

Improved Multi-target Tracking Algorithm Based on Gaussian Mixture Particle PHD Filter

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Abstract

The paper proposes Gaussian mixture particle probability hypothesis density filter (PHD) algorithm, which can effectively solve the problem that the object number is changing or unknown, based on particle PHD filter. This algorithm calculates the object number and state by recursive procedure, avoiding the uncertainty of target state estimation caused by particle sampling and clustering. Gaussian mixture particle is introduced to effectively maintain the multi-modal distribution of each target, reducing the complexity of calculation.

Keywords: Gaussian Particle Filter, Multi-target Tracking, Probability Hypothesis Density, Mixture Particle Filter

1. Introduction

Particle filter is considered to be one of the most effective ways to handle nonlinear non-Gaussian dynamic systems, in the field of target tracking. It is widely used in the field of tracking. But in the multi-target environment, particle filters cannot consistently and effectively maintain the multi-modal distribution of the target, which hinders its development in the field of tracking. Regarding this question, [1] introduced the idea of Mixture Bayesian tracking, proposing the Mixture Particle Filter (MPF), using the multi-modal of multi-target probability density to realize multi-target tracking, drawing near each model of the target with the component probability density. However, in complex multi-target tracking scenarios, overlap, merge and separation among targets is unavoidable, and the influence of clutter and noise does exist, which leads to uncertainty between measurement and targets [2]. Approaches in literature [1] can be used to track multiple targets, but cannot express the association among multiple targets.

For complex data association problem of multi-target tracking, there are two classic solutions: the first kind is Joint Probability Density Association (JPDA) Algorithm, proposed by Bar-Shalom, *etc.*, [3] and Multiple Hypothesis Tracking (MHT) Algorithm proposed by Reid, *etc.*, [4], which has been used in the video multiple targets tracking system. Due to the complexity and huge computing, this kind of method is difficult to realize in practice. The other one is multi-target tracking method which is based on Random Finite Set (RFS) theory. One of the most representatives is Probability Hypothesis Density (PHD) filtering method, proposed by Mahler [5]. Ba-Ngu, *etc.*, [6] also proposed a method of PHD particle filter to achieve the practical application of the PHD filter. This method obtains the approximate value of the first moment statistics of the target's posterior probability density through PHD filter, simplifying the multi-target tracking, greatly reducing the computational complexity. Meanwhile PHD [5, 7]

filter calculates probability hypothesis density of multi-target random set recursively, getting the number of the target and the status of each object. The proposed algorithm applies K-means or EM algorithm, *etc.*, to the tracking algorithm [8, 9] to maintain high sampling rate and keep the distribution of original data. If some feature is used alone, erroneous judgment may occur, with the independent component of PHD filter tracking wrong object, reducing the tracking accuracy.

Recently, many researchers have proposed and realized GM-PHD algorithm, based on PHD filter algorithm. This algorithm is merely suitable for linear Gaussian systems, evaluating the mean value and variance of Gaussian elements with Kalman filter. Cheng Ouyang, *etc.*, [10] improved the traditional algorithm to achieve nonlinear non-Gaussian target tracking. This method solved the problem of overestimate brought by cluster, increasing the computational complexity at the same time. This paper proposes Gaussian Mixture Particle PHD algorithm (GMP-PHD) in the framework of Particle Filter [11] (PHD), introducing Hybrid Particle Filter and Gaussian Particle Filter. This algorithm fully takes the advantages of GPF and particle PHD, using the sequence Monte Carlo method to approximate the posterior probability density of objects, providing the mean variance and weight of posterior PHD Gaussian elements through target probability distribution approximated by a set of Gaussian components with weights, avoiding the inaccuracy caused by clustering. This algorithm reduces the computational complexity in some degree. Particle filter is used for each particle swarm independently, avoiding modal loss of standard particle filter [1].

2. Hybrid Particle Filter

Assuming that the status of the object is x_k at time k , $z_{1:k} = \{z_1, \dots, z_k\}$ represents the sequence of observations from time 1 to time k . We define a set of hybrid model consisted of M components. Then the posterior probability at time k is

$$p(x_k | z_{1:k}) = \sum_{m=1}^M \pi_{m,k} p_m(x_k | z_{1:k}), \quad (1)$$

where $\pi_{m,k}$ represents the weight of the m -th component ($\sum_{m=1}^M \pi_{m,k} = 1$), and $p_m(x_k | z_{1:k})$ denotes the m -th modal approaching the posterior probability density. The posterior probability density of multimodal is obtained by recursive procedure based on (1), in the Bayesian framework. The recursive procedure involves two steps: forecast and update.

Forecast:
$$p(x_k | z_{1:k-1}) = \sum_{m=1}^M \pi_{m,k-1} p_m(x_k | z_{1:k-1}), \quad (2)$$

where $p_m(x_k | z_{1:k-1})$ is the predictive distribution of the m -th component. The formula can be also presented as

$$p_m(x_k | z_{1:k-1}) = \int D(x_k | x_{k-1}) p_m(dx_{k-1} | z_{1:k-1}). \quad (3)$$

Update:

$$p(x_k | z_{1:k}) = \sum_{m=1}^M \left[\frac{\pi_{m,k-1} \int L(z_k | x_k) p_m(dx_k | z_{1:k-1})}{\sum_{n=1}^M \pi_{n,k-1} \int L(z_k | x_k) p_n(dx_k | z_{1:k-1})} \right] \times \left[\frac{L(z_k | x_k) p_m(x_k | z_{1:k-1})}{\int L(z_k | x_k) p_m(dx_k | z_{1:k-1})} \right], \quad (4)$$

in which the weight of the new component is

$$\pi_{m,k} = \frac{\pi_{m,k-1} \int L(z_k | x_k) p_m(dx_k | z_{1:k-1})}{\sum_{n=1}^M \pi_{n,k-1} \int L(z_k | x_k) p_n(dx_k | z_{1:k-1})}. \quad (5)$$

The posterior probability density of the m-th component is

$$p_m(x_k | z_{1:k}) = \frac{L(z_k | x_k) p_m(x_k | z_{1:k-1})}{\int L(z_k | x_k) p_m(dx_k | z_{1:k-1})}, \quad (6)$$

where $L(z_k | x_k)$ and $D(x_k | x_{k-1})$ represent likelihood function and state transition function. Because (3), (5) and (6) is difficult to calculate, literature [1] gives an approximate calculation method based on serialized Monte Carlo.

$$p(x_k | z_{1:k}) = \sum_{m=1}^M \pi_{m,k} \sum_{i \in I_m} w_k^i \delta_{x_k^i}(x_k), \quad (7)$$

in which $I_m = \{i \in \{1 \dots N\} : t_k^i = m\}$ (N is the total number of particles and $t_k^i \in \{1 \dots M\}$). If the i-th particle belongs to the m-th mixture component, then we may know $t_k^i = m$. Represents the particle collection of the m-th component. The weights of mixture components and the weights of particles meet: $\sum_{m=1}^M \pi_{m,k} = 1, \sum_{m=1}^M \pi_{m,k} = 1$. The weights of particles are calculated by the following formula:

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j \in I_m} \tilde{w}_k^j} (\tilde{w}_k^i = \frac{w_{k-1}^i L(z_k | x_k^i) D(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, z_k)}). \quad (8)$$

We estimate $p_m(x_k | z_{1:k})$ after getting the new particle sample set $\{x_k^i, w_k^i\}_{i \in I_m}$. In order to obtain the weight of new mixture component, we have to figure out $p_m(z_k | z_{1:k-1}), m = 1 \dots M$ of each component. We use Monte Carlo method to estimate the likelihood function of the m-th component:

$$p_m(z_k | z_{1:k-1}) \approx \sum_{i \in I_m} \tilde{w}_k^i. \quad (9)$$

Substituting (9) into equation (5), we use

$$\pi_{m,k} \approx \frac{\pi_{m,k-1} \tilde{w}_{m,k}}{\sum_{n=1}^M \pi_{n,k-1} \tilde{w}_{n,k}}, (\tilde{w}_{m,k} = \sum_{i \in I_m} \tilde{w}_k^i) \quad (10)$$

to represent the approximated weight of mixture components. The weight of new particles is $w_k^i = 1/|I_m|, i \in I_m$ after the resample of the components.

3. PHD Filter

PHD filter principle can reference [5]. Probability hypothesis density is first moment of multi-objective posterior probability density, belonging to the random sets optimal Bayesian multi-target filtering method, calculating the PHD of multi-objective finite random sets by recursion. Recursive process includes two steps: forecast and update.

Forecast:

$$\begin{aligned}
 v_{k|k-1}(x) &= v_{S,k|k-1}(x) + v_{\beta,k|k-1}(x) + \gamma_k(x) \\
 &= \int p_{S,k}(\xi) f_{k|k-1}(x|\xi)_{k|k-1}(x, \xi) v_{k-1}(\xi) (d\xi) \\
 &\quad + \int \beta_{k|k-1}(x|\xi) v_{k-1}(\xi) (d\xi) + \gamma_k(x)
 \end{aligned} \tag{11}$$

Update:

$$\begin{aligned}
 v_k(x) &= [1 - p_D(x)] v_{k|k-1}(x) \\
 &\quad + \sum_{z \in Z_k} \frac{p_{D,k}(\xi) g_k(z|x) v_{k|k-1}(x)}{K_k(z) + \int p_{D,k}(\xi) g_k(z|\xi) v_{k|k-1}(\xi) d\xi}
 \end{aligned} \tag{12}$$

The iterative equation evaluates the status and number of the objects. The specific derivation can be seen in lecture [12, 13]. γ_k represents the PHD of stochastic finite set Γ_k of naturally occurring target at time k. $b_{k|k-1}(\cdot|\xi)$ is the PHD of stochastic finite set $B_{k|k-1}(\{\xi\})$ of new target generated by the target whose status is ξ at time k-1. $e_{k|k-1}(\xi)$ is the probability that the target whose status is ξ at time k-1 survives at time k. c_k shows the probability density of the clutter and λ_k expresses the mean value of clutter points in the Poisson distribution of each scan. Besides, p_D represents the detection probability.

Assume that the objectives under the multi-target tracking model satisfy the Gaussian distribution, the transition probability density $f_{k|k-1}(\cdot|\cdot)$ and observation probability density $g_k(\cdot|\cdot)$ of each target meet:

$$f_{k|k-1}(x|\xi) = N(x; F_{k-1}\xi, Q_{k-1}) \tag{13}$$

$$g_k(z|x) = N(z; H_k x, R_k), \tag{14}$$

where $N(\cdot; \mu, \Sigma)$ indicates Gaussian density function whose mean value and variance are μ and Σ . F_{k-1} , Q_{k-1} , H_k , R_k represent state transition function, process noise covariance, observation matrix and measurement noise covariance.

4. Gaussian Mixture PHD Particle Filter Algorithm

The mean-variance and weight of Gaussian Elements recursively update, using the probability distribution of GPF approximation target, on the basis of Gaussian mixture framework and particle filter PHD recursive process. Suppose that prior density at moment k is in the form of Gaussian model and the densities of targets naturally occurring and transforming to new ones fulfill Gaussian mixture model. Therefore, the prediction and update processes [14] at time k are as follows:

Prediction:

$$\begin{aligned}
 v_{k|k-1}(x) &= v_{S,k|k-1}(x) + v_{\beta,k|k-1}(x) + \gamma_k(x) \\
 &= \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^i N(x; \mu_{k|k-1}^i, \Sigma_{k|k-1}^i),
 \end{aligned} \tag{15}$$

where $J_{k|k-1}$ is the number of predicted Gaussian component and $\omega_{k|k-1}^i$, $\mu_{k|k-1}^i$, $\Sigma_{k|k-1}^i$ respectively represents the weight, the mean value and the covariance of predicted Gaussian component. Besides, $v_{S,k|k-1}(x)$, $v_{\beta,k|k-1}(x)$, $\gamma_k(x)$ are the PHD of the survival targets, derivative ones and new ones between moment k-1 and moment k, which can be shown as follows:

$$v_{s,k|k-1}(x) = p_{s,k|k} \sum_{i=1}^{J_{k-1}} w_{k-1}^i N(x; \mu_{s,k|k-1}^i, \Sigma_{s,k|k-1}^i), \quad (16)$$

$$v_{\beta,k|k-1}(x) = \sum_{j=1}^{J_{k|k-1}} \sum_{\ell=1}^{J_{\beta,k}} w_{k-1}^j w_{\beta,k}^\ell N(x; \mu_{\beta,k|k-1}^{(j,\ell)}, \Sigma_{\beta,k|k-1}^{(j,\ell)}) \quad (17)$$

$$\gamma_k(x) = \sum_{i=1}^{J_{\gamma,k}} w_{k|k-1}^i N(x; \mu_{k|k-1}^i, \Sigma_{k|k-1}^i). \quad (18)$$

Formula 15 represents predicted PHD, then PHD at moment k after updating is:

$$\begin{aligned} v_k(x) &= (1 - p_{D,k})v_{k|k-1}(x) + \sum_{z \in Z_k} v_{D,k}(x; z) \\ &= (1 - p_{D,k})v_{k|k-1}(x) + \sum_{z \in Z_k} \sum_{j=1}^{J_{k|k-1}} w_k^j(z) N(x; \mu_{k|k}^j(z), \Sigma_{k|k}^j) \end{aligned} \quad (19)$$

Learning particle tag technology of Cluster-Indexed SMC-PHD [8] filter, add corresponding tag to each particle swarm, then the weight of each particle swarm, mixture component weights and the mean and variance of the Gaussian distribution at the moment k can be obtained through the following steps:

Step 1: predicting new targets and survival targets.

Sampling: $\tilde{x}_k^i = \begin{cases} q_k(\cdot | x_{k-1}^i, Z_k), i = 1, \dots, L_{k-1} \\ p_k(\cdot | Z_k), i = L_{k-1} + 1, \dots, L_{k-1} + J_k \end{cases} \quad (20)$

Step 1.1: predicting new (newly born → hatch) targets.

Newly born: $w_{k|k-1}^i = w_{\gamma,k}^i, \mu_{k|k-1}^i = \mu_{\gamma,k}^i, \Sigma_{k|k-1}^i = \Sigma_{\gamma,k}^i \quad (21)$

for $j = 1, \dots, J_{k-1}$
for $\ell = 1, \dots, J_{\beta,k}$

Hatch: $w_{k|k-1}^i = w_{k-1}^\ell w_{\beta,k}^j \quad (22)$
 $\mu_{k|k-1}^i = d_{\beta,k-1}^j + F_{\beta,k-1}^j \mu_{k-1}^\ell$
 $\Sigma_{k|k-1}^i = Q_{\beta,k-1}^j + F_{\beta,k-1}^j \Sigma_{\gamma,k}^i (F_{\beta,k-1}^j)^T$

Step 1.2: predicting the survival targets.

We can get the new particles $\{x_k^i\}_{i=1}^{L_k}$ after resampling the survival targets, then do prediction:

$$\begin{aligned} \text{for } j = 1, \dots, J_{k-1} \\ w_{k|k-1}^j &= p_{s,k} w_{k-1}^j \\ \mu_{k|k-1}^j &= F_{k-1} \mu_{k-1}^j \\ \Sigma_{k|k-1}^j &= Q_{k-1} + F_{k-1} \Sigma_{k-1}^j (F_{k-1})^T \end{aligned} \quad (23)$$

Step 2: updating

Step 2.1: updating the weights of each particle $i = 1, \dots, L_{k-1} + J_k$:

$$w_k^j(z) = \frac{p_{D,k} w_{k|k-1}^j q_k^j(z)}{K_k(z) + p_{D,k} \sum_{\ell=1}^{J_{k|k-1}} w_{k|k-1}^\ell q_k^\ell(z)} \quad (24)$$

Step 2.2: updating the weights of each particle from particle swarm $N_{R,k-1}$ according to (8):

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j \in I_{R,k-1}^n} \tilde{w}_{n,k}^j} \quad (25)$$

where $I_{R,k-1}^n = \{t_{k-1}^i = T_{R,k-1}^n\}_{i=1}^{L_{k-1}}$, $n = 1, \dots, N_{R,k-1}$.

Step 2.3: updating the corresponding component weights of each particle swarm:

$$\pi_{n,k} \approx \frac{\pi_{n,k-1} \tilde{w}_{n,k}}{\sum_{t=1}^{N_{R,k-1}} \pi_{t,k-1} \tilde{w}_{t,k}}, (n = 1, \dots, N_{R,k-1}) \quad (26)$$

where $\tilde{w}_{n,k} = \sum_{i \in I_{R,k-1}^n} \tilde{w}_k^i$.

Step 3: the estimated number of target is the sum of the number of targets not detected and the number of observations:

$$\tilde{N}_k = \tilde{N}_{k|k-1} \left(\sum_{i=1}^{J_{\beta,k}} w_{\beta,k}^i + p_{S,k} \right) + \sum_{i=1}^{J_{\gamma,k}} w_{\gamma,k}^i \quad (27)$$

Step 4: obtaining the new particle swarm $\{w_k^i, x_k^i, l_k^i\}_{i=1}^{L_k}$ after re-sampling.

5. Experiment Result

The algorithm in this paper is developed in the platform of Microsoft Visual Studio2008and OpenCV2.1, using C++. The video in the experiment is [cameral.mov](#)750-1100 frame, which comes from ftp://ftp.pets.rdg.ac.uk/pub/PETS2001/DATASET1/. This algorithm automatically initializes the target tracking window according to the inter-frame difference method in literature [15], combining regional minimum bounding rectangle method. New targets are kept until ultimately determined to be hatched target. Then carry on state estimation and target estimates over them. The judgments of the disappearance of the target and the appearance of new ones relay on the posterior distribution of target state, comparing the detection probability of the object in the observation scene with empirical data 1/3 and 2/3. If the detection probability is less than 1/3, we think that the target is lost. If the detection probability is larger than 1/3, we think that the target does still exist [16, 17]. The tracking performances of MPF filter and GMP-PHD filter are compared in the scene where the object number is changing and targets crossover and separation often occur. The performance of MPF algorithm is shown in picture one, where particles aggregate when the objects cross and the particles transfer to track one of the objects when they are separated. The performance of GMP-PHD filter is shown as picture 2. This algorithm can solve correlation between complex particles, so steady tracking is kept at the intersection and cross-post, which can be seen in picture 2(e) 943 frames.

GMP-PHD filter models and approaches the stochastic finite set of naturally occurring targets, newborn targets and survival targets in the framework of stochastic finite set theory, recursively calculating the probability hypothesis density function of multi-objective random sets, drawing the target number and state for effective tracking of multiple targets when the target number is unknown or changing. In picture 2(b) and (c) the new human targets have been successfully captured and tracked; in picture (d) multiple targets are merged when overlap happens between them. Picture (e) and (f) indicate the situation where target separation occurs and object runs out of the scene.

Table 1. Experiment Parameter

parameter	value	parameter	value
N_b	300	M	1
p_D	0.95	p_S	0.67

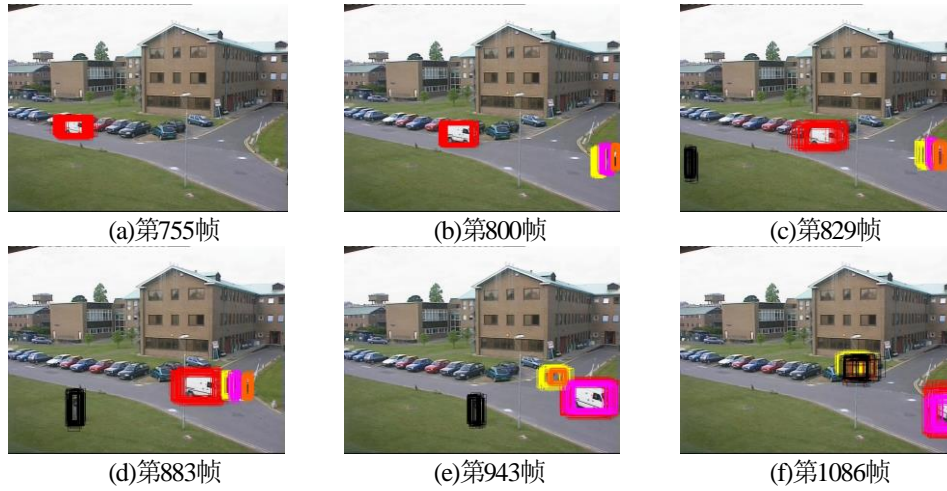


Figure 1. MPF Multi-target Tracking in Variable Target Number

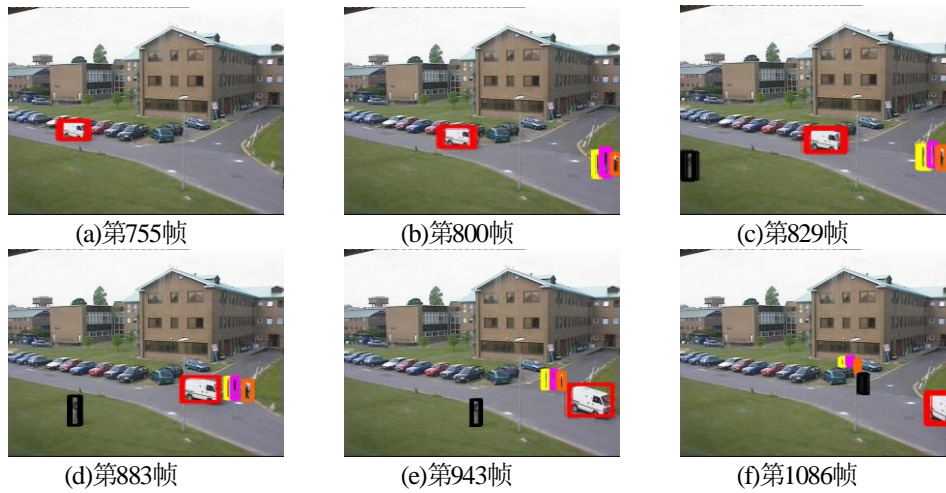


Figure 2. GMP-PHD Multi-target Tracking in Variable Number

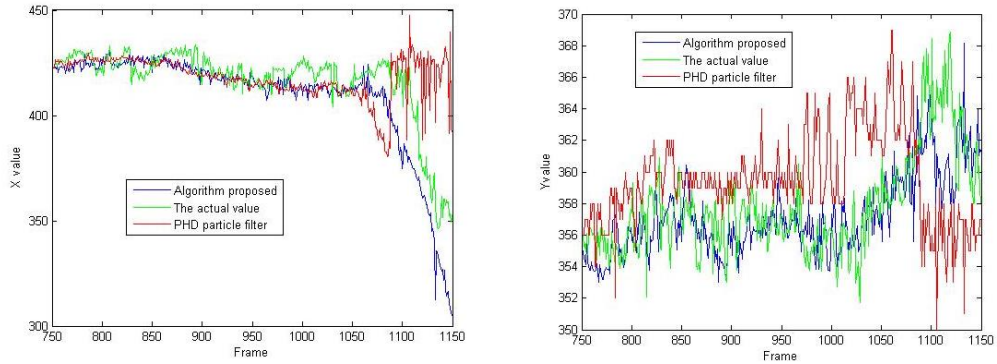


Figure 3. The X and Y Coordinates of a Moving Target

Figure 3 shows the x and y coordinates of a moving target, with the red, blue, green respectively on behalf of PHD particle filtering algorithm, actual value and the algorithms proposed in the paper. As can be seen from the figure, the overall effect of this algorithm has been improved. Figure 4 is a quantitative analysis of the actual effects. Chart 5 shows the comparison of traditional method and one proposed in this paper in evaluating the object number, using video (frame 765-frame 835).

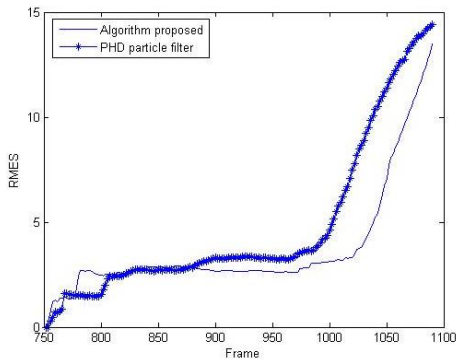


Figure 4. RMES Value

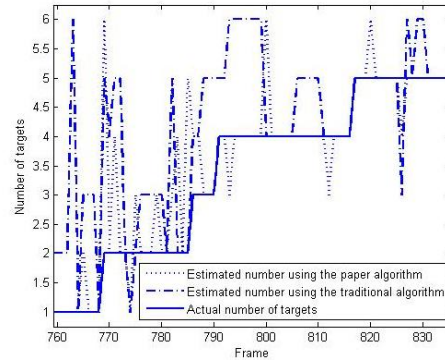


Figure 5. Target Number estimated

6. Conclusion

Multi-target detection and tracking of video in complex scenarios is successfully simulated. And Gaussian mixture particle filter PHD algorithm is proposed in the paper based on MP-PHD and GPF algorithm. Experimental results show that this method not only effectively maintain the multi-modal distribution of targets, but also have good performance in the scenes where object number is unknown or changing.

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