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Robust Fuzzy Variable Structure Control of T-S Model for a Quadrotor Unmanned Air Vehicle

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Abstract

This paper considers the fuzzy modeling and robust fuzzy variable control for quadrotor in uncertain environment. The nonlinear system of quadrotor is firstly analyzed in this paper by utilizing laws of motion and force. Then, a Takagi-Sugeno (T-S) fuzzy model is achieved to approximate the system. LMI method is used to acquire an improved sliding surface. On this basis, a fuzzy variable structure controller is designed to force the system state trajectory toward sliding surface and maintain on it then. The controller ensures the resulting closed-loop quadrotor system is asymptotically stable. Finally, a simulation is shown to verify the effectiveness of the proposed algorithm.

Keywords: *quadrotor; T-S fuzzy model; variable structure control; attitude control*

1. Introduction

In recent years, unmanned aerial vehicles (UAVs) have attracted more attention of many research groups in military and civilian areas [1]. As one type of unmanned aerial vehicle, a quadrotor can take off and land vertically without any environmental requirements, and they are able to easily hover in flight form [2]. Because of these advantages, they are used in various situations, like remote inspection, rescue and research, surveillance, forest fire detection etc. In addition, quadrotor as inexpensive aerial robotic platforms have attracted more and more interesting from researchers in control science and swarm robotics.

Quadrotor system is an under-actuated system with four independent inputs and six coordinate outputs. In the meantime, the quadrotor has nonlinear, time-variant, highly uncertain dynamics which makes it difficult to design the corresponding controller to stabilize the quadrotor. Quadrotor system is made up of many subsystems such as attitude, altitude, and position subsystem. In these subsystems, attitude subsystem is the most basic because of its under-actuated characteristic. In recent years, many method have been used to control a quadrotor, like neural network algorithm [3], sliding control [4] and adaptive control [5], but these method are almost based on linear control. Nonlinear control is more effective relative to linear control. Ke [6] proposed a nonlinear controller based on feedback linearization. G V Raffo [7] certified that nonlinear integral predictive H infinity control has robustness with uncertain parameters and environment. Fuzzy control is one control method that imitate human thinking [8] [9]. T-S fuzzy system is widely utilized nonlinear system modeling method [10]. T-S fuzzy model can well approach to the dynamic behavior of nonlinear system. However, we need consider using robust method, when there exits uncertainty in the system.

In this paper, combining sliding mode variable structure control and T-S fuzzy model, a new fuzzy variable structure control is proposed, which utilized for the attitude control of quadrotor air vehicle. This paper is organized as follows. T-S fuzzy model is designed by analyzing the dynamic model of quadrotor in section 2. In section 3, a sliding

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surface is build and robust fuzzy variable structure controller is given. Matlab simulation is shown in section 4. Finally, we conclude our work in section 5.

2. Model Description and Problem Formulation

A quadrotor helicopter mainly consists of four DC motors with bolted propellers, which are arranged to the terminal of a crisscross frame as depicted in Figure 1.

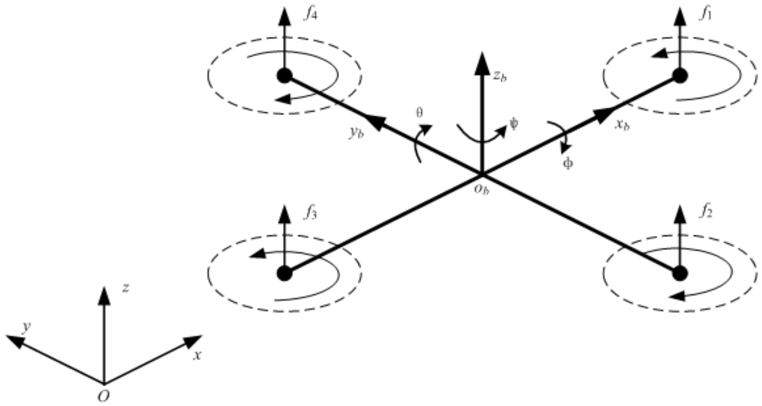

Figure 1. Quadrotor Model

By changing the rotating speed of four DC motors, quadrotor achieve moving in roll, pitch and yaw freely. A suitable dynamic system modelling is the element task of quadrotor control development. Before the mathematical model is developed, we gave some basic assumptions for this lightweight flying system.

• Earth fixed frame is inertial coordinate, gravitational acceleration doesn't change with altitude changing.

• Quadrotor body is rigid and its weight doesn't change with movement.

• Configuration and weight distribution of air vehicle are symmetrical, and center of mass is the center of quadrotor body.

• There are not external effects on quadrotor body such as air friction, wind pressure, etc. According these assumptions, the inertial matrix can be given as diagonal matrix

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

Earth fixed frame $\{E\}$ (O, x, y, z) and body fixed frame $\{B\}$ (O_b, x_b, y_b, z_b) are shown in figure 1, where O_b is supposed to be at the mass center of the quadrotor. A vector $\xi = [x \quad y \quad z]^T$ described the position of the center of gravity in $\{E\}$. Euler angles vector $\eta = [\phi \quad \theta \quad \varphi]^T$ describe the rotorcraft orientation representing three independent angles, respectively roll, pitch and yaw. The transformation matrix R representing the relationship between $\{B\}$ and $\{E\}$ is

$$R = \begin{bmatrix} C_\phi C_\theta & C_\phi S_\theta S_\varphi - S_\phi C_\varphi & C_\phi S_\theta C_\varphi + S_\phi S_\varphi \\ S_\phi C_\theta & S_\phi S_\theta S_\varphi + C_\phi C_\varphi & S_\phi S_\theta C_\varphi - C_\phi S_\varphi \\ -S_\theta & C_\theta S_\varphi & C_\theta C_\varphi \end{bmatrix}$$

, $C_{(*)}$ and $S_{(*)}$ are used to abbreviate $\cos(*)$ and $\sin(*)$

According newton's laws of motion, kinetic equations of the rotorcraft can be given as

$$\begin{cases} m\ddot{\xi} = F_{\xi} - F_{g} \\ I\dot{w} + w \times (Iw) = \sum M_{b} \end{cases} \tag{1}$$

where F_{ξ} is the translational force, *m* denotes the mass of the rotorcraft, $\sum M_b$ is torque on airframe body, $w = \begin{pmatrix}\dot{\phi} & \dot{\theta} & \dot{\varphi}\end{pmatrix}^T$ is the body angular speed. The small body forces is ignored, then we write

$$F_{\xi} = R\overset{*}{F} = R\begin{bmatrix}0\\0\\u\end{bmatrix},\ F_g = \begin{bmatrix}0\\0\\mg\end{bmatrix} \tag{2}$$

where $u = f_1 + f_2 + f_3 + f_4$ and $f_i = b\Omega_i^2, i = 1,\cdots 4$, Ω_i is rotor speed, b is the thrust factor. The generalized moments on the η variable are

$$\sum M_b = M_T + M_G \tag{3}$$

where M_T is lifting moment, M_G is gyroscopic moment which are written as

$$M_T = \begin{bmatrix} l(f_2 - f_4) \\ l(f_3 - f_1) \\ \sum M_{Di} \end{bmatrix},\ M_{Di} = (-1)^i b\Omega_i^2$$

$$M_G = \sum_{i=1}^{4} w \times (J_r v_i)$$

where J_r denotes rotor inertia, $v_i = \begin{bmatrix}0 & 0 & (-1)^{i+1}\Omega_i\end{bmatrix}^T$, *l* is the distance from pivot to motor, The rotors are driven by DC-motors which equation is given as

$$L\frac{di}{dt} = V_i - Ri - K_v\Omega_i\,.$$

Because of inductance and resistance of the small motor that we used are very low, the equation can be simplified as

$$\Omega_i = KV_i$$

where V_i is the voltage applied to the propeller, *K* is the transformation constant. The full quadrotor dynamic model can be given as,

$$\begin{cases} \ddot{x} = (\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi)\frac{1}{m}u_0 \\ \ddot{y} = (\cos\phi\sin\theta\sin\varphi - \sin\phi\sin\varphi)\frac{1}{m}u_0 \\ \ddot{z} = (\cos\phi\cos\theta)\frac{1}{m}u_0 - g \\ \ddot{\phi} = \dot{\theta}\dot{\varphi}(\frac{I_y - I_z}{I_x}) + \frac{J_r}{I_x}\dot{\theta}\Psi + u_1 \\ \ddot{\theta} = \dot{\phi}\dot{\varphi}(\frac{I_z - I_x}{I_y}) - \frac{J_r}{I_y}\dot{\phi}\Psi + u_2 \\ \ddot{\varphi} = \dot{\phi}\dot{\theta}(\frac{I_x - I_y}{I_z}) + u_3 \end{cases} \tag{4}$$

where

$$\Psi = K(V_1 + V_3 - V_2 - V_4),\ u_0 = bK^2(V_1^2 + V_2^2 + V_3^2 + V_4^2)$$

$$u_1 = \frac{blK^2(V_2^2 - V_4^2)}{I_x},\ u_2 = \frac{blK^2(V_3^2 - V_1^2)}{I_y}$$

$$u_3 = \frac{bK^2(V_2^2 + V_4^2 - V_1^2 - V_3^2)}{I_z}$$

Considering the attitude subsystem, the state vector is defined as $x = \left[\dot{\phi}, \dot{\theta}, \dot{\varphi}, \phi, \theta, \varphi\right]^T$ and the control input vector as $u = \left[u_1, u_2, u_3\right]^T$, then according to the nonlinear dynamic (4), the model of quadrotor can be given as

$$\begin{cases} \dot{x}(t) = A(x)x(t) + Bu(t) \\ z(t) = Cx(t) \end{cases} \tag{5}$$

where

$$A(x) = \begin{bmatrix} 0 & \frac{J_r}{I_x}\Psi & \frac{I_y - I_z}{I_x}x_3 & 0 & 0 & 0 \\ -\frac{J_r}{I_y}\Psi & 0 & \frac{I_z - I_x}{I_y}x_1 & 0 & 0 & 0 \\ 0 & \frac{I_x - I_y}{I_z}x_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \left[I_{3\times 3} \quad 0_{3\times 3}\right]^T, \; C = \left[0_{3\times 3} \quad I_{3\times 3}\right].$$

It is shown that three nonlinear terms are in the matrix $A(x)$, which is $\Psi, x_1(t), x_3(t)$. Because that the attitude angles and input voltage to the propellers are bounded, these terms are bounded. Therefore, the nonlinear dynamic (3) can be approximated by a T-S fuzzy dynamic model. The T-S fuzzy model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem for nonlinear systems. The following sector nonlinearity approach is utilized. The term Ψ is bounded by $[\Psi_{\min}, \Psi_{\max}]$ with $\Psi_{\min} = 4KV_{\min}$, $\Psi_{\max} = 4KV_{\max}$ according to (4). Then the weighting function of Ψ can be chosen as

$$w_1 = \frac{\Psi_{\max} - \Psi(t)}{\Psi_{\max} - \Psi_{\min}}$$

The term Ψ can be rewritten as $\Psi(t) = w_1\Psi_{\min} + (1 - w_1)\Psi_{\max}$. It is noted that $0 \le w_1 \le 1$. Similarly, we have $x_1(t) = w_2\overline{x_1} + (1 - w_2)\underline{x_1}$, $x_3 = w_3\overline{x_3} + (1 - w_3)\underline{x_3}$ where

$$\overline{x_1} = \overline{x_3} = \dot{\phi}_{\max} = \dot{\varphi}_{\max} = \alpha_{\max},$$

$$\underline{x_1} = \underline{x_3} = \dot{\phi}_{min} = \dot{\varphi}_{\min} = \alpha_{\min},$$

$$w_2 = \frac{\overline{x_1} - x_1(t)}{\overline{x_1} - \underline{x_1}}, \; w_3 = \frac{\overline{x_3} - x_3(t)}{\overline{x_3} - \underline{x_3}}.$$

Then we can rewrite the nonlinear dynamic in (3) as the following T-S fuzzy model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i[z(t)]\left\{A_i x(t) + Bu(t)\right\} \\ z(t) = Cx(t) \end{cases} \tag{6}$$

where

$$\mu_1 = w_1(1 - w_2)(1 - w_3), \; \mu_2 = w_1 w_2 w_3$$

$$\mu_3 = w_1(1 - w_2)w_3, \; \mu_4 = w_1 w_2(1 - w_3)$$

$$\mu_5 = (1-w_1)(1-w_2)(1-w_3)\,,\ \mu_6 = (1-w_1)w_2 w_3$$
$$\mu_7 = (1-w_1)(1-w_2)w_3\,,\ \mu_8 = (1-w_1)w_2(1-w_3)$$

$$A_1 = \begin{bmatrix} 0 & \frac{J_r}{I_x}\Psi_{\max} & \frac{I_y - I_z}{I_x}\underline{x_3} & 0 & 0 & 0 \\ -\frac{J_r}{I_y}\Psi_{\max} & 0 & \frac{I_z - I_x}{I_y}\underline{x_1} & 0 & 0 & 0 \\ 0 & \frac{I_x - I_y}{I_z}\underline{x_1} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

We can similarly acquire the other expression of the corresponding $A_i, i = 2,\cdots,8$, and the derivation can be left out because of the limited space. However, we only considering that every parameters of quadrotor are definite in the system (6). In practice, these uncertainties of the quadrotor control system all can't be ignored, so T-S fuzzy system can be rewritten as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i[z(t)]\left\{A_i x(t) + Bu(t) + \Phi_i(x,t)\right\} \\ z(t) = Cx(t) \end{cases} \tag{7}$$

where $\Phi i(x,t)$ represents nonlinear uncertainties and external disturbances of the system. It is assumed that the uncertainty is admissibly norm-bounded and it satisfies the following inequation,

$$\left\|\Phi_i(x,t)\right\| \le \varepsilon_i \left\|x(t)\right\| \tag{8}$$

where $\varepsilon > 0$ is the known constant.

Lemma 1 [11]: Assume that nonlinear uncertain function $f(x,t)$ is norm-bounded: for some $\varepsilon > 0$, $\left\|f(x,t)\right\| \le \varepsilon \left\|x(t)\right\|$ for all $x \in R^n$, then $f(x,t)$ satisfies

$$f(x,t) = \varepsilon N(t)x(t)\,,$$

where $N(t)$ is uncertain terms and it satisfies $N^T(t)N(t) \le I$.

By lemma 1, the system (7) can be shown as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i[z(t)]\left\{A_i x(t) + Bu(t) + \varepsilon_i N_i(t)\right\} \\ z(t) = Cx(t) \end{cases} \tag{9}$$

3. Design of Variable Structure Control Algorithm

Although a T-S fuzzy can implement representing a nonlinear function at any precision, there are always existing systems modeling uncertainties in practice. Considering the indeterminacy of aerodynamic parameter and electronic system parameter of quadrotor, it is difficult to complete the attitude control. In this article, we used the variable structure control to eliminate the uncertainty. The algorithm design of fuzzy variable structure control mainly is formed of two steps. The first step is to select suitable sliding surface to make the system motion have the wanted behavior when the system enters the sliding surface. The next step is the designing of the variable structure controller which satisfies the reaching condition. This condition make it be guaranteed that the system trajectories will be forced to reach toward the sliding surface in finite time with this control law, then keep them on the surface. First we need to design the sliding surface on which the system motion is asymptotically stable. In the past process of sliding surface design, we rarely consider the ideal control robustness of the system including mismatched uncertainty. The means of LMI can be utilized in the design of sliding surface to improve the system robustness. Firstly, for getting a regular form of system function (7), a nonsingular matrix could be chosen as

$$T=\begin{bmatrix}T_1\\T_2\end{bmatrix}=\begin{bmatrix}(Q^T PQ)^{-1}Q^T\\(B^T P^{-1}B)^{-1}B^T P^{-1}\end{bmatrix} \tag{10}$$

where $Q^T B=0$ and $T\in R^{2\times n}$. We chose the sliding surface as the follow form

$$S(t)=B^T X^{-1}x(t)=Hx(t) \tag{11}$$

where X is a symmetric and positive definite matrices that need to be designed. Our aim is to choose an appropriate X for making the system motion be asymptotically stable on the sliding surface. To achieve the dynamics on the sliding surface, we give a transform for $x(t)$ as

$$z(t)=\begin{bmatrix}z_1(t)\\z_2(t)\end{bmatrix}=Tx(t)=\begin{bmatrix}T_1x(t)\\T_2x(t)\end{bmatrix} \tag{12}$$

We can easily find that the following equation about $z_2(t)$ is satisfied

$$\begin{aligned}z_2(t)&=(B^T X^{-1}B)^{-1}B^T X^{-1}x(t)\\&=(B^T X^{-1}B)^{-1}S(t)\\&=(HB)^{-1}S(t)\,.\end{aligned}$$

It is satisfied that $S(t)=0$, once the state trajectory of system reach the sliding surface. Then we can get that $z_2(t)=0$. According (10) and (12), we find that $x(t)=XQz_1(t)$. It is obvious that the following function represented the sliding mode dynamics can be written as

$$=(Q^T XQ)^{-1}Q^T\sum_{i=1}^{r}h_i\left\{(A_i+\varepsilon_iN_i)x(t)+Bu(t)\right\}\;=(Q^T XQ)^{-1}Q^T\sum_{i=1}^{r}h_i\left\{(A_i+\varepsilon_iN_i)XQz_1(t)\right\}.$$

To analyze the asymptotic stability of sliding mode dynamics, the following lemmas are necessary.

Lemma 2 [12]*:* Given constant matrices D,E and unknown matrix $N(t)$ of appropriate dimensions, if $N(t)$ satisfies $N^T(t)N(t)\le I$, for arbitrary scalar $\delta>0$ the following inequality holds

$$DN(t)E+E^TN^T(t)D^T\le\delta DD^T+\delta^{-1}E^TE\,.$$

Lemma 3: (Schur complement) The matrix $\begin{bmatrix}G & R\\R^T & S\end{bmatrix}<0$ is symmetrical matrix, then the two equivalent functions are as follows,

$$S<0,\;G-RS^{-1}R^T<0\ \text{or}\ G<0,\;S-RG^{-1}R^T<0\,.$$

The result of asymptotic stability of sliding mode dynamics can be stated as follows.

Theorem 1: The sliding mode dynamics (7) would be asymptotically stable, if there is symmetric and positive definite matrix X and scalar $\delta>0$ such that the following inequation is satisfied.

$$\begin{bmatrix}\Theta & *\\XQ & -\delta I\end{bmatrix}<0 \tag{13}$$

where $\Theta=Q^TXA_i^TQ+Q^TA_iXQ+\delta\varepsilon^2Q^TQ$, and $*$ denotes the transposed elements in the symmetric positions.

Proof: Choose the Lyapunov functions as

$$V_i(t) = z_1^T(t)Q^T XQz_1(t)$$

where X is symmetric and positive definite matrix.
It follows that the Lyapunov derivative corresponding to the sliding mode dynamic, arrange as follows:

$$\dot{V}_i(t) = \dot{z}_1^T(t)Q^T XQz_1(t) + z_1^T(t)Q^T XQ\dot{z}_1(t)$$

$$= z_1^T(t)Q^T X\sum_{i=1}^{r}\mu_i\left\{(A_i^T + \varepsilon_i N_i^T)\right\}Q(Q^T XQ)^{-T}Q^T XQz_1(t) +$$

$$z_1^T(t)Q^T XQ(Q^T XQ)^{-1}Q^T\sum_{i=1}^{r}\mu_i\left\{(A_i + \varepsilon_i N_i)XQz_1(t)\right\}$$

$$= \sum_{i=1}^{r}\mu_i z_1^T(t)Q^T X(A_i^T + \varepsilon_i N_i^T)Q + Q^T(A_i + \varepsilon_i N_i)XQz_1(t).$$

The condition is the inequality as follows holds, which make the sliding mode dynamics be asymptotically.

$$Q^T X(A_i^T + \varepsilon_i N_i^T)Q + Q^T(A_i + \varepsilon_i N_i)XQ < 0 \tag{14}$$

This inequality can easily be written as

$$Q^T XA_i^T Q + Q^T A_i XQ + DN_i E + D^T N_i^T E^T < 0,$$

where $D = Q^T\varepsilon_i$, $E = XQ$. According to lemma 2, the form (13) will be hold for all $N_i(t)$ which satisfied $N_i^T(t)N_i(t) \le I$, if there exits $\delta > 0$ make the following inequality be satisfied.

$$\Theta + (XQ)^T\delta(XQ) < 0.$$

Using Schur complement, we can achieve the equivalent inequation (13). The proof is completed. The reaching condition should be satisfied in the control method designing, for making the system trajectories reach to the sliding surface in limited time and keep them on the sliding surface after reaching [13]. Traditional condition is $S(t)\dot{S}(t) < 0$, where $S(t) \ne 0$. But there may be large chattering and long reaching time in the reaching phase used the condition. An improved condition have been proposed as

$$\dot{S}(t) \le -\gamma S(t) - \lambda signS(t), S(t) > 0$$

$$\dot{S}(t) \ge -\gamma S(t) - \lambda signS(t),\ S(t) < 0 \tag{15}$$

where $\gamma > 0$ and $\lambda > 0$ are constants. This reaching condition accelerated the reaching rate and diminishes chattering. But (15) could not insure the system state to reach to the balance point finally. For overcoming the shortcoming, a novel reaching condition is given as follows

$$\dot{S}(t) \le -\gamma S(t) - \lambda(1 - e^{-\|x(t)\|})signS(t), S(t) > 0$$

$$\dot{S}(t) \ge -\gamma S(t) - \lambda(1 - e^{-\|x(t)\|})signS(t), S(t) < 0 \tag{16}$$

where $\gamma > 0$ and $\lambda > 0$ are constants. We easily know that $S(t)$ in (15) satisfies the condition that $S(t)\dot{S}(t) < 0$, when $S(t) \ne 0$. And if the range of system uncertainty is narrow, the value of $\left|\dot{S}(t)\right|$ is near to zero, which make the reaching rate adjacent balance point become very slow. These acquire smaller system motion inertia adjacent equilibrium point to reduce the chattering. So we consider designing the robust variable structure controller according to the reaching condition (16). It is said that system state trajectories by the control can reach the sliding surface with finite time and maintaining on the surface. At the same time, we can acquire faster reaching rate and lower chattering.
Theorem 2: For the system (7), the reaching condition (16) can be satisfied, if the controller is written as follows

$$u(t) = -(HB)^{-1}\sum_{i=1}^{r}\mu_i(z(t))\left\{v_1 + v_2 + v_3 signS(t)\right\} \quad (17)$$

where

$$v_1 = HA_i x(t)$$
$$v_2 = \gamma S(t) + \lambda(1 - e^{-\|x(t)\|})signS(t)$$
$$v_3 = \varepsilon_i \|H\|\|x(t)\|$$

$\gamma > 0$, $\lambda > 0$, $\varepsilon_i > 0$ $(i = 1, 2, \cdots, r)$ are constants.

Proof: According to (9), the time derivative of $S(t)$ on the state trajectories of system (8) by (17) control can be written as

$$\begin{aligned}\dot{S}(t) = H\dot{x}(t) &= \sum_{i=1}^{r}\mu_i(z(t))\Big\{v_1 + H\Phi_i(x,t)v_2 \\ &\quad - \sum_{i=1}^{r}\mu_j(z(t))\left[v_4 + v_2 + v_5 signS(t)\right]\Big\} \\ &= \sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\big\{v_1 - v_4 - v_2 \\ &\qquad + H\Phi_i(x,t)v_2 - v_5 signS(t)\big\}\end{aligned} \quad (18)$$

where

$$v_4 = HA_j x(t),\ v_5 = \varepsilon_j \|H\|\|x(t)\|$$

According to (8), we can get the following result

If $S(t) > 0$,

$$\begin{aligned}&\sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\left\{H\Phi_i(x,t) - v_5 signS(t)\right\} \\ &\le \sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\left\{\|H\|\|\Phi_i(x,t)\| - v_5 signS(t)\right\} \le 0\end{aligned} \quad (19)$$

We can also get the following equality as

$$\sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\left\{v_1 - v_4\right\} = 0 \quad (20)$$

According to (18) and (19), we can achieve that

$$\dot{S}(t) \le -v_2 = -\gamma S(t) - \lambda(1 - e^{-\|x(t)\|})signS(t) \quad (21)$$

If $S(t) < 0$

$$\begin{aligned}&\sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\left\{H\Phi_i(x,t) - v_5 signS(t)\right\} \\ &\ge \sum_{i=1}^{r}\mu_i(z(t))\sum_{i=1}^{r}\mu_j(z(t))\left\{-\|H\|\|\Phi_i(x,t)\| - v_5 signS(t)\right\} \ge 0\end{aligned} \quad (21)$$

According to (18) and (21), we can achieve that

$$\dot{S}(t) \ge -v_2 = -\gamma S(t) - \lambda(1 - e^{-\|x(t)\|})signS(t) \quad (22)$$

From above proving process, the reaching condition can be satisfied by using controller (17). The control law can make the system state track reach to the sliding surface in limited time and maintain on the surface. At the same time, faster reaching rate and lower chattering are acquired. The proof is completed.

4. Simulation Results

In this section, we use the simulation results of the attitude control of the quadrotor aircraft to validate the effectiveness of T-S fuzzy modeling and the fuzzy variable structure control method advised in this paper. We use the parameters of the quadrotor that are shown as follows

$$V_i \in [\text{-7V} \quad \text{7V}],\ K = 53.685(\text{rad/s})/\text{V}\ J_r = 5\times10^{-5}\text{kgm}^2,$$

$$I_x = 0.0563\text{kgm}^2, I_y = 0.0563\text{kgm}^2, I_z = 0.1126\text{kgm}^2,$$

$$b = 3.8965\times10^{-6}\ \text{N/Volt},\quad l = 0.1759\text{m}, T\text{=}0.005\text{s}$$

$$\dot{\phi}_{\min} = \dot{\theta}_{\min} = \dot{\varphi}_{\min} = -\pi/4\ rad/s,\quad \dot{\phi}_{\max} = \dot{\theta}_{\max} = \dot{\varphi}_{\max} = \pi/4\ rad/s$$

$$\phi_{\max} = \theta_{\max} = \pi/2,\quad \varphi_{\max} = \pi .$$

The simulation objective is to build T-S model of the quadrotor and design a fuzzy variable structure controller (FVSC). According to the method shown in section 2, we can achieve the fuzzy model, and we can acquire the matrix of the sliding surface as follows using theorem 1.

$$X = \begin{bmatrix} -5.1438 & 0.7400 & -0.7548 & -0.7548 & 5.1438 & 0.7400 \\ 0.7400 & -0.1349 & -0.0785 & -0.0785 & -0.7400 & -0.1349 \\ -0.7548 & -0.0785 & -0.1050 & -0.1050 & 0.7548 & -0.0785 \end{bmatrix}$$

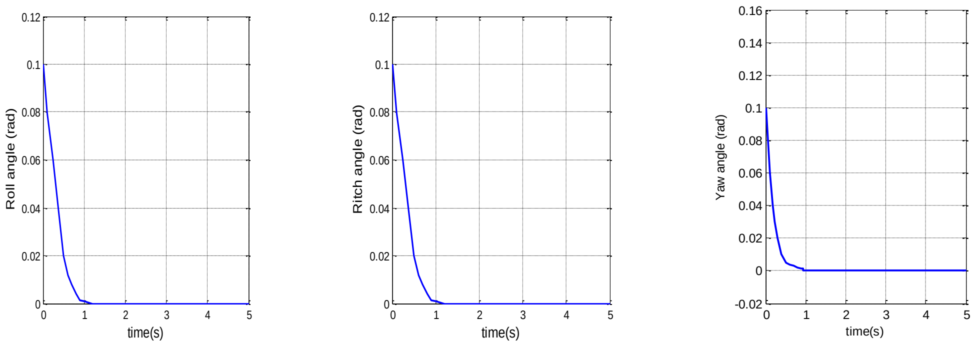

Figure 2. Simulation Results

The finally results based on the FVSC algorithm are shown in Figure 2. Simulation results show that the FVSC method can stabilize the used quadrotor.

4. Conclusion

In this paper, the T-S fuzzy modeling and robust fuzzy variable structure controller design problem of the nonlinear quadrotor aircraft is discussed. A T-S fuzzy model approach nonlinear is proposed system design method and the system uncertainty is considered. By means of LMI, the stable sliding surface is designed which effectively decrease the effect of uncertainty. Utilizing matlab simulation, the effectiveness of the control algorithm is verified.

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