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Static Hand Gesture Recognition Method Based on Depth Images with Shape Context Feature

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Abstract

Static hand gesture recognition is very important for human-computer interaction systems, which is widely used in human-computer interaction systems. Using visible RGB images as the input method would be more stable, but the illumination and skin color are big problems; by contrast the depth images is a better choice. A novel algorithm which recognizes static hand gesture based on depth images using 3d shape context feature and improves the performance by arm major axis correction and contour-center sampling is proposed. Arm major axis correction can solve the rotation problem and the sampling, which increases the recognition rate. Besides, using 3d space information makes the algorithm more stable. Experimental results show that the average recognition rate gets to 95.8%, and the performance and speed are superior to existing algorithms. Recognition results can be widely used in the follow-on real scene human-computer interaction (HCI) operations.

Keywords: *Hand gesture recognition, Shape Context Feature 3d shape context, arm major axis correction, contour-center sampling*

1. Introduction

Static hand gesture recognition is very important for HCI systems. For example, when hand gesture, position and distance information are acquired, it can be used in the interaction game to replace the traditional mouse operation. Although it is not as accurate as the mouse, it is very user-friendly in non-contact display operation. Most of the earlier works were based on the visible RGB images. B. Stenger [1] created a model of the palm to identify the hand. L. Sha [2] collected samples and extracted features to identify. The main advantage of the visible RGB images is that color models could be used to determine the location and hand's region, but there are some existing drawbacks such as colors, facial interference, sensitivity to light, and so on.

Gesture recognition on the depth image is not sensitive to light and does not have color distortion. Depth image generation methods include the time-of-flight camera [3] and Microsoft Corp Kinect device [4], seen in Figure. 1. The resolution of time-of-flight camera is low, so Kinect device which uses the speckle imaging technique is more convenient. The high resolution depth image at 640 × 480 could be obtained by Kinect. So many algorithms based on this depth image equipment are published, which are mainly used in the human posture recognition [5].

There are several hand gesture recognition algorithms based on the depth image [6-11]. These algorithms use different frameworks or features to recognize the hand gesture. Although some of the algorithms can get good performance, there are still some shortcomings. For example, the recognition rate is not high enough, or the optimization complexity is very high and difficult to run in real-time, or not easy to extend to more gestures.

In this paper, in order to perform static hand gesture recognition in real-time with high recognition rate, the 3D Shape Context feature is used on the depth image and combined with the random forest classifier to recognize static hand gesture. The original shape context feature [12] was mainly based on contour information, and used in shape matching, which can describe objects effectively. So it can be matched to the shape of the object and character recognition, and can reach a higher level. Kanaujia [13] proposed the 3D visual hull for human posture recognition and achieved good results. To be used in the body posture cannot yet compare with the gesture. The depth image directly provides distance depth information of each pixel and the actual usage of three-dimensional position is more accurate, which is the main reason of using the 3D Shape Context feature in this research. But Shape Context is not rotation invariant, so feature calculation should be combined with arm major axis detection and correction. For the contour sampling, this paper proposes contour-center points sampling, which can further improve the gesture recognition rate. The main advantages of the proposed algorithm are high recognition rate and low requirement of the actual condition. The algorithm has good adaptability for the light, color, distance, and rotary. The experimental results also show that the performance and speed are superior to existing algorithms.

This paper is organized as follows. The second section reviews the related work. The third section introduces the proposed framework of hand gesture recognition algorithm. Each module of the framework is described in details, which focuses on 3D shape context features and corresponding improvements. The forth section gives the specific results and analysis. The fifth section summarizes this paper briefly.

Figure 1. TOF Camera and Kinect Device

Table 1. The Main Parameters of TOF Camera and Kinect

Technical parameters	TOF camera	Kinect
Image resolution	176 (h) × 144 (v)	640 × 480, 16bit
Frame rate	54fps	30fps
Field angle	43.6 (h) × 34.6 (v)	57 (h) × 43 (v)
Measuring range	0 - 5 m	1.2 - 3.5 m
Precision	10mm	5mm
SDK software function	Adjust camera parameters in real time 3D images.	Kinect can simultaneously track up to six players, but only two players can simultaneously operate a game with speech recognition.

2. Related Work

Most of the earlier hand recognition works were based on the visible RGB images. B. Stenger [1] proposed a new practical technique based on 3D model of the hand tracking. Hand posture was estimated with an unscented Kalman filter, minimized geometric errors between contours and edge extracted from images. Sha [2] discussed a new framework to recognize the hand postures in successive video frames. Gaussian Mixture model was constructed for hand region detection and hand tracking after the particle filter. Zou [18] suggested gestures model updates and results prediction algorithm based on mean shift which should first use the method background difference and skin color detection to detect gestures model. These works on RGB images usually adopted the skin color model. But the RGB images are not very stable and easy changed by the influence of illumination, camera, occlusion, *etc.* If the color values are distorted, the effect of skin color model will decline and it is difficult to recognize or track the hand, seen in Figure. 2.

Figure 2. The Skin Color Problem of RGB Images

Depth image generation methods include the TOF camera, and Microsoft Kinect device, seen in Figure. 1. The resolution of time-of-flight camera is low, and Kinect can obtain high resolution depth image at 640 × 480, which is more convenient to use. The main parameters of TOF camera and Kinect are listed in Table 1. The resolution and precision of Kinect are better than TOF camera. Many algorithms based on Kinect were published in recent years, mainly in the human posture recognition. Shotton [5] proposed a method to predict the 3D position of the body joint from an input depth image quickly and accurately, using no temporal information. The performance is better than former posture recognition methods of RGB images.

There are several hand gesture recognition algorithms based on the depth image. These algorithms use different frameworks or features to recognize the hand gesture. Although some can get good performance, there are still shortcomings.

The palm model algorithm [6] described a fitting algorithm based on the skeleton of the hand depth image using a part of the object recognition method. The authors created a realistic 3D hand model represented different parts. The algorithm was similar [5], but the recognition rate was not high enough.

Ref. [7] proposed a new shape decomposition method called Minimum Near-Convex Decomposition (MNCD), which decomposes arbitrary shapes into minimum number of near-convex parts. The decomposition was very robust to large local distortions and shape deformation, formulated as a combinatorial mathematics optimization problem by minimizing the number of non-intersection cuts. The optimization complexity is very high; so it is difficult to run I awsdn real-time.

Ref. [8] proposed a measure named Finger-Earth Mover's Distance (FEMD) for gesture recognition. The system used the depth and color information from Kinect to detect the shape of a hand, and then analyzed the profile curve point relative to the center point.

Contour tracking algorithm [9] was a gesture recognition method to detect the presence of a finger, and recognized the significance in a predefined gesture popular program nine gestures. The algorithm was established to identify polygons with a fingertip position. Although the recognition rate is high, it failed to use most of the description information.

Ref. [10] proposed a motion gesture recognition descriptor. Difference value for each pixel in the depth image was quantized as Depth Difference codes. Target area was divided into several sub-regions. In each sub-region, a description of each DD vector generated code distribution. Each code vector for each sub-area to a different was cascaded as final Depth Difference Distribution. The descriptor is a combination of both motion and shape information, but it is not easy to extend to more gestures.

Complex-Valued Neural Network (CVNN) algorithm [11] used a system consisted of three components: real-time tracking of the hands, tree construction, and hand gesture recognition. The main contributions were simple hand gesture on behalf of the road after thinning algorithm applied to the image, and using a model of complex-valued neural networks for real-valued classification. The algorithm defined 20 kinds of finger change gesture, but for several commonly used gestures the recognition rate is not high.

Belongie [12] suggested a method to measure the similarity between shapes, and used it into target recognition. Shape context in a reference point to capture the remaining points relative to its distribution, thus providing the discriminative feature worldwide. There will also be a similar shape context of corresponding points in the two similar shapes. Kanaujia [13] propose a method using 3D visual hull reconstruction, multi-angle images from the contour estimated target human bones and posture. The main contribution of this work was an extension of the human body pose estimation algorithm to predict the data in 3D visual hull Kinect gaming system.

Compared to be used in the body posture, shape context feature is more convenient to be used in hand gesture. The depth image directly provides distance depth information of each pixel and the actual use of 3D position is more accurate, which is the main reason of adopting the 3D Shape Context feature in our research.

3. Gesture Recognition Algorithm Based on 3D Shape Context

This paper presents a static hand gesture recognition algorithm based on depth image using the 3D shape context feature. The overall system block diagram is shown in Figure. 3. Depth image from the Kinect device usually contains some redundant background data, so the first processing is simple background segmentation to obtain the hand area. The arm axis direction correction is necessary because human interactive operation arm will have a certain angle and affect the feature distribution. Then we obtain the hand area, detect the contour, and extract 3D Shape Context features. In this paper, the random forest [14-15] classification is applied. Tree structure is simple and practical, but sometimes one tree's results reliability is relatively low, so the usage of the multiple trees can make a forest. The forest is constructed in a supervised learning method of parameter selection process by adding certain random. Random forest can be used in the segmentation, classification, regression and other algorithms, this paper uses directly in the classification.

3.1 The Input Depth Image Preprocessing: Foreground Segmentation, Arm Axis Correction

Generally, the depth image also contains the scene information, which can be seen in Figure. 4(a). The original image cannot be directly used for feature extraction, so the first step of algorithm is hand region segmentation. Visible RGB light can use color model, but the depth image does not have color information. In order to be better segmented, this paper tries the Gaussian Mixture Model (GMM) algorithm [16] to establish the foreground and background GMM model:

$$P(y|\ \theta) = \sum_{k=1}^{K} \alpha_k \phi(y|\ \theta_k)\,. \tag{1}$$

α_k is the probability of different Gaussian Model ϕ, where

$$\phi(y|\ \theta_k) = P(y\,|\,z=k,\theta) \tag{2}$$

Figure. 4(a) can be processed to Figure. 4(b). But the image still cannot be used directly for feature extraction, because the direction of human arm in the actual interaction changes, while the shape context feature does not maintain invariance to the direction of rotation. For consistency, it needs to correct the arm direction after foreground segmentation.

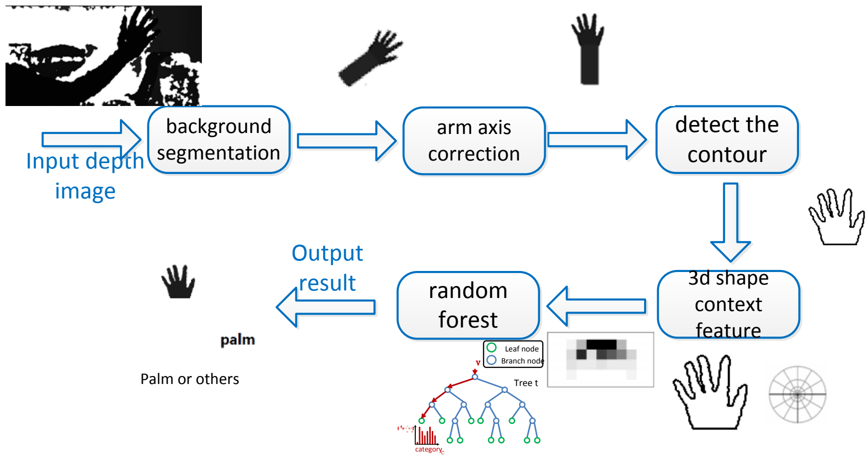

Figure 3. The Flowchart of 3D Shape Context Hand Gesture Recognition

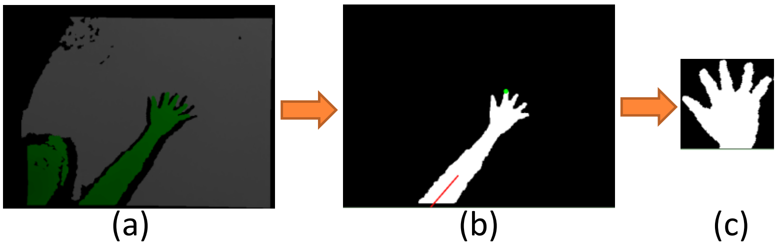

Figure 4. Hand Gesture Segmentation, (a) The Original Depth Image with Background; (b) Extract Palm Foreground; (c) Rotate the Hand

This paper carries on the arm region using Principal Component Analysis (PCA) [17], by rotating the image to make sure the arm show vertically in the image, as the red line shown in Figure. 4(b). This step is very important for the feature extraction, which may affect the recognition effect. One another operation step is rotating the hand region, to get the image only with palm area, as shown in Figure. 4(c).

3.2. The 3D Shape Context

Shape Context is proposed by Belongie in 2000 for shape matching application description. Firstly the feature need get the object contours in the image, then use a certain number of points as a collection, calculate the location information of each point, including the distance and angle. The distance and angle are used as a quantitative joint variables histogram statistics. Shape context features can describe object shape information effectively, resulting in well application in the shape matching. This research is about the static hand gesture recognition based on depth image. Compared with the visible RGB light image, one advantage is the direct depth information. Combined with the planar images information we can get the 3D coordinate information, so we can use 3D Shape Context feature in 3D depth image. The distance and angle information is closer to the actual object description. As shown in Figure. 5, the forefinger point as an example, a three-dimensional space of distance and direction information to several contour points can be calculated.

The 3D Shape Context feature proposed in this paper can be calculated as follows:

a) Hypothesis sampling N points from the contour, the pixel coordinates (x, y) adding the 3D coordinates of each contour value from depth image to the real space coordinate $p(x, y, z)$;

b) For each sampling point i, calculate the distance and direction to other $N-1$ points, described by vectors, including the radius d_{ij} and angle θ_{ij}, using polar coordinates to easy analysis;

$$d_{ij} = \left\| p_i(x, y, z) - p_j(x, y, z) \right\| \tag{3}$$

$$\theta_{ij} = \theta(p_i \to p_j) \tag{4}$$

c) For sampling point i, calculate on the $(N-1)$ data, to quantify the radius and angle, and project to a histogram normalization;

d) Integrate N feature data from all sampling points histogram as finally features;

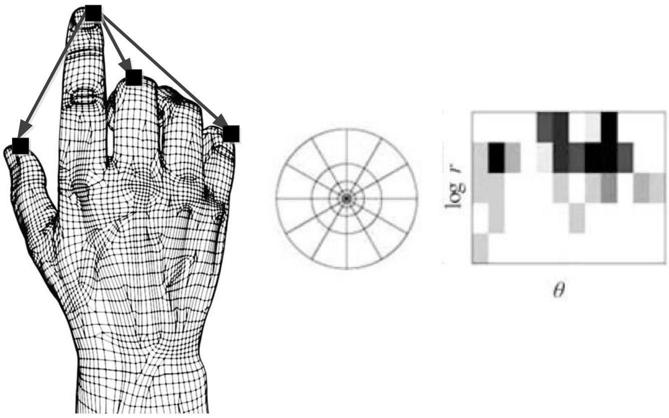

Figure 5. The 3D Shape Context Feature: the Forefinger Point to Several Other Points; the Corresponding Histogram

Because the depth values are real space distance, feature is invariant, meaning the same gesture in different distance actually results in the same feature basically, which can be seen as the advantage with a good adaptability in the distance scale and distance change.

Because the Shape Context feature is not rotation invariant feature, histograms to the same object are different before and after rotation. The numerical values may be similar, but the point sequence causes difference distribution, as shown in Figure. 6. Therefore the arm axis direction correction is very necessary. After correction we set the wrist point as the starting point in order to calculate each sampling contour point features.

Figure 6. Histograms Before and After Angle Correction

There are many contour points, so it needs the points sampling. The ordinary sampling method treats the whole contour points as a collection, using method of interval sampling to get a certain number of points, but the disadvantage of this method is that each finger's position information may not be balanced. For example, if only putting out one forefinger while others are closed, at this time the contour of forefinger is longer while the other fingers with less points. This paper presents method for contour-center point sampling, as shown in Figure. 7. For the gesture shapes after correction, we sample at intervals of equal angle in a concentric circle, and get same number of contour points for same angle.

Figure 7. Sampling Method of Contour Center Point

The advantage of this sampling method is the average sampling to each finger regions, better for different gestures. The experimental results also show that the new method can improve the recognition rate. After the point sampling and calculation to the histogram, 3D Shape Context feature data are extracted successfully, which are set as the input to the machine learning module of random forest classifier.

3.3. The Random Forest

Random forest can be used in the segmentation, classification, regression and other algorithms, this paper uses directly in the classification. A random forest is obtained by decision trees combination in accordance with the rules of a classifier, and tagged a large number of samples in various parameters. The general training of the algorithm proceeds as follows:

a) Randomly select parts samples from all sample data as training samples for the first tree.

b) For samples of each tree $Q=(I,x)$, randomly select number of characteristic θ and threshold τ, divide the sample into two classes; parameters are recorded as $\phi=(\theta,\tau)$; for each ϕ, sample collection Q can be divided into two subsets: the left one and the right one

$$Q_l(\phi)=\{(I,x)\mid f_\theta(I,x)<\tau\} \tag{5}$$

$$Q_r(\phi) = \{(I, x) \mid f_\theta(I, x) \geq \tau\} \tag{6}$$

c) For each selected feature ϕ, search into two collection-best of division classification results measured by entropy

$$\phi^* = \arg\max_{\phi} G(\phi) \tag{7}$$

$$G(\phi) = H(Q) = H(Q_l, Q_r) \tag{8}$$

d) If the optimal values are consistent and do not reach the maximum tree depth, then save ϕ^* as a classification of this node in the tree. The two collections $Q_l(\phi^*)$ and $Q_r(\phi^*)$ are trained recursively.

e) When finished training a tree, randomly select additional training samples for another tree.

The algorithm can be finished after one reaches a certain depth. Random forest classifier contains T trees, as shown in Figure. 8.

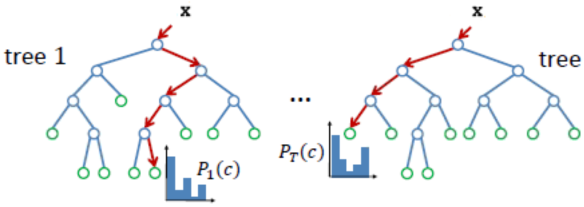

Figure 8. The Random Forest

4. Experimental Results and Analysis

4.1. The Experimental Configuration

The Kinect depth image resolution is 640 × 480. For the palm, fist, ‘V shape’, ‘three’ and ‘four’ gesture we collected about 300 pictures. Each kind is randomly selected half as training samples, the remaining samples for testing. Parts of the samples are shown in Figure. 9, Color change in the image represents the distance of different depth value. Image distance is between 1-2 meters, because if the distance is less than one meter, the Kinect cannot normally access the depth data; if the distance is too far, the hand area is too small and loss of detail for training and recognition.

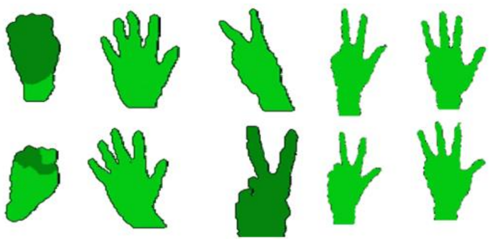

Figure 9. Some Gestures Samples in the Experiment

In experiment, the hardware configuration of the PC is Intel Core Dual 2.5 GHz CPU and 4 GB memory environment. The radius in 3D shape content feature is quantized into 5 bins, and angle in 3D shape content feature is quantized as 12 × 6 bins. It costs about 10 minutes in one training process.

4.2. Analysis and Comparison of Static Hand Gesture Recognition Performance

In order to compare with other algorithm, this paper lists the frequently-used fist, palm, V gesture. We can get the recognition rate 99% for the fist, palm samples, recognition rate about 91% for V gesture. The results are shown in Table 2. Each column here is the number of sample identification to all the categories. We can see the fist, palm recognition rate is around 99%, and V gesture is between the actions of fist and palm. There are small amounts of miscarriage of justice, so the recognition rate is about 91%.

Table 2. Recognition Rate of Three Kinds Gestures

	Fist	Palm	V
Fist	130	1	5
Palm	2	130	7
V	0	0	122
Rate(%)	99%	99%	91%

Next, the paper lists the result before and after improvements mentioned as comparison, shown in Table 3. First, if using the same data set, the feature is degraded into ordinary 2D shape context features for training and testing, we can see the recognition rate is around 71%-90%, and if using 3D shape context feature the original recognition rate is around 87%~96%. Although the calculation complexity of 3D shape context feature will be higher than two dimensional, but it can effectively improve the gesture recognition rate. Then adopting the proposed new contour point sampling method, the new method can improve recognition rate about 3%. The reason of two dimensional methods with low recognition rate is that feature extraction in two-dimensional plane is equivalent to project hand onto a plane, losing the dimensional information. 3D shape content feature of this paper tries to use more information in original image and embodies the advantages of 3D shape context feature.

Table 3. Gestures Recognition Rate, Improved Contrast

	Fist	Palm	V
2d shape content	90%	90%	71%
3d ordinary sampling	96%	95%	87%
3d contour-center sampling	99%	99%	91%

Table 4. Comparison of 3dSC with RGB Image Algorithm, MNCD, the Contour Tracking Algorithm, CVVN Algorithm

different algorithm	Fist	Palm	V	Average
3d SC	99%	99%	91%	96.3%
RGB image, complex scenes	70%	71%	68%	69.7%
RGB image, simple scenes	85%	87%	75%	82.3%
MNCD	96%	98%	90%	94.7%
contour tracking	98%	99%	89%	95.3%
CVNN	--	96%	81%	88.5%

Then we compare with the recognition algorithm on the visible RGB image. The average recognition rate has a great advantage because the visible light image halts on illumination, color, background influence. The paper lists in two scenes, first in the complex scene, may also have facial occlusion; the palm area sometimes cannot be accurately segmented, and make the average identification rate to about 70%. If using the white wall as the background, the recognition rate will increase and can reach about 82%, which is shown in Table 4.

There are other good performance gesture recognition algorithms on depth image, like convex decomposition. Compared with MNCD results, the average recognition rate of this paper is higher. More ever, the MNCD method for calculating and complexity is very high, so runs more slowly. One frame takes several seconds, so the actual interactive is not good enough.

With respect to the contour tracking algorithm, the recognition rate of this paper is slightly higher, but this algorithm only uses the fingertip and position information, while 3D shape context feature describes more comprehensive information.

The CVNN algorithm used the bifurcation tree structure of a hand skeleton model, because they identified more than 20 definitions of finger movement posture, fist gesture is not included in. The overall rate is lower than our algorithm.

The results of mentioned algorithms are also listed in Table 4. Although 3D Shape Context improves computation complexity than two-dimensional feature, but still can be run 50ms for the depth image of 640 × 480 (20 frames / second), so can be completed smoothly in the actual interaction system.

4.3. Future Work

A thin finger may cause the reflection area to be too small after Kinect emission speckle pattern. Then the image may be not clear, with more noise, and sometimes even miss fingers. It may cause recognition errors and reduce the identification rate. So error compensation on this algorithm is also necessary. We could use the lack of integrity of visible light image to complete corresponding regions.

The calculation of Shape Context features are based on floating-point computation, so the amount of calculation is higher compared to some simple features. At present we can almost run in real-time, but we still need to reserve time for each frame to the interactive operation. The cost of recognition module should down to 15ms below. We can use some multi core parallel optimization methods to accelerate it.

5. Conclusion

This paper presents a new static hand gesture recognition algorithm on depth images. The shape context feature is effective for shape matching. For the depth image, shape context features are improved to three dimensional in this paper. At the same time, training using random forest classifier can get better recognition rate after the detection and correction of arm major axis. The run speed on depth image of 640 × 480 is about 50ms per frame. This paper solves the problem of various changes in static hand gesture recognition such as the light, color, rotation. Our algorithm also has stronger adaptability and can complete the application of HCI effectively. Our algorithm performs better than several other mentioned algorithms. There is noise problem of Kinect output depth image. In order to be able to identify more gestures in the future, we need to add the depth image compensation calculation. We should also enhance the speed of the algorithm by using some multi core parallel optimization methods.

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