

## VPIT: An Improved Range-Free Localization Algorithm Using Voronoi Diagrams for Wireless Sensor Networks

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### Abstract

*Localization systems have been identified as key issues in the development and operation of wireless sensor networks. The global positioning system (GPS) solves the problem about localization, but it is not suitable for wireless sensor networks. Due to range-free localization approaches requiring low equipped hardware and easy to implement, it is widely used in WSNs localization system. When the node communication radius increases, the accuracy of area-based localization is reduced. It is important to improve the localization accuracy in this situation. In this paper, to improve the accuracy of the node localization, we propose a new range-free localization using Voronoi diagrams based approximate point-in-triangulation test (APIT) algorithm. We compare our algorithm with APIT. Simulation results show that the VPIT improved the precision of localization by narrowing the node's geometry region.*

**Keywords:** *range-free localization, Voronoi diagrams, wireless sensor networks*

### 1. Introduction

Localization is a fundamental issue of wireless sensor networks and it's vital to many applications, such as battlefield surveillance [1], environmental monitoring [2], target tracking [3]. Furthermore, many routing and management protocols, proposed such a network is based on the assumption that the geographic parameters of the sensor nodes are available. Due to the severe limited resource available at each tiny low-cost sensor node, node self-localization is a challenging problem.

Range-based methods measure the distance among the nodes with diverse ranging techniques [4-5]. Although range-based approach can be accurate, hardware are expensive. Range-free approach proposed smart ideas to balance the expenditure and localization accuracy. As considering the former, range-free localization applies widely. A well-known localization algorithm is approximate point-in-triangulation test (APIT). APIT localization system works through reducing the possible area in which a target unknown node resides with anchor nodes. The edge effect leads to In-To-Out Error and Out-To-In Error the major error. This work is motivated by the need for accurate location information when the communication radius of nodes increases. When the node communication radius increases, the overlapping area will also increase. The reason why is that positions of nodes are defined by the center of mass of the polygon currently. When the polygon becomes bigger, the lower accuracy of the localization will be. To make sure that the result of localization is more precise, we propose an improved range-free localization algorithm using Voronoi diagrams (VPIT).

The rest of the paper is organized as follows: The Section 2 briefly reviews the previous work in localization for WSNs. In the next Section, we describe the design of

VPIT. The Section 4 describes the setting of our simulation and we evaluate our scheme with APIT in the Section 5. Finally, we conclude in Section 6.

## 2. Related Work

In wireless sensor network, previous work about localization is falling into categories: range-based and range-free localization.

Range-based methods estimate absolute distances or angles among randomly deployed sensor nodes with certain ranging techniques and then calculated with triangulation or multilateration. In this category, there are many solutions such as time of arrival (ToA) [6], time difference of arrival (TDoA) [7-8] and angle of arrival (AoA) [4, 9]. All those accurated approaches require extra hardware support.

Range-free methods try to estimate node location with a low-cost system design. Such as Centroid [10], DV-Hop [11] and APIT [12], mainly depend on connectivity measurements from anchor nodes to the others.

Boukerche *et al.* [13] have proposed a difference DV-Hop localization system: Distributed Voronoi Localization (DV-Loc). DV-Loc shows how Voronoi diagrams can be applied efficiently for scaling a DV-Hop algorithm while maintaining and reduce further DV-Hop's localization error.

The basic idea of the APIT localization algorithm: each unknown node monitor the information of nearby anchor nodes, assume that the number of nearby anchor nodes is  $n$ , every three anchor nodes out of the  $n$  anchor nodes form a triangle, there are  $C_n^3$  kinds of different combinations. Individually test whether the unknown node is located inside the triangle or not, finally find the centroid of the coincidence region of all the triangles including the unknown node. Then the location of the centroid can be regarded as the estimated location of the unknown node.

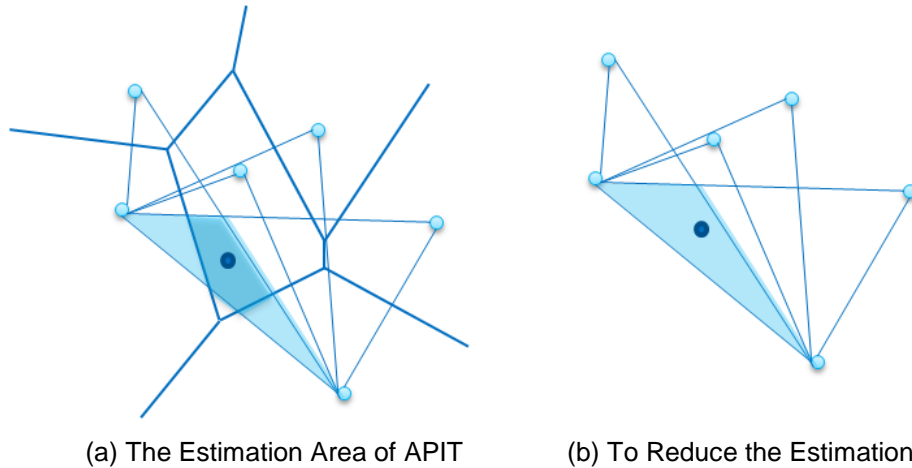
## 3. Main Algorithm

### 3.1. Motivation

Although RSSI is closely related to the proximity of nodes, directly converting RSSI to physical distance estimation is unacceptable in many scenes because of unknown radio path loss factors, multipath effects, hardware discrepancies, antenna orientation, *etc.* The distance-related information according to RSSI measurements instead of the directly ranging information will be more reliable.

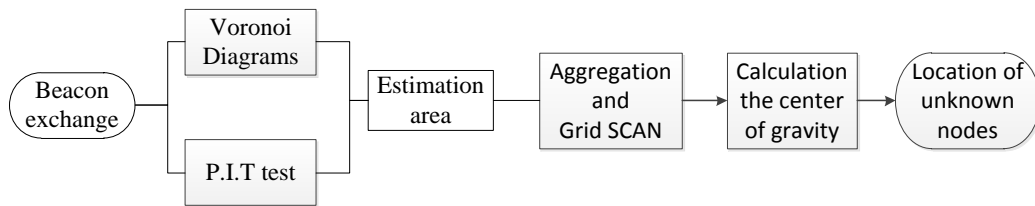
The area-based localization approach use the location and proximity information of anchor nodes to give the residence area of the remaining nodes. The residence area represents a geometric region which the node is in. The basic idea of area estimation is to compute the intersection of all overlapping coverage regions and choose the centroid as the location estimate.

APIT is an area-based localization algorithm leveraging the potential proximity estimation of RSSI. By utilizing the geometric relationship, thus reduce the area of all the possible area, eventually the precision of estimation position of target node would be improved. An example is shown in Figure 1, light blue point is the anchor nodes, and dark blue point is the unknown node (*i.e.* target node). The shadow part in Figure 1(a) is obtained by APIT algorithm possible area. By using the Voronoi diagrams to reduce the shadow area, as shown in Figure 1 (b), the possible area of target node diminished.



**Figure 1. Making Full Use of the Geometric Relationships as Motivation**

We consider locating a network of wireless nodes on a two dimensional plane by using the connectivity information and RSSI readings. A few sensor nodes called anchors which know their own location information via GPS or manual pre-loading. The design of VPIT mainly consist of P.I.T test and Voronoi diagrams, then calculate the overlapping region center of gravity. Figure 1 illustrates the VPIT workflow.



**Figure 1. The Workflow of VPIT**

### 3.2. Anchor Exchange Beacon Message for Voronoi Diagrams

Each anchor broadcasting a beacon message which including Anchor ID, Location, Signal Strength for each anchor heard. Before describing the VPIT scheme proposed in this paper, it is appropriate to introduce two definitions relating to the Voronoi sites.

#### 3.2.1. Problem Formulation

Definition1: Let  $P$  be a set of points in a two-dimensional Euclidean plane. These points are called sites.

$$P = \{p_i, i = 1, \dots, n\}$$

Definition 2: Let the half plane  $H(p_i, p_j)$  be defined such that the Euclidean distance between the point  $p_i$  and any point  $x$  is shorter than that between any other point  $p_j$  and the same point  $x$  in the two-dimensional plane.

In wireless sensor network, the set of anchors  $A$  is treated as the set of Voronoi sites  $P$  in the Voronoi diagram. Namely, the location of the sensors  $L\{a_i\}$  is given by  $L\{p_i\}$  for all  $i$ , that is

$$H(p_i, p_j) = \{x | d(p_i, x) \leq d(p_j, x), i = 1, \dots, |P|, i \neq j\}$$

In the following part, we briefly describe the procedure for constructing the Voronoi diagram. The Voronoi cell  $V(c_i)$  for anchor  $a_i$  is created by the intersection of all the

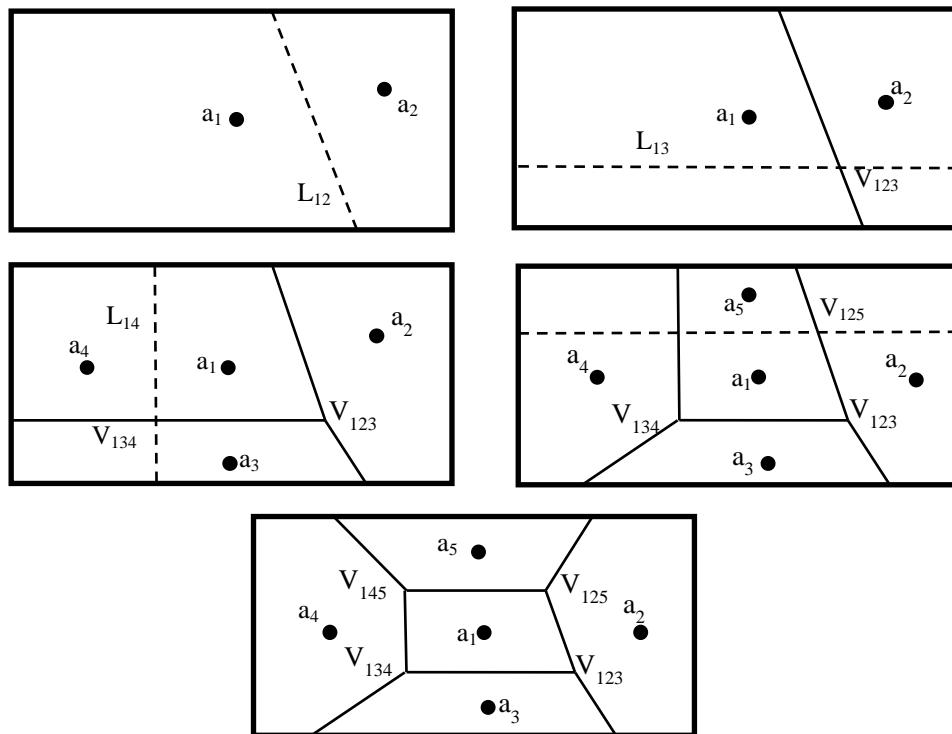
half planes of  $a_i$ . From the previous definitions, the Voronoi cell  $V(c_i)$  is formed by the intersection of all the half planes of the sites  $a_i$ , where  $i = 1, \dots, |A|$  that is

$$V(c_i) = \bigcap_{i=1, \dots, |A|, i \neq j} H(a_i, a_j)$$

Since then, the plane is divided by the full Voronoi diagram  $V(C)$ , into  $|S|$  independent cells for which each site  $a_i$  and any point  $x$  within the Voronoi cell  $V(c_i)$  satisfy, that is

$$V(C) = \{V(c_i), i=1, \dots, |S|\}$$

### 3.2.2. Formulation of Voronoi Diagrams



**Figure 2. Formulation of Voronoi Diagrams**

Each sensor determines the Voronoi cell within, using a neighboring node discovery procedure. A simple example is shown in Figure 3. In this Figure, after anchor node traverse the network, the forming produce of the Voronoi diagram involves the following steps:

First, anchor  $a_1$  determines the perpendicular bisector  $L_{12}$  after discovering neighboring anchor  $a_2$ .

Second, anchor  $a_1$  determines the intersection of  $L_{12}$  and  $L_{13}$  after discovering neighboring anchor  $a_3$ . The intersection of  $L_{12}$  and  $L_{13}$ , that is  $V_{123}$ , is a shared Voronoi vertex of anchor  $a_1, a_2$  and  $a_3$ .

Third, vertices  $V_{145}$  and  $V_{125}$  are determined by using the same procedure after anchor  $a_1$  has discovered  $a_4$  and  $a_5$ . Finally, the Voronoi cell of sensor  $a_1$ , that is  $V(c_1)$ , is defined by the Voronoi vertices  $V_{123}, V_{134}, V_{145}$  and  $V_{125}$ .

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**Algorithm 1. Formulation of Voronoi diagrams**

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**Input:** A set  $A := \{a_1, \dots, a_n\}$  of points sites on a 2D plane.

**Output:** The Voronoi diagram  $V(a)$  given inside a bounding box in an edge list.

**Steps:**

1. Initialize the event queue  $Q$  with all site events, initialize an empty status structure  $T$  and an empty edge list  $D$ .
  2. Let  $R_a$  be the region covered by site  $a$ .
  3. Let  $C_{pq}$  be the boundary ray between sites  $p$  and  $q$ .
  4. Let  $a_1, a_2, \dots, a_m$  be the sites with minimal  $y$ -coordinate, ordered by  $x$ -coordinate  
 $Q \leftarrow S - a_1, a_2, \dots, a_m$
  5. Create initial vertical boundary rays  
 $C_{a_1, a_2}^0, C_{a_2, a_3}^0, \dots, C_{a_{m-1}, a_m}^0 \quad T \leftarrow *(R_{a_1}), C_{a_1, a_2}^0, *(R_{a_2}), C_{a_2, a_3}^0, \dots, *(R_{a_{m-1}}), C_{a_{m-1}, a_m}^0, *(R_{a_m})$
  6. **while not** IsEmpty( $Q$ ) **do**
  7.      $a \leftarrow DeleteMin(Q)$
  8.     **if**  $a$  is a site in  $*(V)$
  9.         Find the occurrence of a region  $*(R_a)$  in  $T$  containing  $a$ ,
  10.         bracketed by  $C_{rq}$  on the left and  $C_{qs}$  on the right
  11.         Create new boundary rays  $C_{pq}^-$  and  $C_{pq}^+$  with bases  $a$
  12.         Replace  $*(R_a)$  with  $*(R_q), C_{pq}^-, *(R_p), C_{pq}^+, *(R_q)$  in  $T$
  13.         Delete from  $Q$  any intersection between  $C_{rq}$  and  $C_{qs}$
  14.         Insert into  $Q$  any intersection between  $C_{rq}$  and  $C_{pq}^-$
  15.         Insert into  $Q$  any intersection between  $C_{pq}^+$  and  $C_{qs}$
  16.     **else**  $a$  is a Voronoi vertex in  $*(V)$
  17.         Let  $a$  be the intersection of  $C_{qr}$  on the left and  $C_{rs}$  on the right
  18.         Let  $C_{uq}$  be the left neighbor of  $C_{qr}$  and let  $C_{sv}$  be the right neighbor of  $C_{rs}$  in  
 $T$
  19.         **if**  $q_y = s_y$
  20.             Create a new boundary ray  $C_{qs}^0$
  21.         **else**
  22.             **if**  $p$  is right of the higher of  $q$  and  $s$ , create  $C_{qs}^+$
  23.             **else** create  $C_{qs}^-$
  24.         Replace  $C_{qr}, *(R_r), C_{rs}$  with newly created  $C_{qs}$  in  $T$
  25.         Delete from  $Q$  any intersection between  $C_{uq}$  and  $C_{qr}$
  26.         Delete from  $Q$  any intersection between  $C_{rs}$  and  $C_{sv}$
  27.         Insert into  $Q$  any intersection between  $C_{uq}$  and  $C_{qs}$
  28.         Insert into  $Q$  any intersection between  $C_{qs}$  and  $C_{sv}$
  29.         Record  $a$  as the summit of  $C_{qr}, C_{rs}$  and the base of  $C_{qs}$
  30. **endwhile**
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### 3.3. The Proximity Area of Unknown Nodes

After the anchor Voronoi diagram has formed, the unknown nodes decided which Voronoi cell belongs to them. In two-dimensional plane, the Voronoi diagram of discrete sites partitions the plane into a set of convex polygons so that all points inside a polygon are closest to only one site. Each anchor broadcasts a beacon message contains its location information and those unknown sensors which can deliver the message among each other within the Voronoi cell (polygon vertices). Upon receiving the beacon, each unknown sensor  $s$ , constructs its neighbor anchors list denoted by ALs. Each row in the ALs includes: the Anchor's ID, the Anchor's Location, the anchor's Voronoi cell and the RSSI corresponding to the received beacon message from the anchor.

The Voronoi cell of the nearest anchor, which has the strongest RSSI in ALs, represents the initial sensor's residence area. Let's present an example to explain our VPIT algorithm.

Having received beacons from anchors  $a_1, a_2, a_3$  and so on, each node maintains a table in the ALs (Table 1).

**Table 1. Table of Heard Anchors**

	(X,Y)		SS
$a_1$	18	19	1mv
$a_2$	34	46	2mv
$a_3$	24	57	3mv
· ⋮	⋮	⋮	⋮

Node 1

Each node beacons once to exchange anchor tables with its neighbors. After every node has maintained neighborhood state, these tables are merged as Table2.

The algorithm runs on every column of the node's table to determine whether a neighboring node has consistently larger/smaller signal strengths from the three anchors  $a_1, a_2$  and  $a_3$ . If such a neighbor is found, M assumes that it is outside triangle composed of randomly three anchors  $a_1, a_2$  and  $a_3$ . If no such neighbor is found, M assumes it is inside this region.

**Table 2. Combined Table**

	(X,Y)		$SS_1$	$SS_2$	...	$SS_n$
$a_1$	18	19	1mv	2mv		4mv
$a_2$	34	46	2mv	3mv		7mv
$a_3$	24	57	3mv	1mv		5mv
⋮	⋮	⋮	⋮	⋮		⋮

Node M

Each node repeats the step for varying combinations of three anchors, and then used to determine the area with maximum overlap. Finally, the center of gravity of this area is used as the final location estimation.

### 3.4. Calculation the Center of Gravity Based on Grid SCAN

After the individual tests of unknown node finish, VPIT aggregates the results the proximity area of unknown node through a grid SCAN algorithm. In this algorithm, a grid array is used to represent the maximum area in which a node will likely reside. In our experiments, the length of a grid side is set to  $0.1R$ , to guarantee that estimation accuracy is not noticeably compromised.

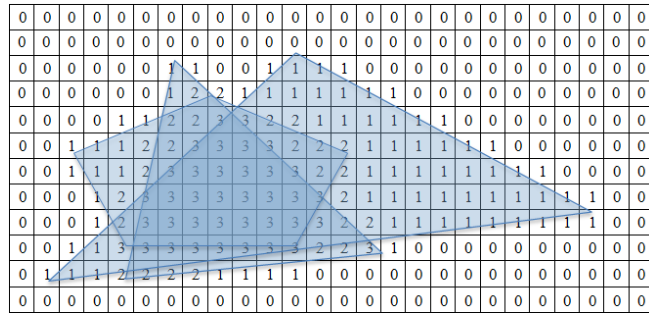


Figure 5. Grid SCAN Approach

For each VPIT inside decision (a decision where the test determines the node is inside a particular region) the values of the grid regions over which the corresponding polygon resides are incremented. For an outside decision, the grid area is similarly decremented. Once all the polygon regions are computed, leading to finding the maximum overlapping area and calculating the center of gravity for position estimation.

#### 4. Simulation Settings

This section describes the simulation settings we use in our evaluation.

##### 4.1. Radio Transmission Model

In our simulation, two common transmission models are investigated, respectively Regular Model and RIM Model.

Regular Model:

$$P_R(d) = P_T - PL(d_0) - 10\eta \log_{10}(\bar{d}_0)$$

RIM Model:

$$P_R(d) = P_T - PL(d_0) - 10 \log_{10}(\bar{d}_0) * K_i + X_\sigma$$

$$P_R(d) = P_T - PL(d_0) - 10\eta \log_{10}(\bar{d}_0) * K_i$$

Where PR is the received signal power, PT is the transmit power, and  $PL(d_0)$  is the path loss for a reference distance of  $d_0$ .  $X_\sigma = N(0, \sigma^2)$ , a random variation, is expressed as the fading component of RSS.

##### 4.2. System Parameters

In our experiments, we study several system-wide parameters those have a direct effect on estimation error in range-free localization algorithms. The description of these parameters is as follows:

Suppose the anchor node no error of GPS.

Node Density (ND): The average number of nodes within the scope of a node communication.

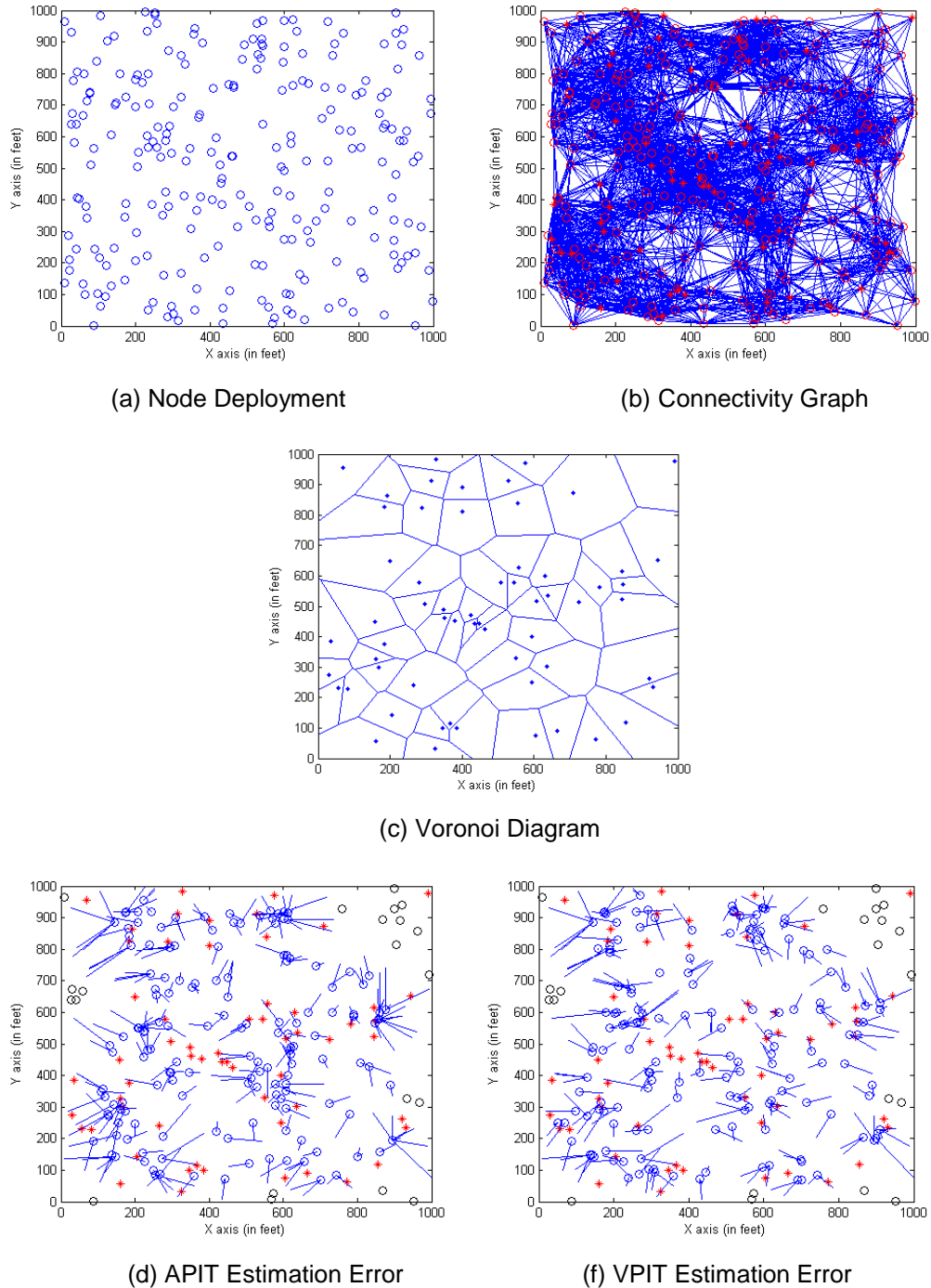
Anchor Heard (AH): Average number of anchors heard by a node and used during estimation.

Anchor to Node Range Ratio (ANR): The average distance an anchor beacon travels divided by the average distance a regular node signal travels. When this value equals one, the anchor and nodes have the same average radio range. The larger this value, the fewer anchors required to maintain a desired AH value.

## 5. Experiment and Evaluation

This section provides a detailed quantitative analysis compared to the performance of the APIT algorithms described in above.

Figure 6 (a) illustrates 300 nodes randomly deployed in the rectangular area. Figure 6 (b) shows 1-hop links of the network with line segments and Figure 6 (c) shows the constructed Voronoi cells. Figure 6 (d) and (e) show the result from APIT and VPIT. The obvious metric for comparison when evaluating localization schemes is location estimation error. We have conducted a variety of experiments to cover a wide range of system configurations including varying 1) Anchors Heard, 2) Node Density.

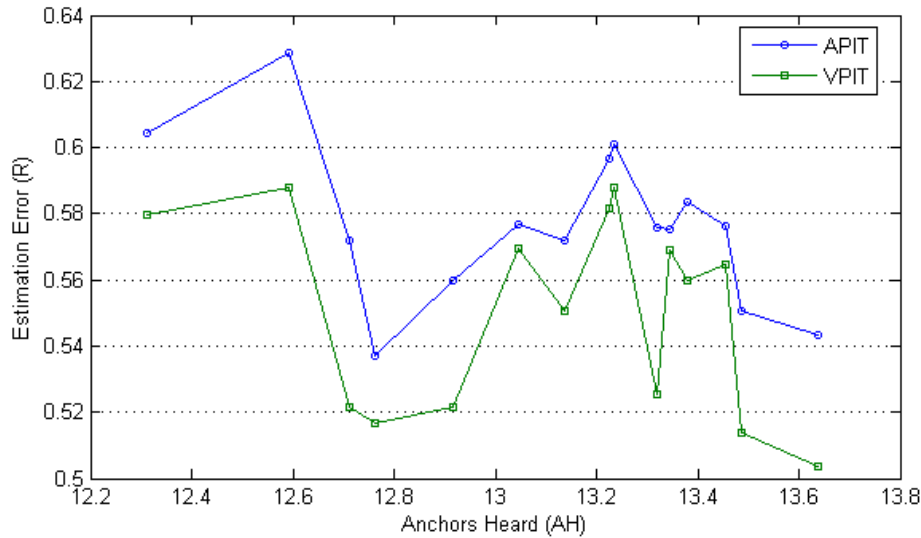


**Figure 6. Localization: APIT Versus VPIT**



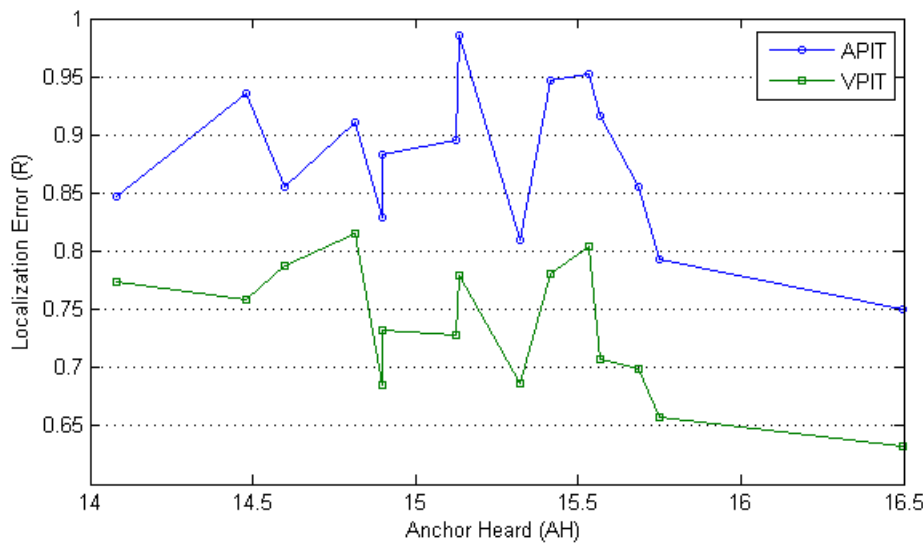
### 5.1. Localization Error when Varying AH

In this experiment, we scrutinize the consequences of fluctuating number of anchors heard (AH) at a node to determine its effect on the localization error. To avoid the effects of other parameters, we set the same parameter of the experiment and multiple randomized trials to obtain the result.



**Figure 7. Regular Model, ANR = 0.01, Random**

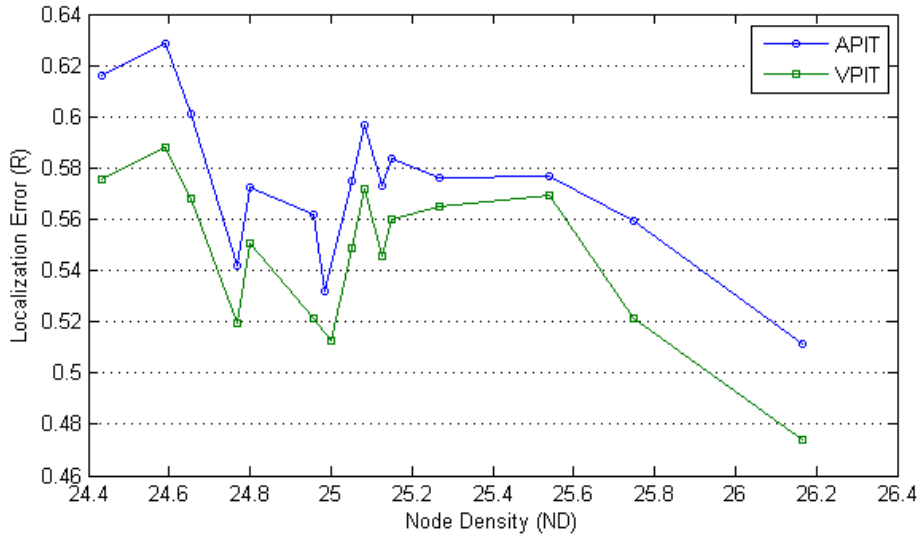
In this simulation, 300 nodes are randomly deployed in a rectangular area, and including 20 percent of anchor nodes. Respectively under the different communication models, we compare our algorithm with APIT in estimation error (R) with the fixed parameters of ANR (Anchor to Node Range Ratio). We used two difference radio transmission models. The Figure 7 shows the result with the regular model and the Figure 8 with the RIM model. However, both Figure 7 and Figure 8 show that comprehensive estimation error decreases as the number of anchors heard increases. Compared with APIT, the estimation error of our algorithm is lesser and performs better.



**Figure 8. RIM Model, DOI = 0.01, ANR = 1.5, Random**

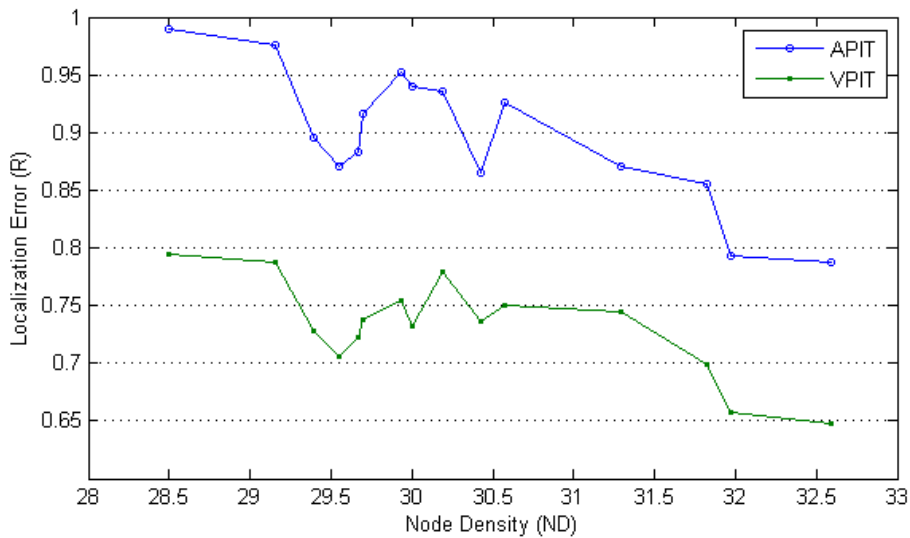
### 5.2. Localization Error when Varying ND

We scrutinize the consequences of fluctuating number of node density (ND) at a node to determine its effect on localization error. By setting the same parameter of the experiment and multiple randomized trials avoid the effects of other parameters.



**Figure 9. Regular Model, ANR = 0.01, Random**

We use two difference radio transmission models. The Figure 9 shows the result of the regular model and the Figure 10 with the RIM model. The parameter of ANR is setted to 0.01. As setted the same parameter and the abscissa value changing relatively dense, leading to volatile estimation error change. However, both Figure 9 and Figure 10 show that comprehensive estimation error decreases as the number of node density increases. Compared with APIT, the estimation error of our algorithm is lesser and performs better.



**Figure 10. RIM Model, DOI = 0.01, ANR = 1.5, Random**

## 6. Conclusion

This paper proposes VPIT, an improved range-free localization algorithm based on traditional APIT algorithm and Voronoi diagrams.

Given the inherent constraints of the sensor devices envisioned and the estimation accuracy desired by location dependent applications, range-free localization schemes are regarded as a economical and sufficient solution for localization in sensor networks. From our extensive comparison study, we identify preferable system configurations of range-free localization schemes as a design guideline for further research. We have improved APIT scheme particularly, which proposed in this paper is better.

## Acknowledgements

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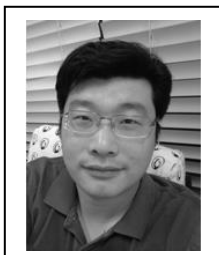
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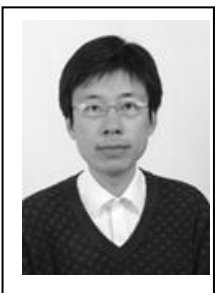
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