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Ultrasonic Signal De-noising Based on Wavelet Entropy and Inter-Scale Correlation

Zhi Cui * and Xianpu Cui

School of Communication and Electronic Engineering, Hunan City University
zhicui@yeah.net

Abstract

In this paper, we proposed an adaptive threshold de-noising method by combining wavelet entropy and inter-scale correlation. Different from the traditional wavelet threshold based de-noising methods, our method can be divided into three steps. First, we decompose the noisy signal by using discrete wavelet transform (DWT), calculate the value of inter-scale correlation of the decomposed wavelet coefficients, and delete the high frequency coefficients which are smaller than the value of inter-scale correlation. Secondly, we equally divide the processed high frequency coefficients into several subintervals, calculate the wavelet entropy of each subinterval, and decide the threshold of high frequency coefficients by combining wavelet entropy and adaptive threshold rules. Finally, we de-noise the high frequency coefficients by using logarithmic smoothing threshold function, and reconstruct the ultrasonic signal. Experiment results have shown that the proposed method is better than some other de-noising methods in terms of SNR (signal noise ratio) and SDR (signal distortion ration).

***Keywords**: wavelet entropy; correlation; ultrasonic signal; de-noising*

1. Introduction

Ultrasonic testing is one of the most effective methods of nondestructive testing. It has the advantages such as high adaptability to complicated tested components, strong ability of identification and characterization of defects and high efficiency of detection, so it has been widely used in many fields which have high request for rapid test and field test [1]. In practice, because of the intrinsic characteristics that ultrasonic in heterogeneous interface has high reflectivity and low transmission rate, and the influence of instrument noise, environmental noise and the random noise in the process of signal transmission, the reliability of test result has been seriously affected [2]. Therefore, the ultrasonic signal de-noising is a hot topic [3-5].

The wavelet threshold de-noising method is an effective method for signal de-noising. It assumes that the noise in the signal is mainly distributed in the high frequency section, while useful content is distributed in the low frequency part, and then by finding the spectral distribution pattern in the signal and distinguishing content from noise through frequency domain, the goal of de-noising could be achieved [6]. To design a threshold function is one of the critical question in wavelet threshold de-noising. Donoho proposed two kinds of threshold function such as hard threshold and soft threshold [7]. However, soft threshold would cause constant bias between estimated value and actual value of the wavelet coefficients, and hard threshold would cause reverberate of the reconstructed signal [8-9]. With the above reasons in mind, some improved approaches have been proposed to address the problem of ultrasonic signal de-noising. F. Ykhlef *et al.* proposed a compromised index threshold function [10]. In this method, the deviation of wavelet coefficients caused by soft threshold could be regulated by a variable factor. However, the threshold function is discontinuous. S. Emeterio *et al.* proposed a continuous threshold

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function [11]. In this method, an evolving factor was set to get a better noise reduction. Whereas, setting the gradual factor is difficult. X. Yin *et al.* proposed a threshold function with exponential factor [12]. With this method, a smooth function curve through a set of exponential term with throttling capability could be realized, meanwhile, the vibration could be reduced. However, the scope of index factor is difficult to determine. W. X. Ren *et al.* proposed a de-noising method based on wavelet entropy [13]. In this method, the threshold of high frequency wavelet coefficients could be decided by wavelet entropy. Nevertheless, this method is short of adaptability.

From the research course of the signal de-noising based on wavelet transform, it is known that most of the current de-noising methods are on the basis of soft threshold and hard threshold, and more or less has some shortcomings such as uncertainty of adjust factor, or lack of adaptability. Due to the complex work environment, the ultrasonic signal contains rich details and low signal-to-noise ratio. In order to avoid the disadvantages of the above criterions, in this paper, we propose a new method for the ultrasonic signal de-noising. Motivated by works in [13], the proposed method has two improvements. First, the proposed method uses inter-scale correlation of the decomposed wavelet coefficients to decide the high frequency coefficients which completely consist of noise. Second, the proposed method decides the threshold of high frequency coefficients by combining wavelet entropy and adaptive threshold.

The rest of this paper is organized as follows. The description of related work is briefly presented in Section 2. The proposed method is described in Section 3. The simulation and experimental results and comparisons are given in Section 4 and Section 5. The conclusions are drawn in Section 6.

2. Related Work

Research results have shown that, in the ultrasonic non-destructive testing, noise in the echo always shows characteristic of flat broadband, so it can considered to be additive white gaussian noise [14]. For this reason, we represent a noisy ultrasonic signal as follows:

$$y(t) = x(t) + z(t) \quad (t = 1, 2, \cdots N) \quad (1)$$

Where $y(t)$ is the noisy signal, $x(t)$ is the original signal, $z(t)$ is noise.

The goal of de-noising is to eliminate the noise $z(t)$ as far as possible, and make the de-noised signal $\hat{x}(t)$ as close as possible to the original signal $x(t)$. The difference between $\hat{x}(t)$ and $x(t)$ can be represented as follows:

$$\Delta(\hat{x}(t), x(t)) = \frac{1}{N}\left\|\hat{x}(t) - x(t)\right\|^2 = \frac{1}{N}\sum_{t=1}^{N}(\hat{x}(t) - x(t))^2 \quad (2)$$

Where N is the length of signal.

When the signal $y(t)$ is processed by discrete wavelet transform, the formula (2) can be described as follows:

$$\Delta(\hat{x}(t), x(t)) = \frac{1}{N}\sum_{t=1}^{N}(\hat{x}(t) - x(t))^2 = \frac{1}{N}\sum_{j,k}(\hat{\psi}(j,k) - \psi(j,k))^2 \quad (3)$$

Where $\hat{\psi}(j,k)$ and $\psi(j,k)$ represent the coefficients of true value and estimated value, respectively. From the above analysis, it is known that the de-noising in wavelet domain is translated into the problem of making formula (3) get minimum solution.

In the wavelet domain, energy of signal is concentrated in coefficients with large range. Moreover, because of the dispersion of the frequency and energy spectrum of noise, the absolute value of wavelet coefficient of noise is small. Therefore, the wavelet coefficients of useful signal will be greater than the wavelet coefficients of noise after wavelet decomposition. Then the de-noising can be achieved through threshold shrinkage.

3. The De-Noising Method Based on Wavelet Entropy and Inter-Scale

In this section, we describe the proposed method. One core of our method is to use inter-scale correlation of the decomposed wavelet coefficients to decide the high frequency coefficients which completely consist of noise. This is because coefficients of useful information have strong inter-scale correlation, while coefficients of noise do not [15]. In addition, we decide the threshold of high frequency coefficients by combining wavelet entropy and adaptive threshold.

3.1. Wavelet Entropy

In information theory, entropy shows the mean value of information and uncertainty in the information source.

Assuming that the high frequency coefficient in k^{th} point of j^{th} scale is $d(j,k)$ when the signal $f(t)$ is decomposed by DWT, we can formulate the relevant wavelet entropy as follows:

$$E_j = \sum_k |d(j,k)|^2 \quad (k = 1, 2, \cdots N) \quad (4)$$

Where j $(j = 0, 1, \cdots, M)$ represents the scale of the decomposition. Then the total energy of signal can be described as follows:

$$E = \|f(t)\|^2 = \sum_j \sum_k |d(j,k)|^2 = \sum_j E_j \quad (5)$$

The relative wavelet energy can be described as follows:

$$P_j = E_j / E \quad (6)$$

Where P_j shows the relative wavelet energy. When equally divide the high frequency coefficient $d(j,k)$ of the j^{th} scale into n subintervals, the wavelet entropy in each subinterval of this scale can be described as follows:

$$E_{j,k} = \sum_k^{m/n} |d(j,k)|^2 \quad (7)$$

Where m is the sampling number. Then the relative wavelet energy in each subinterval can be calculated as follows:

$$P_{j,i} = E_{j,i} / E_j \quad (8)$$

Where E_j is the wavelet entropy with the constraint of $E_j = \sum_{i=1}^{n} E_{j,i}$. Finally the wavelet entropy in the i^{th} subinterval can be solved as follows:

$$W_i = -\sum_j P_{j,i} \ln(P_{j,i}) \quad (9)$$

3.2. Inter-Scale Correlation

Motivated by work in [16], the definition of inter-scale correlation can be described as follows:

$$C(j,k) = d(j,k) \cdot d(j+1,k) \quad (10)$$

Where $C(j,k)$ is the correlation coefficient in the k^{th} point of the j^{th} scale, $d(j,k)$ and $d(j+1,k)$ shows the high frequency coefficients in the k^{th} point of the j^{th} and $j+1^{th}$ scale, respectively.

The equation (10) can be changed as follows after being normalized:

$$N_{C(j,k)} = C(j,k)\sqrt{E_j / E_{C_j}} \quad (11)$$

Where $N_{C(j,k)}$ is the value of normalized correlation coefficient, EC_j is the energy of the correlation coefficient in the j^{th} scale with the constraint of $E_{C_j} = \sum_k |C(j,k)|^2$.

As equation (11) is hard to solve, this paper proposes a new correlation coefficient as follows:

$$C_{new}(j,k) = \frac{d_{max}(j,k)}{d_{min}(j,k)} \quad (12)$$

Where $d_{max}(j,k)$ is the maximum of high frequency coefficient in the k^{th} point under j^{th} scale, and $d_{min}(j,k)$ signifies the minimum.

From the above equation, it is known that when the correlation coefficient is in line of $C_{new}(j,k) \in (1,\varepsilon)$, where ε is a constant greater than 1. The grater it is, the bigger the difference between $d_{max}(j,k)$ and $d_{min}(j,k)$ is. This means more of the wavelet coefficients which represent noise will be reserved. On the other side, when it is closer to 1, the wavelet coefficients which represent useful information will be removed.

3.3. Adaptive Threshold and Logarithmic Smoothing Threshold Function

In the wavelet threshold de-noising, the choice of the threshold and the design of the threshold function are the key factor. The traditional threshold can be described as follows [17]:

$$T = \sigma\sqrt{2\ln N} \quad (13)$$

Where T is the threshold, σ is the noise level and N is the length of signal. From equation (13), it is known that the traditional threshold has not addressed the scale of wavelet decomposition, which results in low precision. For this reason, this paper adopts the adaptive threshold as follows [18]:

$$T_{new} = \sigma\sqrt{\frac{2\ln N}{\ln(j+1)}} \quad (14)$$

Where j is the scale of wavelet decomposition, N is the sampling number.

Logarithmic smoothing threshold function is a new wavelet threshold function proposed by Zheng et al. [19], which can be described as follows:

$$d_{new}(j,k) = \begin{cases} d(j,k) + \mathrm{sgn}(d(j,k))T_{new}\log_2\left(\left|\frac{T_{new}}{d(j,k)}\right|^n + 1\right), d(j,k) \geq T_{new} \\ 0, d(j,k) < T_{new} \end{cases} \quad (15)$$

Where n is a constant.

3.4. The Proposed Method

On the basis of the above analysis, the specific process of our method is summarized as follows:

(1) The first step

To obtain the high frequency coefficients and the low frequency coefficients, we choose the wavelet primary function and decompose the ultrasonic signal by using DWT.

(2) The second step

We calculate the value of inter-scale correlation of high frequency in each noise dominant interval, and compare it with high frequency in the same positions. Then we mark the position in which the value of inter-scale correlation are smaller than the high frequency coefficients, and set the high-frequency coefficients at that location as $d'(j,k)$, while the others are set as zero.

(3) The third step

We fix the position of $d'(j,k)$ on the premise that the sampling number remain unchanged, and build the new high-frequency wavelet coefficients of each scale by them.

(4) The fourth step

We equally divide the new high frequency coefficients into n subintervals, calculate the wavelet entropy of each subinterval by formula (9), find the subinterval i which has the biggest wavelet entropy, and regard it as the noise dominant interval.

(5) The fifth step

When the noise dominant interval has been decided in step 4, we start calculating the threshold using equation (14). Specifically, we set the parameter σ as $\sigma = \frac{\mathrm{median}(|d''(j,k)|)}{0.6745}$, where σ is the noise variance, $d''(j,k)$ is the high frequency coefficient in the noise dominant interval.

(6) The sixth step

In this step, the high frequency coefficients are processed by combining the logarithmic smoothing threshold function and new threshold in each scale.

(7) Finally, the de-noised signal is reconstructed by the low frequency coefficients in the highest scale and the high frequency coefficients obtained from step 6.

4. Simulation and Analysis

In this section, we compare the performance of the proposed method with other three different methods. The parameters of the proposed method are set as $\varepsilon = 1.25$, and $n = 5.4$. For the methods to be compared with, the parameters are set to the default value to reach their best results. In our experiments, SNR and SDR are adopted as the objective indices to evaluate the quality of de-noised signal. All the experiments are implemented on a Core i5(R) 2.6 GHz PC with 4 GB RAM.

Specifically, the SDR can be described as follows:

$$SDR = \sqrt{\sum_{n=1}^{N} \left|x(t) - \hat{x}(t)\right|^2 \Big/ \sum_{n=1}^{N} \left|x(t)\right|^2} \quad (16)$$

Where $x(t)$ is the original signal, $\hat{x}(t)$ is the de-noised signal.

SNR and SDR show the distortion degree of the de-noised signal. It is clear that either a higher SNR or a lower SDR represents the effective of the de-noising.

The test signal can be described as follows:

$$h(t) = \rho e^{-\mu(t-\tau)} \cos[2\pi f(t-\tau)] \quad (17)$$

In the first simulation experiment, we compare the output SNR and SDR results of the proposed method with those of wavelet transform using stationary model (WT-SM) [20], wavelet transform using translation invariant model (WT-TIM) [21], and the method proposed by W. X., Ren *et al.*, [13]. The parameters of the test signal are set as $\rho = 1$, $\mu = 1.4 \times 10^{13}$, $\tau = 1.25 \times 10^{-6}$, and $f = 2.5 \times 10^{6}$. When the input SNR is set as 0dB, 5dB and 10dB, the output SNR and SDR results are given in Table 1.

As can be observed in Table 1, the proposed method outperforms the other methods on all output SNR and SDR results. For instance, with the input SNR of 0 to the original signal, the output SNR result obtained by the proposed method is 9.023, with the gain of 3.051, 2.405 and 0.516 respectively over the values obtained by using S-M, TI-M and the method in [13]. It can also be found that the results obtained by the proposed method are better than the others for original signal with different input SNR, with the average rise at 4.931, 3.884 and 0.832 on average respectively than using WT-SM, WT-TIM and the method in [13]. It shows that when the signal was de-noised by using the compared methods, the noise could not be completely removed. Our method is to de-noise signal by using wavelet entropy and inter-scale correlation, which can help effectively keep the useful information in ultrasonic signal.

Table 1. Performance of the De-Noising Methods by Output SNR and SDR

Input	Output SNR (dB)				SDR			
SNR (dB)	WT-SM	WT-TIM	Method [13]	Proposed	WT-SM	WT-TIM	Method [13]	Proposed
0	5.972	6.618	8.507	**9.023**	0.125	0.121	0.115	**0.096**
5	10.510	11.653	14.976	**15.883**	0.116	0.114	0.107	**0.089**
10	12.455	13.806	17.749	**18.823**	0.107	0.103	0.097	**0.082**

We also compare the visual effects got in the proposed method and the others for the test signal with the input SNR of 0dB, 5dB, and 10dB. The result is given in Figure 1. From the visual results, it can be found that de-noised waveform obtained by the proposed method has the smallest distortion, which means the proposed method can better preserve the features than the other methods.

In the second simulation experiment, we test the de-noised effect with the same signal: $h(t) = \rho e^{-\mu(t-\tau)} \cos[2\pi f(t-\tau)]$. The parameters of the test signal are set as $\rho = 3.24$, $\mu = 1 \times 10^{12}$, $\tau = 5 \times 10^{-6}$, and $f = 2 \times 10^{6}$. When the input SNR is set as the same value 0dB, 5dB and 10dB, the output SNR and SDR results are given in Table 2. It can be found that the results obtained by the proposed method are better than the others for the original signal with different input SNR, with the average rise at 3.37, 2.96 and 0.641 on average respectively than using WT-SM, WT-TIM and the method in [13].

Table 2. Performance of the De-Noising Methods by Output SNR and SDR

Input	Output SNR (dB)				SDR			
SNR (dB)	WT-SM	WT-TIM	Method [13]	Proposed	WT-SM	WT-TIM	Method [13]	Proposed
0	7.196	7.468	8.995	**9.417**	0.113	0.108	0.101	**0.097**
5	11.269	11.695	14.085	**14.746**	0.108	0.107	0.095	**0.091**
10	14.338	14.879	17.920	**18.761**	0.102	0.101	0.094	**0.083**

The visual comparison is given in Figure 2. From the visual results, it can be found that the proposed method can get better de-noised waveform than the other methods.

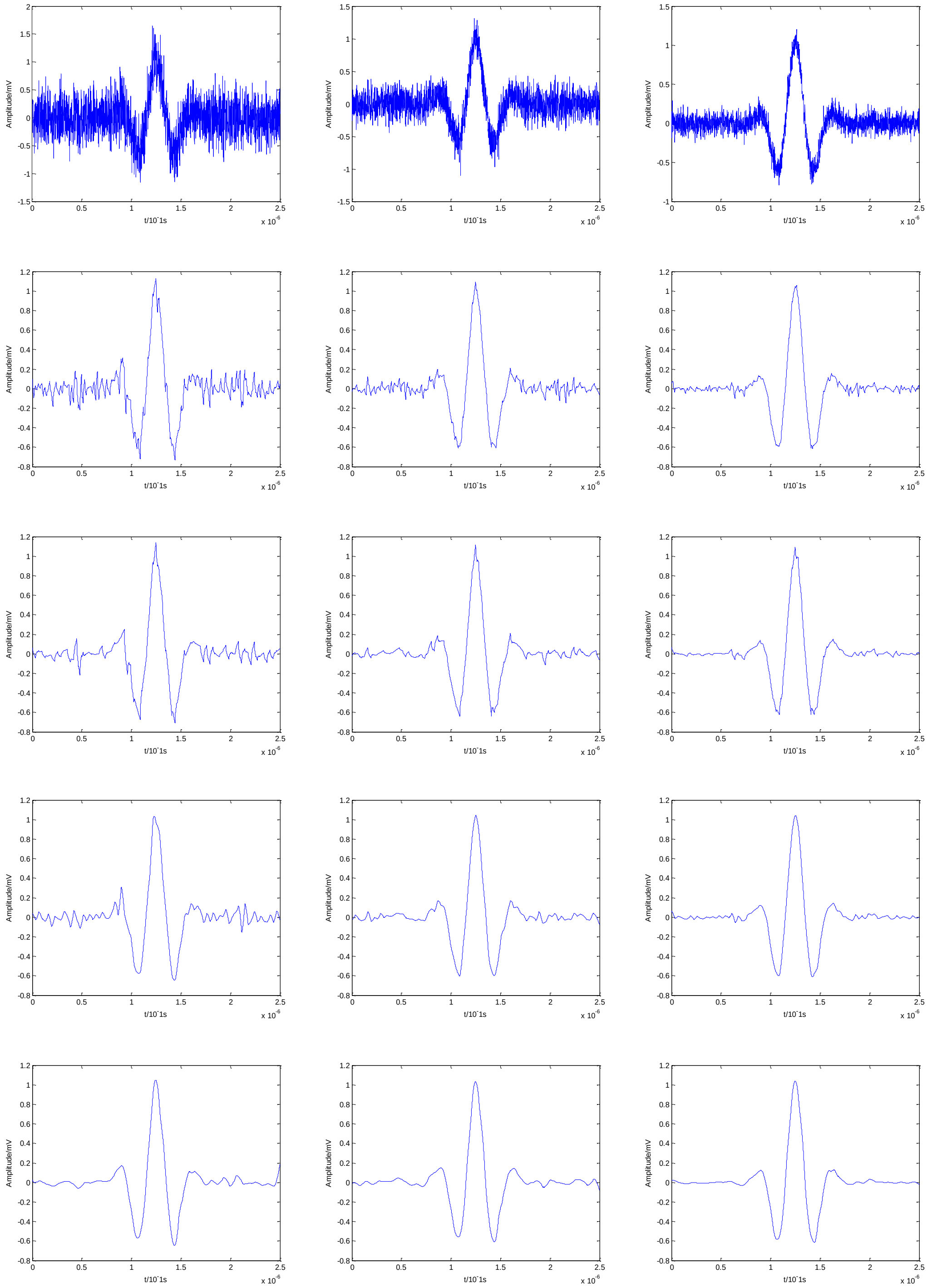


Figure 1. De-Noising Results Obtained by Four Compared Methods on Test Signal with Different Input SNR

The first line is the test signal with input SNR of 0, 5, and 10. The second line is the de-noised waveform by using WT-SM. The third line is the de-noised waveform by using

WT-TIM. The fourth line is the de-noised waveform obtained by method in [13]. The final line is the de-noised waveform obtained by the proposed method.

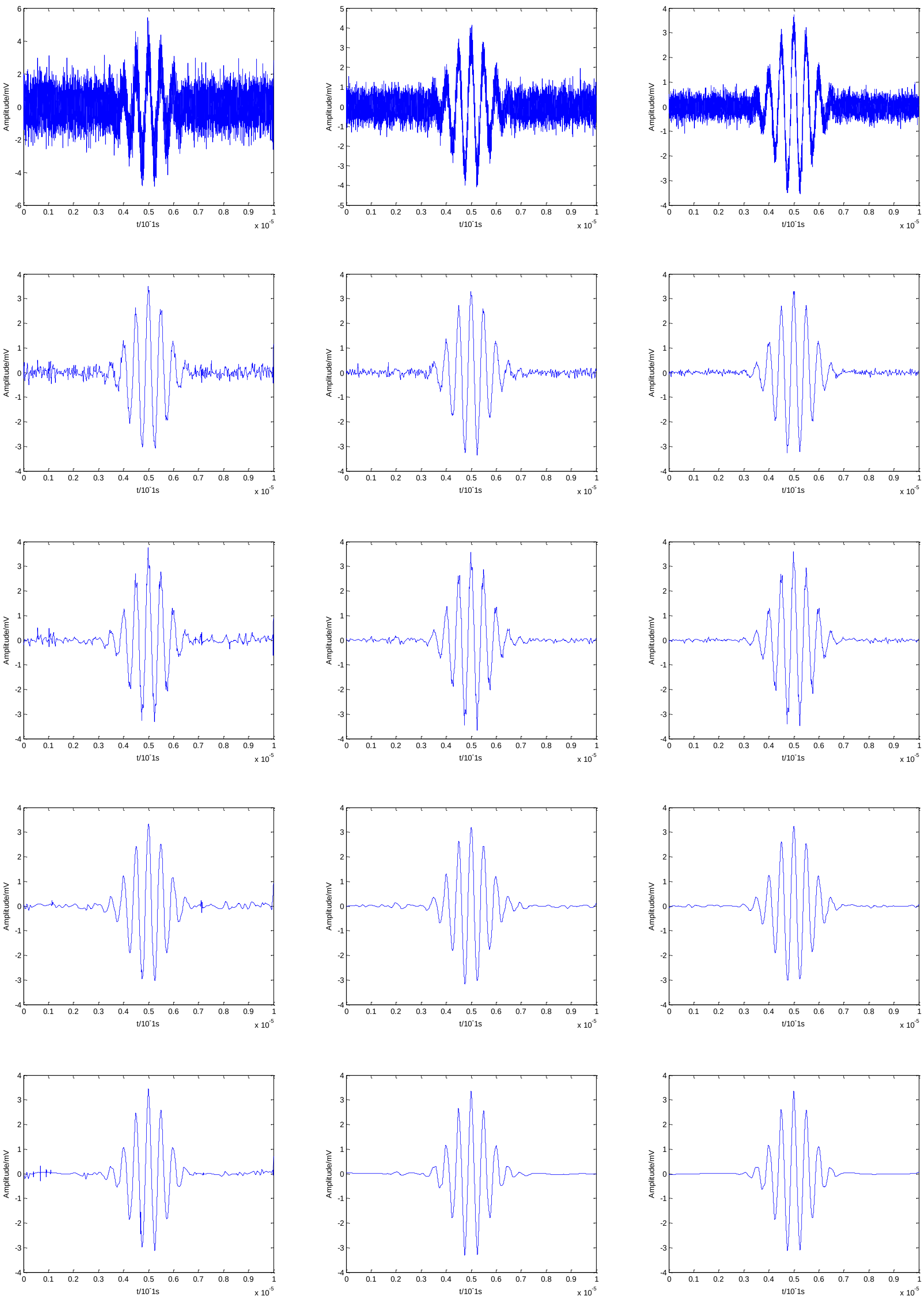
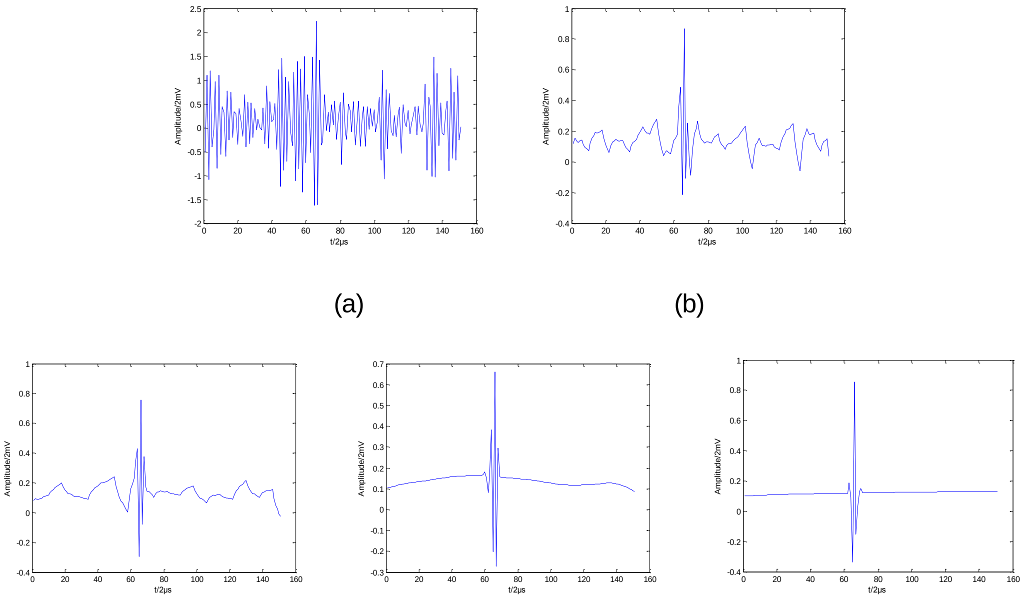


Figure 2. De-Noising Results Obtained by Four Compared Methods on Test Signal with Different Input SNR

The first line is the test signal with input SNR of 0, 5, and 10. The second line is the de-noised waveform by using WT-SM. The third line is the de-noised waveform by using WT-TIM. The fourth line is the de-noised waveform obtained by method in [13]. The final line is the de-noised waveform obtained by the proposed method.

5. Experiment Results and Analysis

In this section, we discuss the experiment results. The specimen in the experiment is a defective slab of γ-TiAl alloy with the thickness of 1cm and deformation of $\pm 0.5\%$. The signal is collected using A-scan display by a probe with center frequency of $5\,MHz$. The visual comparison is given in Figure 3. As we can see in Figure 3, the de-noised results obtained by using WT-SM and WT-TIM are undesirable. The de-noised result obtained by method [13] is better than the above methods, therefore, it can be found that there is some deviation between the 65th and 70th sampling point. Our method can get better de-noised effect, meanwhile, the identification rate of ultrasonic signal can be improved effectively by the proposed method.



(a) (b)

(c) (d) (e)

Figure 3. De-Noised Signal Waveform Using Different Method (A) Ultrasonic Original Signal (B) The De-Noised Waveform by Using WT-SM (C) The De-Noised Waveform by Using WT-TIM. (D) The De-Noised Waveform Obtained by Method in [13]. (E) The De-Noised Waveform Obtained by the Proposed Method

6. Conclusion

In this paper, we have proposed an improved de-noising method for the ultrasonic signal de-noising. The proposed method can overcome the drawback of the traditional methods based on wavelet transformation. We separate the useful information from noise of high frequency by calculating the value of inter-scale correlation of wavelet coefficients, and decide the threshold of high frequency coefficients by combining the wavelet entropy and adaptive threshold, so that the de-noised signal can be effectively reconstructed by this method. Experiment results have shown that the proposed method has better de-noising performance than some other methods in terms of output SNR and SDR. In the future, we will focus on the constructing of threshold function to improve the efficiency and effect of the proposed method.

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Authors

Zhi Cui, He received his M.Sc. in Electronic and Communication Engineering (2012) from Hunan University. Now he is a lecturer at School of Communication and Electronic Engineering, Hunan City University. Science 2014 he is a visiting scholar at the College of Electronic and Information Engineering, Hunan University, supported by the Hunan Provincial Education Department. His current research interests include signal processing, pattern recognition and sparse representation.

Xian-Pu Cui, He is an associate professor at School of Communication and Electronic Engineering, Hunan City University. His research interest is signal processing.