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Highly Reliable Product Code for Error Correction

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Abstract

In storage and communication systems, the product and concatenation codes, which consist of Reed–Solomon (RS) codes and other error-correction codes, can be utilized to effectively correct burst errors by performing erasures-and-errors decoding. However, the RS codes have two drawbacks: 1) the RS decoders may not detect received erroneous codewords, and 2) the RS decoders may generate valid codewords which are not equal to the transmitted codewords. In the implementation of the product and concatenation codes, such decoding drawbacks can cause the RS code to provide incorrect error-detection information or erasure information to other error-correction codes, resulting in unreliable error-correction performance of the systems. In this study, we propose a new product code to overcome these drawbacks by studying the RS product code (RSPC). The new code combines CRC code with RSPC which is called as CRSPC. In addition, the results of experiments show that the new CRSPC can significantly reduce the probabilities of these drawbacks, thereby improving the performance of erasures-and-errors decoding. With minor modifications, the CRSPC algorithm can be applied to Blue-ray disc systems which have powerful recording functions in multimedia.

Keywords: *Reed Solomon; Product code; Decoding error; Erasures-and-errors decoding*

1. Introduction

The demand for large-scale storage recording systems increases rapidly due to the strong growth of digital communications. To satisfy this demand, the capacity and recording density of storage systems should be increased. However, the same scratches on the surfaces, the intertrack interference and the missynchronization of those storage systems can cause more serious burst errors than those of the conventional storage systems [1-3]. Moreover, in multimedia communications, packet loss may lead to burst errors. Therefore, powerful burst-error correction codes are required.

The Reed–Solomon (RS) codes have been proven as good compromises between implementation complexity and error correction capability [4]. RS codes have been exploited in many applications such as Wireless Sensor Networks [5-6], digital broadcasting systems [7-8] and other networks [9]. In order to correct burst errors, some product codes based on RS codes have been proposed. For instance, DVD systems utilize the RS product-code (RSPC) as an error-correction code [10]. Blue-ray disc systems employ the Picket code to correct errors which consists of two RS codes: a long distance code (LDC) and a burst indicator subcode (BIS) [11]. Moreover, a Blue-ray disc can store 50GB of high-definition information and plays an important role in Multimedia recording.

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Yang proposed a product code to protect NAND flash memories, in which RS codes and Hamming codes are used along rows and columns respectively [12]. For future magnetic recording channel applications, Van proposed a product code which consists of RS codes and LDPC codes [13]. In addition, some communication systems utilize concatenation codes based on RS codes to improve the error-correction performance. For instance, Sethakaset presented a concatenation of marker and RS code to correct insertion/deletion errors in differential pulse-position modulation over optical wireless communications [14]. Gho proposed a rate-adaptive optical transmission scheme utilizing a concatenated RS-RS code as variable-rate error-correction code [15]. Liu presented an LDPC-RS product code for digital terrestrial broadcasting transmission systems [16].

In these cases, the product and concatenation codes require RS codes to provide reliable error-detection or erasure information. In particular, the RSPC and Picket codes employ erasures-and-errors decoding which requires that some RS codes should supply accurate erasure information to others [17-18]. It should be noted that the erasures-and-errors decoding plays an important role in RS codes [19].

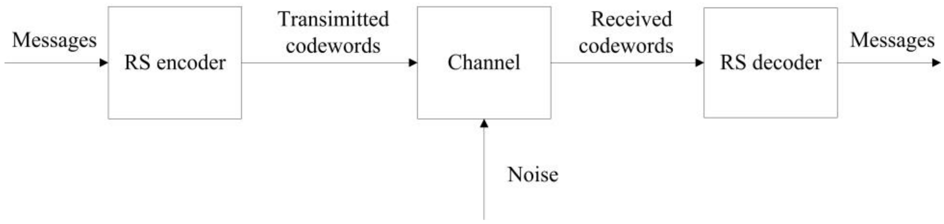


Figure 1. Simple Diagram of Codeword Transmission

However, the RS codes have two drawbacks: an undetected error and a decoding error. Next we briefly introduce these drawbacks. A diagram of codeword (a codeword is a set of fixed length message) transmission based on RS codes over a noisy channel is depicted in Figure 1. First, if a received codeword is disturbed by some noise to be a valid codeword that differs from the transmitted codeword, the RS decoder regards no errors occurring and cannot detect the received erroneous codeword. This drawback is referred to as an undetected error [20]. Second, if a received erroneous codeword is decoded by the RS decoder to a valid codeword that is not equal to the transmitted codeword, the RS decoder regards this decoding as a successful decoding. This drawback is called as a decoding error [21]. The occurrence of either of two drawbacks can cause the RS codes to supply inaccurate error-detection or erasure information, which can degrade the performance of product codes. Recently, many new schemes have been proposed to improve the performance of RS codes. Almawgani introduced a three-user RS coded cooperation scheme to have simple encoding/decoding complexity [22]. Wachter-Zeh proposed an approach to increase the error correcting capability of interleaved RS codes and derived the upper bound failure probability [23]. In this study, we focus on how the two drawbacks impact the error-correction performance of RSPC, and propose a CRC and RS product code (CRSPC) to combat the drawbacks.

The rest of this study is organized as follows. In Section 2, we show the upper bound probabilities of undetected errors and decoding errors in RS codes. In Section 3, we demonstrate how the drawbacks degrade the performance of RSPC and propose the new CRSPC. Section 4 provides theoretical and experimental comparisons between the RSPC and CRSPC. The conclusion is given in Section 5.

2. The Upper Bound Probabilities of the Two Drawbacks

An RS code can be represented by (n, k), where n is the number of encoded symbols and k is the number of message symbols. A symbol has m bits. The RS code (n, k) can correct $t = (n - k)/2$ symbol errors.

2.1. The Upper Bound Probability of Undetected Error

To derive the probability of an undetected error, we give an explanation why the undetected error occurs. The RS code (n, k) has 2^n – 1 nonzero erroneous codewords and 2^k – 1 nonzero valid codewords. Thus, there are 2^k – 1 erroneous codewords that are identical to the 2^k – 1 nonzero codewords. The occurrence of any of these erroneous codewords in a channel will convert a transmitted codeword **C** into another valid codeword **U.** In such case, the RS decoder accept **U** as the transmitted codeword, resulting in an undetected error.

Then, we consider an RS (n, k) code transmitting over a q-ary symmetric channel. In addition, each transmitted symbol is received correctly with probability 1 – p and converted with equal probability $p/(q – 1)$ to any of other q – 1 symbols. The probability of undetectable error of an RS code is given by

$$P_{ud}(p) = \sum_{w=1}^{n} A_w \left(\frac{p}{q-1} \right)^w (1-p)^{n-w} \quad , \tag{1}$$

Where A_w is the number of codewords with Hamming weight w. When $p = (q – 1)/q$, the upper bound probability of undetectable error P_{ud} is $q^{-(n-k)}$ [24].

2.2. The Upper Bound Probability of Decoding Error

As discussed above, we first provide an explanation of why a decoding error occurs. The decoding area of an RS code (n, k) can be pictured as a sphere of radius t, with the center at the RS code. If a transmitted codeword **C** is received erroneously as codeword **R**, and **R** reaches into the decoding area of another valid codeword **D** that is not equivalent to **C**, the RS decoder will decode **R** to **D** rather than the transmitted codeword **C**, resulting in a decoding error.

Second, we present a specific example of decoding error. The RS code we employed is RS (15,11), which can correct up to two symbols. The symbol has 4 bits.

For RS (15,11), let the primitive polynomial be

$$p(x) = x^4 + x + 1 \ , \tag{2}$$

and the generator polynomial be

$$g(x) = (x-a)(x-a^2)(x-a^3)(x-a^4) \ . \tag{3}$$

Let the original message be [1-3,11]. After RS (15,11) encoding, the transmitted codeword is [1-11 11 10 14 6]. And then, we add three random errors to the transmitted codeword, resulting in the received codeword [1-11 11 0 0]. Applying RS (15,11) decoding, the decoded codeword is [1 2 12 0 5 6-10 11 11 0 0 0], which has five symbol errors compared with the transmitted codeword. But the RS (15,11) decoder regards this decoding process as a successful decoding. This example shows that the decoding error occurs and results in error propagation (leading to more errors).

At last, the probability of decoding error of an RS code is given by [25]

$$P_{de}(p) = \sum_{w=0}^{n} \left(\frac{p}{q-1} \right)^w (1-p)^{n-w} \sum_{s=0}^{t} \sum_{v=1}^{n} A_v N(n,v,w,s) \ , \tag{4}$$

Where A_v is the number of codewords with Hamming weight v and $N(n,v,w,s)$ is the number of erroneous codewords of weight v that are at a distance s from codewords of weight w:

$$N(n,v,w,s)=\sum_{\substack{0\le i\le n\\0\le j\le n\\i+2j+w=s+v}}\binom{n-v}{j+w-v}\binom{v}{i}\binom{v-i}{j}(q-1)^{j+w-v}(q-2)^{i} \quad (5)$$

According to [26], it is shown that the upper bound of the probability of decoding error P_{de} is $\frac{1}{t!}$.

3. New Product Code

3.1. The Drawbacks Degrade the Performance of RSPC

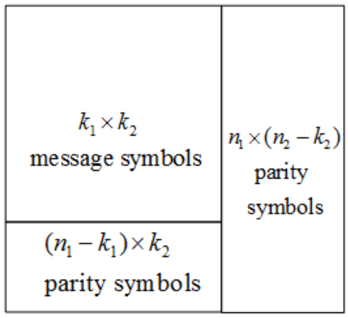


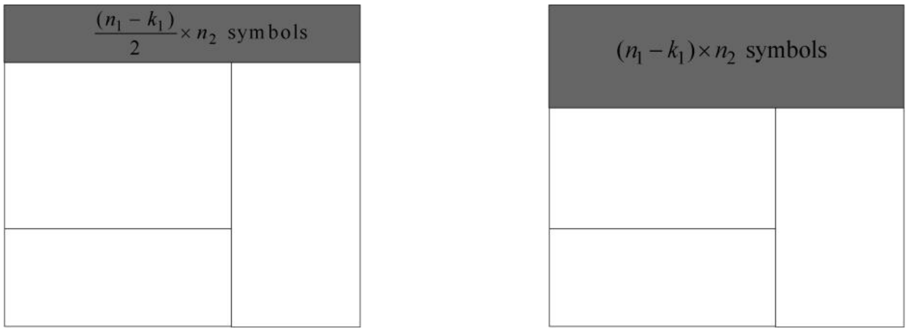
Figure 2. Structure of the RSPC

An RSPC is a two-dimensional code composed of two RS codes: an RS (n_1, k_1) and an RS (n_2, k_2). Each RSPC symbol has m bits. In the structure of the RSPC shown in Figure 2, a set of k_1k_2 message symbols is arranged in a $k_1 \times k_2$ array.

The encoding of RSPC includes two steps. First, by using the encoding of RS (n_1, k_1) in the column direction, $(n_1 - k_1)\times k_2$ parity symbols are generated. The $k_1 \times k_2$ array becomes an $n_1 \times k_2$ array. Second, by using the encoding of RS (n_2, k_2) in the row direction, $n_1 \times (n_2 - k_2)$ parity symbols are generated. The $n_1 \times k_2$ array becomes an $n_1 \times n_2$ array.

The erasures-and-errors decoding of the RSPC is also consists of two steps. First, the RS (n_2, k_2) is applied to decode each row. If the errors occurring in a row cannot be corrected, the row is marked as an erasure. Then, taking these row erasures into account, the RS (n_1, k_1) uses erasures-and-errors decoding to each column.

Next, we provide a specific comparison between error and erasures-and-errors decoding to show the advantage of erasures-and-errors decoding. An RS code with t error correction capability is capable of correcting v errors and e erasures, where $2v+e \le 2t$ [27]. In other words, the maximum correctable burst length (MCBL) by error decoding of RS codes is t, while the MCBL by erasures-and-errors decoding of RS codes is $2t$. Figure 3.a illustrates the $(n_1 - k_1)/2\times n_2$ consecutive symbols corrupted by burst errors. In such case, $(n_1 - k_1)/2$ errors are within the correction capability of RS (n_1, k_1) and can be corrected. However, if more than $(n_1 - k_1)/2$ errors occur, the RS (n_1, k_1) cannot correct the errors only by applying error decoding. Thus, the MCBL of RSPC by error decoding is $(n_1 - k_1)n_2m/2$ bits. Figure 3.b illustrates the $(n_1 - k_1)/2$ consecutive symbols corrupted by burst errors. By erasures-and-errors decoding, the RS (n_1, k_1) can correct the $n_1 - k_1$ errors, but fails to correct more errors. Thus, we derive that the MCBL of RSPC is $(n_1 - k_1)n_2m$ bits by applying erasures-and-errors decoding, at the cost of the requirements of erasure information.



a b

Figure 3. Gray Parts Corrupted by Consecutive Burst Errors

Therefore, it is apparent that the erasure information plays a significant role in the RSPC. However, the occurrence of the two drawbacks results in that the RS (n_2, k_2) cannot supply accurate erasure information for the RS (n_1, k_1). In such case, the error-correction capability of the RSPC decreases, and even the application of iterative decoding may not be able to improve the capability because the two drawbacks may lead to error propagation.

3.2. The Structure of the CRSPC

To lower the probabilities of the two drawbacks, we propose a CRSPC in which the CRC algorithm is applied to the RSPC. Figure 4 shows the structure of the proposed CRSPC.

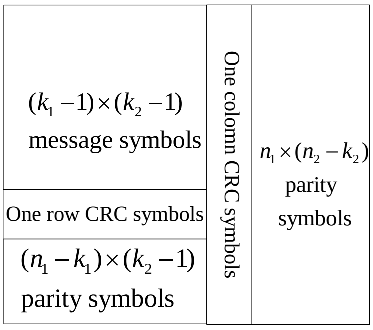


Figure 4. Structure of the CRSPC

CRC is not only simple to implement, but has the benefit of being particularly well-suited to the detection of burst errors. CRC can detect all single, double errors, any odd number of errors and most burst errors. A CRC code can be represented by (s, r), and the upper bound of undetected error probability of CRC is 2^{s-r}, where an s-bit message is taken to create an r-bit codeword. The parameters of row and column CRC used in CRSPC are $((k_1 - 1)m, k_1m)$ and $((k_2 - 1)m, k_2m)$, respectively. In particular, the check bits generated by row and column CRC are m bits.

The encoding procedures of CRSPC are described as follows:

1) A set of $(k_1 - 1)(k_2 - 1)$ message symbols is arranged in a $(k_1 - 1)\times(k_2 - 1)$ array.
2) Apply column CRC encoding to each column of the $(k_1 - 1)\times(k_2 - 1)$ array, generating one row CRC check symbols. Note that a symbol has m bits. Thus, the $(k_1 - 1)\times(k_2 - 1)$ array becomes a $k_1 \times (k_2 - 1)$ array.
3) Apply RS (n_1, k_1) encoding to each column of the $k_1 \times (k_2 - 1)$ array. And then, the $(n_1 - k_1)\times(k_2 - 1)$ parity symbols are generated, forming an $n_1 \times (k_2 - 1)$ array.

4) Apply row CRC encoding to each row of this $n_1 \times (k_2 - 1)$ array, generating one column CRC check symbols. Thus, the $n_1 \times (k_2 - 1)$ array becomes an $n_1 \times k_2$ array.
5) Apply RS (n_2, k_2) encoding to each row of the $n_1 \times (k_2 - 1)$ array, And then, the $n_1 \times (k_2 - 1)$ parity symbols are generated, forming an $n_1 \times n_2$ array.

Then, the encoded data are transmitted row by row. When the encoded data are received, they are stored in an $n_1 \times n_2$ array.

The decoding procedures of CRSPC are described as follows:

1) Apply RS (n_2, k_2) decoding to each row of the $n_1 \times n_2$ array. If the errors occurring in a row are less than $(n_2 - k_2)/2$, the errors can be corrected. Otherwise, the RS (n_2, k_2) decoding fails to correct the row.
2) Apply row CRC decoding to each row of the $n_1 \times n_2$ array. If the CRC check does not agree with the codeword in a row, which demonstrates that the RS(n_2, k_2) decoding is incorrect, the row is marked as a row erasure. The erasure information obtained in this step by RS (n_2, k_2) decoding and CRC check is much reliable, compared with that only obtained by RS (n_2, k_2) decoding in RSPC.
3) According to the row erasure information, apply the erasures-and-errors decoding of RS (n_1, k_1) to each column of the $n_1 \times (k_2 - 1)$ array.
4) Apply column CRC decoding to each column of the $n_1 \times (k_2 - 1)$ array. If the CRC check does not agree with the codeword in one column, we can come to a conclusion that the RS (n_1, k_1) fails to correct the column. In such case, the codeword of the column will not be changed by RS (n_1, k_1), in order to avoid error propagation.

Note that in the CRSPC, the RS (n_1,k_1) and RS (n_2,k_2) are the same as those of RSPC. Thus, the MCBL of the CRSPC is also $(n_1 - k_1)n_2m$ bits.

4. Performance Comparison between the RSPC and CRSPC

4.1. Comparison in Theory

In the RSPC, the erasures-and-errors decoding of RS (n_1, k_1) requires the erasure information supplied by RS (n_2, k_2). The existence of two drawbacks makes the RS (n_2, k_2) supply inaccurate erasure information to degrade the performance of erasures-and-errors decoding. Thus, as for the CRSPC, we need to verify whether the RS (n_2, k_2) improves the performance of erasures-and-errors decoding or not. We provide a parameter comparison between the RSPC and CRSPC based on the upper bound probabilities of the two drawbacks of RS (n_2, k_2), as shown in Table 1.

Next, we give explanations for Table 1. Code rate is the number of messages divided by the total number of transferred symbols. In addition, m is the number of bits in a symbol, and the upper bound probability of undetected error of CRC used in CRSPC is 2^{-m}. According to Section 2, in the RSPC, the upper bound probability of undetected error P_{ud} of RS (n_2, k_2) is $m^{k_2-n_2}$, and the upper bound probability of decoding error P_{de} of RS(n_2, k_2) is $\frac{1}{(\frac{n_2-k_2}{2})!}$.

Note that in the CRSPC, the CRC decoding comes after the RS decoding. In such case, when both the undetected error of RS(n_2, k_2) and the undetected error of CRC occur, the erasure information supplied by RS(n_2, k_2) is inaccurate. Otherwise, if the undetected error of RS(n_2, k_2) or the undetected error of CRC does not occur simultaneously, the RS(n_2, k_2) can detect the error and supply accurate erasure information for RS(n_1, k_1). Thus, we conclude that the upper bound P_{ud} of RS(n_2, k_2)

in the CRSPC is the multiplication of $m^{k_2-n_2}$ and 2^{-m}. Similarly, the upper bound P_{de} of RS(n_2, k_2) is the multiplication of $\frac{1}{(\frac{n_2-k_2}{2})!}$ and 2^{-m}.

Finally, in Table 1, it can be seen that the CRSPC reduces the upper bound probabilities of the two drawbacks of RS (n_2, k_2). In other words, the undetected error and decoding error occurring in the RS (n_2, k_2) decrease. Thus, the RS (n_2, k_2) in the CRSPC can supply accurate erasure information to improve the performance of erasures-and-errors decoding.

Table 1. Parameter Comparison Between RSPC and CRSPC

	Code rate	Upper bound P_{ud} of RS(n_2, k_2)	Upper bound P_{de} of RS(n_2, k_2)
RSPC	$(k_1k_2)/(n_1n_2)$	$m^{k_2-n_2}$	$\frac{1}{(\frac{n_2-k_2}{2})!}$
CRSPC	$((k_1-1)(k_2-1))/(n_1n_2)$	$m^{k_2-n_2}2^{-m}$	$\frac{1}{(\frac{n_2-k_2}{2})!2^m}$

We know that the RSPC used in DVD systems consists of RS(182,172) and RS(208,192). Then, a specific parameter comparison between the RSPC and CRSPC is obtained, as summarized in Table 2.

Table 2. Parameter Comparison in DVD Systems

	Code rate	Upper bound P_{ud} of RS(182,172)	Upper bound P_{de} of RS(182,172)
RSPC	0.87	9.31×10^{-10}	8.33×10^{-3}
CRSPC	0.86	3.64×10^{-12}	3.26×10^{-5}

As seen in Table 2, the upper bounds of P_{ud} and P_{de} of RS (182,172) are reduced by two orders of magnitude in the CRSPC than those in the RSPC. The RS (182,172) can supply accurate erasure information for the RS (208,192).

4.2. Comparison in Experiment

According to Section 3, the MCBL of RSPC is $(n_1 - k_1)n_2m$ bits. In other words, if consecutive burst errors of length $(n_1 - k_1)n_2m$ occur in the received codewords, the RSPC will correct the burst errors. However, it turns out that, the RSPC may fail to correct the burst errors with some probability owing to the impact of the two drawbacks of RS codes. It is necessary to verify whether the CRSPC can decrease the impact of the drawbacks or not by experiments.

Therefore, an experimental comparison between the RSPC and the CRSPC based on the uncorrected probability of burst errors of length $(n_1 - k_1)n_2m$ is presented. In order to lower computational complexity and for convenience, we let RSPC (n_3,k_3,m) and CRSPC(n_3,k_3,m) represent that both the row RS code and the column RS code are RS (n_3,k_3) code. The m expresses the number of bits in a symbol. Furthermore, both the row CRC and column CRC employed by the CRSPC (n_3,k_3,m) are the same CRC code which generates m check bits. Eight RSPC and CRSPC codes are selected and hundreds of thousands of experiments are carried out to compute the uncorrected probability of burst errors of length $(n_3 - k_3)n_3m$, as shown in Table 3. In particular, the row code and column code in the DVD are RS (182,172) and RS (208,192) respectively, and the length of burst errors is (208 – 192) × 182 × 8=23296 (bits).

Table 3. Experiment Results

Code	Burst errors of length $(n_3 - k_3)n_3m$ (bits)	Uncorrected probability
RSPC(15,11,4)	240	0.6047
CRSPC(15,11,4)	240	0.0890
RSPC(31, 21,5)	1550	0.0422
CRSPC(31,21,5)	1550	0.0013
RSPC(63,53,6)	3780	0.0584
CRSPC(63,53,6)	3780	0.0010
RSPC in DVD	23296	0.1160
CRSPC in DVD	23296	0.0004399

As seen in Table 3, the uncorrected probability of burst errors of length $(n_3 - k_3)n_3m$ decreases significantly by applying CRSPC. In particular, the uncorrected probability obtained by applying CRSPC in DVD is reduced by three orders of magnitude than that obtained by applying RSPC.

5. Conclusions

This study proposes a new product algorithm called CRSPC to overcome the two drawbacks of RS codes which are composed of undetected error and decoding error. To validate the proposed CRSPC algorithm, we conduct theoretical and experimental comparisons between the CRSPC and the RSPC. The comparison results demonstrate that the use of CRSPC can significantly decrease the probabilities of the two drawbacks, and can supply enough accurate erasure information to improve the performance of erasures-and-errors decoding, at a little code rate cost. We can infer that if storage and network systems employ concatenation codes or product codes based on RS codes, the CRSPC algorithm may be applied to those systems to improve the error-correction performance.

Note that the DVD systems employ the RSPC consisting of RS (182,172) and RS (208,192) as an error correction code. Thus, we can apply the CRSPC to the DVD systems to improve the error-correction performance. In Blue-ray disc systems, RS (248,216) supplies error location information and RS (62,30) uses the information to perform erasures-and-errors decoding. Thus, the CRSPC can be also applied to Blue-ray disc systems with minor modifications.

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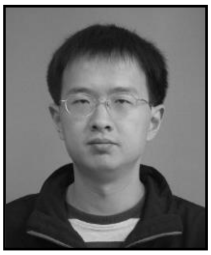

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