  $\mathbf{N} = \{1, 2, \dots, N\}$  is the set of players(secondary users),  $\mathbf{A}_n = \{m \in M : C_{nm} = 1\}$  is the action set (the available channel set) of player  $n$ ,  $U_n(a_n, a_{-n})$  is the utility function of player  $n$ , *i.e.*,

$$U_n(a_n, a_{-n}) = R_n(a_n, a_{-n}) - D_n(a_n, a_{-n}) \quad (9)$$

Nash Equilibrium [12]: An action profile  $a^* = (a_1^*, a_2^*, \dots, a_N^*)$  is a pure strategy NE if and only if no player can improve its utility by deviating unilaterally, *i.e.*,

$$U_n(a_n^*, a_{-n}^*) \geq U_n(a_n, a_{-n}^*) \forall n \in \mathbf{N}, \forall a_n \in \mathbf{A}_n, a_n \neq a_n^*. \quad (10)$$

Theorem 1:  $G$  is an exact potential game which has at least one pure strategy NE.

Proof: We construct the potential function as follows:

$$\Phi(a_n, a_{-n}) = \sum_{n \in \mathbf{N}} \rho_n \left( \log \frac{\tau R_{a_n} \theta_{a_n}}{p_n T e} - \frac{1}{2} \sum_{j \in I_n(a_n, a_{-n})} \rho_j \right) \quad (11)$$

Where  $\rho_k = \log(1 - p_k)$ . Suppose that an arbitrary player unilaterally changes its channel selection from  $a_n$  to  $a_n^*$ , the change in the potential function caused by this unilateral change is given by

$$\begin{aligned} \Phi(a_n^*, a_{-n}) - \Phi(a_n, a_{-n}) &= \rho_n \left( \log \frac{\tau R_{a_n^*} \theta_{a_n^*}}{p_n T e} - \frac{1}{2} \sum_{j \in I_n(a_n^*, a_{-n})} \rho_j \right) - \rho_n \left( \log \frac{\tau R_{a_n} \theta_{a_n}}{p_n T e} - \frac{1}{2} \sum_{j \in I_n(a_n, a_{-n})} \rho_j \right) + \\ &\sum_{j \in I_n(a_n, a_{-n})} \left( \rho_j \left( \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j}^*)} \rho_k \right) - \rho_j \left( \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j})} \rho_k \right) \right) + \\ &\sum_{j \in I_n(a_n^*, a_{-n})} \left( \rho_j \left( \log \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j}^*)} \rho_k \right) - \rho_j \left( \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j})} \rho_k \right) \right) + \\ &\sum_{j \in K, j \neq n} \left( \rho_j \left( \log \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j}^*)} \rho_k \right) - \rho_j \left( \log \frac{\tau R_{a_j} \theta_{a_j}}{p_j T e} - \frac{1}{2} \sum_{k \in I_j(a_j, a_{-j})} \rho_k \right) \right) \\ &= \rho_n \left( \log \frac{\tau R_{a_n^*} \theta_{a_n^*}}{p_n T e} - \frac{1}{2} \sum_{j \in I_n(a_n^*, a_{-n})} \rho_j \right) - \rho_n \left( \log \frac{\tau R_{a_n} \theta_{a_n}}{p_n T e} - \frac{1}{2} \sum_{j \in I_n(a_n, a_{-n})} \rho_j \right) \\ &+ \frac{1}{2} \sum_{j \in I_n(a_n, a_{-n})} \rho_j (\rho_n) + \frac{1}{2} \sum_{j \in I_n(a_n^*, a_{-n})} \rho_j (-\rho_n) \\ &= \left( \rho_n \left( \log \frac{\tau R_{a_n^*} \theta_{a_n^*}}{p_n T e} - \sum_{j \in I_n(a_n^*, a_{-n})} \rho_j \right) \right) - \rho_n \left( \log \frac{\tau R_{a_n} \theta_{a_n}}{p_n T e} - \sum_{j \in I_n(a_n, a_{-n})} \rho_j \right) \\ &= -\rho_n (U_n(a_n^*, a_{-n}) - U_n(a_n, a_{-n})) \end{aligned}$$

A game is called a weighted potential game [12] if it admits a potential function  $\Phi(a)$  such that for every  $n \in \mathbf{N}$  and  $a_{-n} \in \mathbf{M}^{N-1}$ ,

$$\Phi(a_n^*, a_{-n}) - \Phi(a_n, a_{-n}) = w_n (U_n(a_n^*, a_{-n}) - U_n(a_n, a_{-n})) \quad (12)$$

Where  $w_n > 0$  is some positive constant. Since  $0 < p_k < 1$  and hence  $\rho_k = \log(1 - p_k) < 0$ , It is clear that the game is a potential game which has several nice properties and the most important one is that every potential game has at least one pure strategy NE. Based on the property, Theorem 1 is proved.

#### 4. Simulation Results

In this section, simulation results are presented to validate the proposed game-theoretic channel selection mechanism based on spatial reuse of the SLA learning algorithm[13]. The basic simulation parameter settings are as follows:

The number of secondary users is set to be  $N = 9$ , the number of channels is set to be  $M = 3$  (interference graph in Figure 2, where each circle represents a cognitive radio link, the dashed lines indicate the interference between the secondary users), the fixed channel access probability  $p_n$  of user  $n$  is randomly selected from the set  $\{0.1, 0.2, \dots, 0.9\}$ . The simulation results show in Figure 3, Figure 4, Figure 5 and Figure 6.

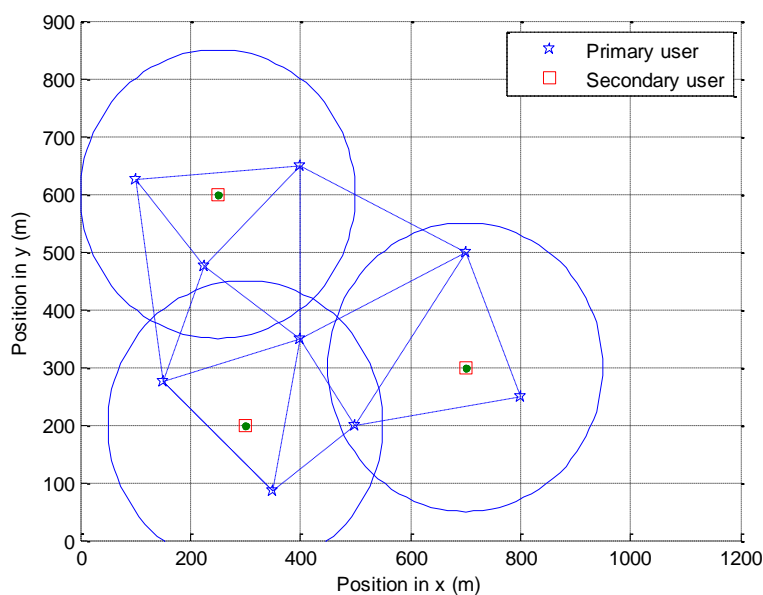


Figure 2. Interference Graph

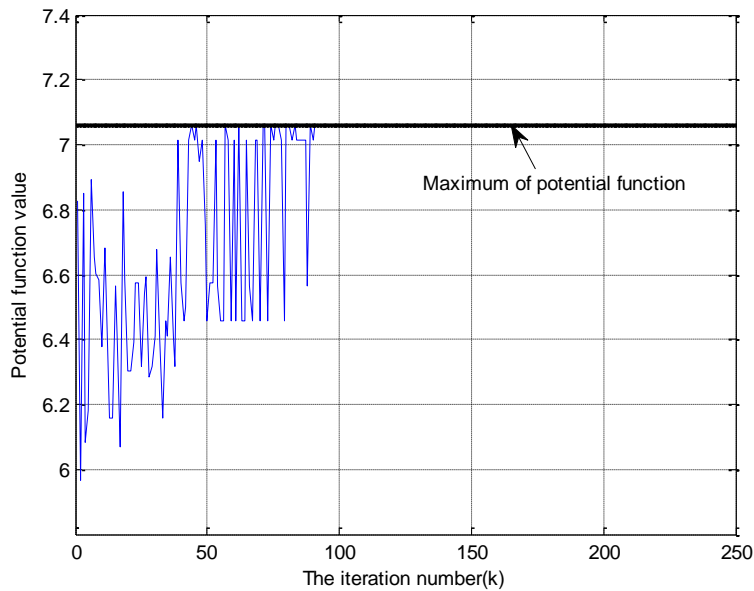
In order to verify the convergence of the proposed game-theoretic channel selection mechanism, Figure 3 shows the dynamic change of potential function  $\Phi$  based on interference graph, it can be seen that with the increasing of the iteration number the potential energy function gradually converges to the maximum value in the 70 iterations, which is a Nash equilibrium according to the property of potential game.

In order to test the performance of the proposed game-theoretic channel selection mechanism, the solution obtained by learning algorithm with the global optimization of  $\sum U_n(a_n, a_{-n})$  is compared in Figure. 4. The performance loss of system utility is not more than 5%, therefore, it can be estimated that the Nash equilibrium solution is global optimal solution, in other words, the game for minimizing system loss throughput has a

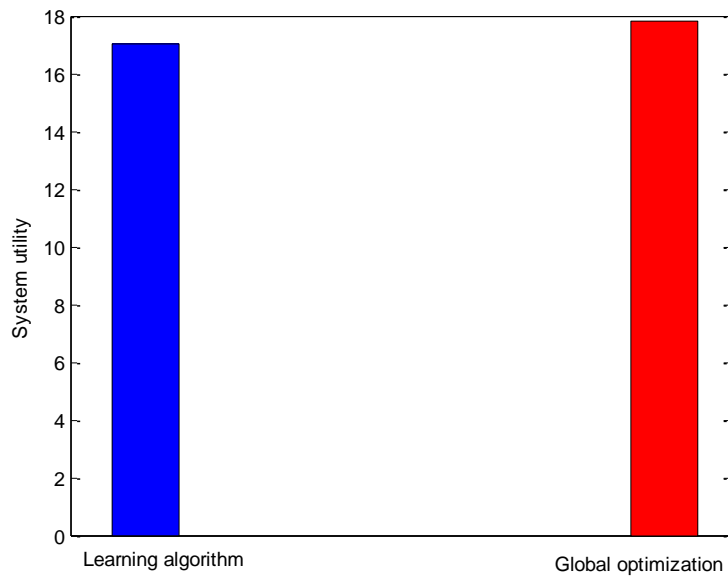
Nash equilibrium which is the optimal solution of the network loss throughput minimization problem.

It can be seen from Figure 5 user satisfaction (the expectation of the calculation-to-need ratio about the access channel probabilities) increases with the increasing of the access channel probabilities, where user satisfaction is 1, if the calculation-to-need ratio greater than or equal to 1, otherwise the user satisfaction is 0. After introducing the user different demands, user satisfaction is all 1, greatly improved. In other words, the calculated access channel probability is not greater than the required access channel probability when not considering user demand. Therefore, user demand is an important indicator to influence user quality of service in cognitive radio network.

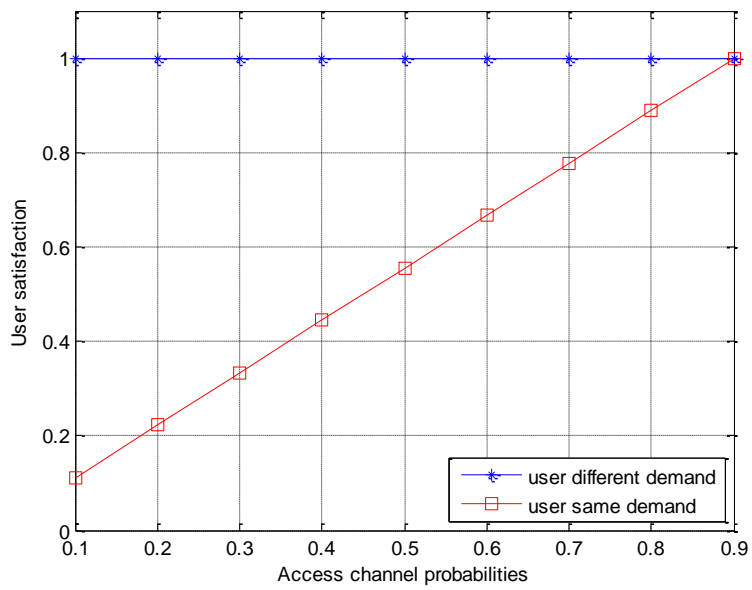
In order to unify the comparison of system throughput, it is assumed that  $p_a = 0.2, R_{a_n} = 1$  in Figure 6. We can see from it that the system throughput based on spatial reuse is greatly improved than traditional system throughput. As taking into account the users' spatial position, obviously the system throughput is greatly improved, thereby improving the spectrum utilization rate, it is because if the users are sufficiently far enough and they can transmit in the same frequency band simultaneously.



**Figure 3. Diagram of Potential Function and the Number of Iterations**

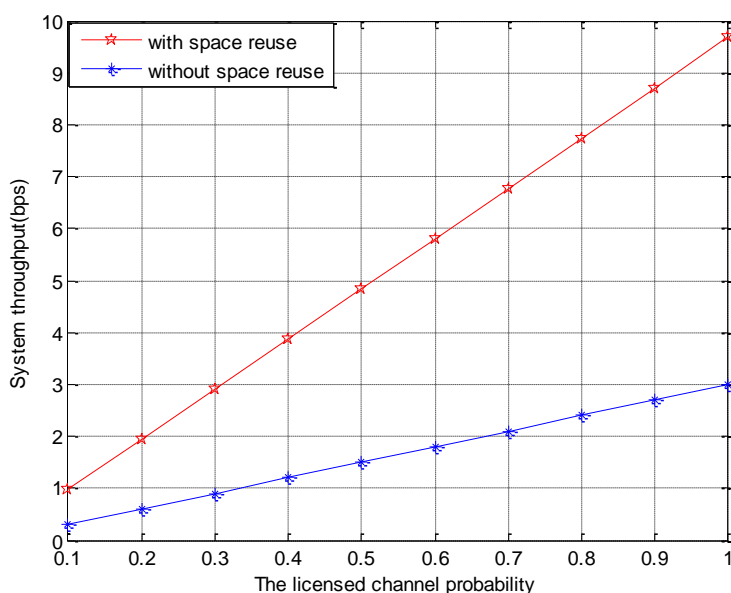


**Figure 4. Comparison of Learning Algorithm and Global Optimization**



**Figure 5. Comparison of User Satisfaction**





**Figure 6. Comparison of System Throughput**

## 5. Conclusions

With the help of the interference graph about the distribution of user-space location, the interference between users is assessed, so based on it we investigated the optimization problem of dynamic spectrum access with spatial reuse. The optimization model of loss throughput is constructed, where the optimal channel selection set is to determine to minimize loss throughput. A game model of maximizing the potential function, which is a potential game that has at least one Nash equilibrium and the Nash equilibrium is the optimal solution of the above optimization model, is proposed in this paper. Finally the simulation results prove the rationality and effectiveness of the optimization problem, improving opportunities access throughput in cognitive network after considering the spatial location information between users. However, there are still many problems need further research such as based on the heterogeneous spatial locations different user demands (telephone, SMS, *etc.*) influence the choice of the optimal channel set from the perspective of the system after considering mutual cooperation between cognitive users.

## Acknowledgements

The financial support of the National Natural Science Foundation of China under grants (No. 61271421 and No. 61301150), the Technology Project of Henan Provincial under grants (No. 112102210507) and the Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) under grants (No. 20134101120001) are gratefully acknowledged.

## References

- [1] D. J. Lee, "Adaptive Random Access for Cooperative Spectrum Sensing in Cognitive Radio Networks", *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, (2015), pp. 831-840.
- [2] S. Janevski, "Radio Network Aggregation For 5G Mobile Terminals in Heterogeneous Wireless Networks", *International Conference on Telecommunication in Modern Satellite, Cable and Broadcasting Services (TELSIKS)*, Nis: IEEE, (2013), pp. 225-228.
- [3] Z. Ning, "Throughput Improvement by Network Coding and Spatial Reuse in Wireless Mesh Networks", *Global Communications Conference, Atlanta, GA: IEEE*, (2013), pp. 4572-4577.

- [4] E. Nekouei, T. Alpcan and D. Chattopadhyay, "Game-Theoretic Frameworks for Demand Response in Electricity Markets", IEEE Transactions on Smart Grid, vol. 6, no. 2, (2015), pp. 748-758.
- [5] M. Felegyhazi, M. Cagalj and J. P. Hubaux, "Efficient MAC in Cognitive Radio Systems: A Game-Theoretic Approach", IEEE Transactions on Wireless Communication, vol. 8, no. 4, (2009), pp. 1984-1995.
- [6] D. Niyato and E. Hossain, "Competitive Spectrum Sharing in Cognitive Radio Networks: A Dynamic Game Approach", IEEE Transactions on Wireless Communication, vol. 7, no. 7, (2008), pp. 2651-2660.
- [7] Y. Xu, J. Wang and Q. Wu, "Opportunistic Spectrum Access in Unknown Dynamic Environment: A Game-Theoretic Stochastic Learning Solution", IEEE Transactions on Wireless Communications, vol. 11, no. 4, (2012), pp. 1380-1391.
- [8] Y. Xu, Q. Wu, L. Shen and J. Wang, "Opportunistic Spectrum Access with Spatial Reuse: Graphical Game and Uncoupled Learning Solutions", IEEE Transactions on Wireless Communications, vol. 12, no. 10, (2013), pp. 4814-4826.
- [9] X. Chen and J. Huang, "Distributed Spectrum Access with Spatial Reuse", IEEE Journal on Selected Areas, vol. 31, no. 3, (2013), pp. 593-603.
- [10] D. Zheng, W. Ge and J. Zhang, "Distributed opportunistic scheduling for ad hoc networks with random access: an optimal stopping approach", IEEE Trans. Inf. Theory, vol. 55, no. 1, (2009), pp. 205-222.
- [11] T. Harks, "Utility proportional fair bandwidth allocation: An optimization oriented approach", Quality of Service in Multiservice IP Networks, (2005), pp. 61-74.
- [12] D. Monderer and L. S. Shapley, "Potential games", Games and Economic Behavior, vol. 14, (1996), pp. 124-143.
- [13] P. Sastry, V. Phansalkar and M. Thathachar, "Decentralized Learning of Nash Equilibria in Multi-Person Stochastic Games with Incomplete Information", IEEE Transactions on Systems Man and Cybernetics, vol. 24, no. 5, (1994), pp. 769-777.

## Authors



**Ying Liu**, received her B. S. degree in Mathematics from Xi'an University of post and Telecommunications in 2013. She is now a graduate student at the Zhengzhou University. Her main research interests are in the areas of dynamic spectrum access.

**Guangqiang Shi**, received his B. S. Degree in Mathematics from Zhengzhou University. He is now a graduate student at the Zhengzhou University. His main research interests are in the areas of dynamic spectrum access.

**Xiaomin Mu**, received her B. S. degree from Beijing Institute of Technology in 1982. She is now a professor and also the PhD supervisor. Her research concerns cognitive radio, multi-antenna wireless communication systems, communication information processing, image signal processing, *etc.*