

# Energy-Efficient Power Allocation in OFDM-based Relaying Networks

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## Abstract

*This study deals with the energy-efficient (EE) power allocation for orthogonal frequency division multiplexing (OFDM)-based two-hop relaying networks under different relaying scenarios including amplify-and-forward (AF) and decode-and-forward (DF) with or without diversity. Firstly, we develop the equivalent channel gain (ECG) for any given subcarrier pair under every relaying scenario, and formulate the optimal EE problem. For the sake generality, a weighted factor is assigned to rate transmitted by the source on every subcarrier to reflect different priorities or quality-of-service (QoS) requirements. Then, by exploring the inherent structure and property of the EE design, we prove the quasiconcave relation between the EE and the transmit power, and provide an optimal solution for the EE maximum. Ordered subcarrier pairing (OSP) scheme is also introduced to further improve the EE for the relaying system. Simulation results are shown to compare the EE performance under different scenarios.*

**Key words:** *energy-efficient, power allocation, relaying networks, OFDM*

## 1. Introduction

To achieve reliable service in wireless communications, it is asked that user equipment (UE) must get its QoS requirement. With the rapid development in broadband wireless access technology, the mobile system can support higher data rates. However, one of the main challenges faced by the mobile system is the provision of high rates for mobiles at the cell edge. UEs at the cell edge often suffer from bad channel conditions, such as the large pathloss and shadowing, so that the signals between the source and the destination are severely impaired. Relaying transmission has become an effective way to overcome this problem as it provides throughput gains and coverage extension [1]. Several cooperation relay strategies have been proposed such as AF and DF [2].

There are many studies about the spectral efficiency (SE) for the OFDM-based relaying networks. In [3-4], optimal power allocation and subcarrier pairing under AF and DF relaying scenarios is studied, and OSP is optimal for both AF and DF relay links when optimal power allocation is applied, which has been proved in [4]. The relay selection, power allocation and subcarrier assignment problem is investigated [5]. In [6], DF and AF relay strategies are studied under different signal-to-noise ratio (SNR). [7] Proposes an opportunistic spectrum sharing protocol in relaying system, making the win-win between the primary and secondary systems. A relay-assisted OFDMA cellular system with joint consideration of direct and relaying paths is studied.

With the explosive growth of the high-data-rate wireless services, energy consumption of the wireless devices is rapidly increasing. Because of the battery-powered wireless station, growing requirement of ‘anytime and anywhere’ multimedia applications, the limited battery

and slow advancement of battery technology [9], the EE, which is one of the important performance measures for wireless system design, has become increasingly critical in wireless communication systems. In recent year, more and more people study the EE in wireless communication. In [10], the authors address EE resource allocation problem, and propose optimal and low-complexity suboptimal algorithms in both downlink and uplink of OFDMA network. The relationship between EE and SE is studied in downlink OFDMA network [11, 12]. Transforms the considered EE problem in fractional form into an equivalent optimal problem in subtractive form, and study the resource allocation for EE communication in multi-cell OFDMA downlink networks. EE radio resource scheduling is studied with QoS guarantees in a multi-user OFDMA system in [13], and the optimal subcarrier assignment is also considered. However, the EE gets little attention in relaying networks as far as the authors know. EE resource allocation in multiuser relay-based OFDMA networks is studied in [14]. The power allocation to maximize the EE for the two-hop AF relay link under QoS requirement is investigated [15].

In this paper, we address the EE power allocation in OFDM-based relaying networks. Two-hop relaying system, namely source-to-destination, the source-to-relay and the relay-to-destination links, is discussed. We study the ECG model for each subcarrier pair under different relaying scenarios, including AF and DF with or without two-hop diversity. In fact, the QoS requirement of every subcarrier may be different, so we consider weighted sum rate as the performance metric. An optimal solution is provided relaying on the quasiconcave relation between the EE and transmit power. Simulation results show the EE performance under different relaying scenarios.

The remainder of this paper is organized as follows. Section 2 introduces the system model and formulates the optimization EE problem. Section 3 develops the optimal power allocation algorithm to maximum the EE under different relaying scenarios. Simulation results are discussed in Section 4, and finally, concluding remarks are drawn in Section 5.

## 2. System Model and Problem Formulation

A two-hop OFDM relaying network is depicted in Figure 1, which consists of three nodes: a source (S), a destination (D) and a relay (R). OFDM with the same spectral occupancy is used for all links. The total system bandwidth is divided into  $K$  subcarriers. For each subcarrier, to avoid interference, only one node (the S or the R) transmits in a given time phase. In relaying scenario, the communication between the source and the destination is carried out in two phases. In the first phase, the source transmits signals over all the subcarriers, and the signals can be received by the destination and the relay simultaneously because of the broadcast nature of radio channels. In the second phase, the relay forwards the received signals to the destination over all the subcarriers. As assumed in [3], the destination can either combine signals copies from the two phases (with diversity) or just use the signals in the second phase (without diversity).

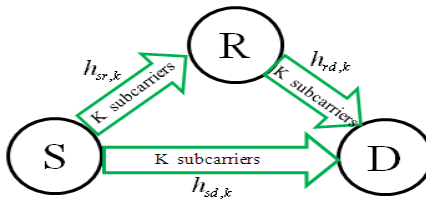


Figure 1. System Model for OFDM-based Two-hop Relaying Networks

In this work, we assume that the relaying system experiences independence and frequency-selective Rayleigh fading. The channel variances of S→D, S→R and R→D links are denoted as  $\delta_0^2, \delta_1^2, \delta_2^2$ . The channel coefficients of S→D, S→R and R→D links over subcarrier  $k(1 \leq k \leq K)$  are denoted as  $h_{sd,k}, h_{sr,k}$  and  $h_{rd,k}$ , respectively. We assume that all the noise terms are complex Gaussian random variables with zero mean and variance  $\delta^2=1$ . Therefore, the channel power gains are defined as  $\alpha_{sd,k}=|h_{sd,k}|^2$ ,  $\alpha_{sr,k}=|h_{sr,k}|^2$  and  $\alpha_{rd,k}=|h_{rd,k}|^2$ , respectively. The channels are assumed to remain constant in a two-hop period.

For different relaying scenarios of concern, the SE at the  $k$ th subcarrier pair is given by [3]

$$R_k^{\text{AF, w/o diversity}} = \frac{w_k}{2} \log_2 \left( 1 + \frac{p_{s,k} \alpha_{sr,k} p_{r,k} \alpha_{rd,k}}{p_{s,k} \alpha_{sr,k} + p_{r,k} \alpha_{rd,k}} \right) \quad (1)$$

$$R_k^{\text{AF, diversity}} = \frac{w_k}{2} \log_2 \left( 1 + p_{s,k} \alpha_{sd,k} + \frac{p_{s,k} \alpha_{sr,k} p_{r,k} \alpha_{rd,k}}{p_{s,k} \alpha_{sr,k} + p_{r,k} \alpha_{rd,k}} \right) \quad (2)$$

$$R_k^{\text{DF, w/o diversity}} = \frac{w_k}{2} \log_2 \left( 1 + \min(p_{s,k} \alpha_{sr,k}, p_{r,k} \alpha_{rd,k}) \right) \quad (3)$$

$$R_k^{\text{DF, diversity}} = \frac{w_k}{2} \log_2 \left( 1 + \max(p_{s,k} \alpha_{sd,k}, \min(p_{s,k} \alpha_{sr,k}, p_{r,k} \alpha_{rd,k} + p_{s,k} \alpha_{sd,k})) \right) \quad (4)$$

where  $p_{s,k}$  and  $p_{r,k}$  represent the transmit power at  $k$ th subcarrier for the resource and the relay, respectively. The factor 1/2 is due to the two-phase transmission, and  $w_k$  is the weighting factor of every subcarrier. Similar with literature [3], the item “1” in the denominator for (1) and (2) has been omitted in order to simplify.

According to [3], we can get the ECG model for the  $k$ th subcarrier pair:

$$R_k^* = \frac{w_k}{2} \log_2 (1 + p_k g_k) \quad (5)$$

where  $g_k$  is the ECG for the  $k$ th subcarrier pair, and  $R_k^*$  is the  $k$ th subcarrier SE under different relaying scenarios.

However,  $g_k$  is different under different relaying scenarios. According to literature [3], we summarize the ECG  $g_k$  at AF or DF scenario in Table 1.

**Table 1. The ECG and Subcarrier Pair Power under Different Relaying Scenarios**

			$g_k$	$p_{s,k}$	$p_{r,k}$
AF	w/o diversity	$\alpha_{sr,k} \neq \alpha_{rd,k}$	$\frac{\alpha_{sr,k} \alpha_{rd,k}}{(\sqrt{\alpha_{sr,k}} + \sqrt{\alpha_{rd,k}})^2}$	$\frac{-\alpha_{rd,k} + \sqrt{\alpha_{sr,k} \alpha_{rd,k}}}{\alpha_{sr,k} - \alpha_{rd,k}} p_k$	$\frac{\alpha_{rd,k} - \sqrt{\alpha_{sr,k} \alpha_{rd,k}}}{\alpha_{sr,k} - \alpha_{rd,k}} p_k$
		$\alpha_{sr,k} = \alpha_{rd,k}$		$p_k / 2$	$p_k / 2$
	diversity	$\alpha_{sd,k} < \alpha_{rd,k}$	$\frac{\alpha_{rd,k} (\alpha_k^* + \alpha_{sd,k})^2}{(\alpha_k^* + \alpha_{rd,k})^2}$	$\frac{\alpha_{rd,k} \alpha_k^* + \alpha_{sd,k} \alpha_{rd,k}}{(\alpha_k^*)^2 + \alpha_{rd,k} \alpha_k^*} p_k$	$\frac{\alpha_{sr,k} \alpha_{rd,k} - \alpha_{sr,k} \alpha_{sd,k}}{(\alpha_k^*)^2 + \alpha_{rd,k} \alpha_k^*} p_k$
		$\alpha_{sd,k} \geq \alpha_{rd,k}$	$\alpha_{sd,k}$	$p_k$	0
DF	w/o diversity		$\frac{\alpha_{sr,k} \alpha_{rd,k}}{\alpha_{sr,k} + \alpha_{rd,k}}$	$\frac{\alpha_{rd,k}}{\alpha_{sr,k} + \alpha_{rd,k}} p_k$	$\frac{\alpha_{sr,k}}{\alpha_{sr,k} + \alpha_{rd,k}} p_k$
	diversity	$\alpha_{sd,k} \leq \alpha_{sr,k}$ $\alpha_{sd,k} \leq \alpha_{rd,k}$	$\frac{\alpha_{sr,k} \alpha_{rd,k}}{\alpha_{sr,k} + \alpha_{rd,k} - \alpha_{sd,k}}$	$\frac{\alpha_{rd,k}}{\alpha_{sr,k} + \alpha_{rd,k} - \alpha_{sd,k}} p_k$	$\frac{\alpha_{sr,k} - \alpha_{sd,k}}{\alpha_{sr,k} + \alpha_{rd,k} - \alpha_{sd,k}} p_k$
		otherwise		$\alpha_{sd,k}$	$p_k$

Consequently, we can get the overall throughput and total transmit power for the relaying

system:

$$R = \sum_{k=1}^K R_k^* = \sum_{k=1}^K \frac{w_k}{2} \log_2(1 + p_k g_k)$$

$$P = \sum_{k=1}^K p_k = \sum_{k=1}^K (p_{s,k} + p_{r,k}) \quad (6)$$

Besides transmit power, the energy consumption also includes circuits consumption incurred by active circuit blocks [10]. So the overall power consumption is given [15]

$$P_{total} = P + P_c \quad (7)$$

where  $P_c$  denotes circuit power of the source and the relay.

Different from previous studies, we consider the energy efficiency maximum under satisfying with user's QoS. Compared with the spectrum efficiency maximum, EE maximum is very meaningful, especially for mobile terminals that are not able to connect to an external charger due to battery capacity.

Accordingly, the optimal EE problem can be formulated as following:

$$\max \frac{\sum_{k=1}^K R_k^*}{\sum_{k=1}^K p_k + P_c} \quad (8a)$$

subject to

$$\sum_{k=1}^K R_k^* \geq R_{min} \quad (8b)$$

$$\sum_{k=1}^K p_k \leq P_T \quad (8c)$$

$$p_k \geq 0, k \in \{1, 2, \dots, K\} \quad (8d)$$

where  $R_{min}$  is the required minimum rate for the relaying system.

### 3. Optimal Power Allocation

In order to solve the problem (8), for a certain total power  $P$ , we define the following problem:

$$\eta(P) @ \max_{p_k \geq 0} \frac{R^*(\mathbf{P})}{P + P_c} @ \max_{p_k \geq 0} \frac{\sum_{k=1}^K \frac{w_k}{2} \log_2(1 + p_k g_k)}{\sum_{k=1}^K p_k + P_c} \quad (9a)$$

subject to

$$\sum_{k=1}^K R_k^* \geq R_{min} \quad (9b)$$

$$\sum_{k=1}^K p_k = P \quad (9c)$$

Based on the problem (9), we summarize the following theorem and give the detailed proof process.

**Theorem.** 1) EE  $\eta(P)$  is continuously differentiable and strictly quasiconcave in  $P$ .

$$2) \frac{d\eta(P)}{dP} \begin{cases} > 0 & \text{if } \eta(P) \leq \frac{dR(P)}{dP} \\ = 0 & \text{if } \eta(P) = \frac{dR(P)}{dP} \\ < 0 & \text{if } \eta(P) > \frac{dR(P)}{dP} \end{cases}$$

Where  $R(P) \square \max_{p_k \geq 0} R^*(P) = \max_{p_k \geq 0} \sum_{k=1}^K \frac{w_k}{2} \log_2(1 + p_k g_k)$  under constraints (9b) and (9c). And its derivative satisfies  $\frac{dR(P)}{dP} \equiv \max_{k \in K} \frac{w_k g_k \log_2 e}{1 + g_k p_k}$ .

**Proof:** 1) First, we prove that  $R(P)$  under the constraint (9b) is strictly concave and continuously differentiable in  $P$ . We can easy get that the transmit power on each subcarrier is nondecreasing with the total transmit power because of the nature of water-filling. Then we consider the limit under the constraint  $\sum_{k=1}^K \Delta p_k = \Delta P$  in (10). The existence of the limit indicates that  $R(P)$  is continuously differentiable in  $P$  and  $\frac{dR(P)}{dP} = \max_{k \in K} \frac{w_k g_k \log_2 e}{1 + g_k p_k}$ . Accordingly,  $\eta(P)$  is continuously differentiable in  $P$ . Moreover  $\frac{w_k g_k \log_2 e}{1 + g_k p_k}$  is nonincreasing with  $P$ , while  $\max_{k \in K} \frac{w_k g_k \log_2 e}{1 + g_k p_k}$  is strictly monotonically decreasing with  $P$ . Thus,  $\frac{d^2 R(P)}{dP^2} < 0$  and  $R(P)$  is strictly concave in  $P$ .

Denote the superlevel sets of  $\eta(P)$  as  $S_{\alpha} = \{P \geq P_{min} | \eta(P) \geq \alpha\}$ . According to [17],  $\eta(P)$  is strictly quasiconcave in  $P$  if  $S_{\alpha}$  is strictly convex for any real number  $\alpha$ . When  $\alpha < 0$ , no points

$$\frac{dR(P)}{dP} = \lim_{\Delta P \rightarrow 0} \frac{R(P + \Delta P) - R(P)}{\Delta P} \quad (10)$$

$$\begin{aligned} &= \lim_{\Delta P \rightarrow 0} \frac{\max_{\Delta p_k \geq 0} \sum_{k=1}^K \frac{w_k}{2} \log_2(1 + g_k(p_k + \Delta p_k)) - \max_{\Delta p_k \geq 0} \sum_{k=1}^K \frac{w_k}{2} \log_2(1 + g_k p_k)}{\Delta P} = \lim_{\Delta P \rightarrow 0} \frac{\max_{\Delta p_k \geq 0} \sum_{k=1}^K \frac{w_k}{2} \log_2 \left( \frac{1 + g_k(p_k + \Delta p_k)}{1 + g_k p_k} \right)}{\Delta P} \\ &= \lim_{\Delta P \rightarrow 0} \frac{\max_{\Delta p_k \geq 0} \sum_{k=1}^K \frac{w_k g_k \log_2 e}{2} \Delta p_k}{\Delta P} = \max_{k \in K} \frac{w_k g_k \log_2 e}{1 + g_k p_k} = \max_{k \in K} \frac{w_k g_k \log_2 e}{2} \frac{1}{1 + g_k p_k} \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d\eta(P)}{dP} &= \lim_{\Delta P \rightarrow 0} \frac{\frac{R(P + \Delta P)}{(P + \Delta P) + P_c} - \frac{R(P)}{P + P_c}}{\Delta P} = \lim_{\Delta P \rightarrow 0} \frac{\frac{R(P + \Delta P) - R(P) + R(P)}{(P + \Delta P) + P_c} - \eta(P)}{\Delta P} = \lim_{\Delta P \rightarrow 0} \frac{\frac{R(P + \Delta P) - R(P)}{(P + \Delta P) + P_c} + \frac{R(P) - \eta(P)((P + \Delta P) + P_c)}{(P + \Delta P) + P_c}}{\Delta P} \\ &= \lim_{\Delta P \rightarrow 0} \frac{\frac{R(P + \Delta P) - R(P)}{(P + \Delta P) + P_c} + \frac{-\eta(P)\Delta P}{(P + \Delta P) + P_c}}{\Delta P} = \lim_{\Delta P \rightarrow 0} \frac{\frac{R(P + \Delta P) - R(P)}{\Delta P} - \eta(P)}{(P + \Delta P) + P_c} = \lim_{\Delta P \rightarrow 0} \frac{\frac{dR(P)}{dP} - \eta(P)}{(P + \Delta P) + P_c} = \frac{\frac{dR(P)}{dP} - \eta(P)}{P + P_c} \end{aligned}$$

exist on the counter  $\eta(P) = \alpha$ . When  $\alpha \geq 0$ ,  $S_{\alpha}$  is equivalent to  $S_{\alpha} = \{P \geq P_{min} | \alpha P + \alpha P_c - R(P) \leq 0\}$ . We have proved that  $R(P)$  is strictly concave in  $P$ , so  $S_{\alpha}$  is strictly convex in  $P$ . Therefore,  $\eta(P)$  is continuously differentiable and strictly quasiconcave in  $P$ .

2) From (11), we can get that  $\text{sgn}\left(\frac{d\eta(P)}{dP}\right) = \text{sgn}\left(\frac{dR(P)}{dP} - \eta(P)\right)$ , where  $\text{sgn}(a)$  denotes the sign of  $a$ . This completes the proof of theorem.

For any strictly quasiconcave function, there is always a unique global maximum. Above theorem guarantees the existence and uniqueness of the global maximum. However, in order to get the optimal power allocation, we need to solve the following problems:

**Problem 1:** Getting the minimum power satisfied with the QoS for the relaying system.

To solve problem 1, we can get the optimal problem as following:

$$\min_{p_k > 0} \sum_{k=1}^K p_k \tag{12a}$$

$$\text{s.t.} \quad \sum_{k=1}^K R_k^* \geq R_{\min} \tag{12b}$$

It is very easy to solve problem (12), we can get the optimal power allocation by water-filling equation:

$$p_{k,\min} = \left( \frac{2^\varphi w_k}{2 \ln 2} - \frac{1}{g_k} \right)^+, \quad \varphi = \frac{2R_{\min} - \sum_{k=1}^K w_k \log_2 \left( \frac{w_k g_k}{2 \ln 2} \right)}{\sum_{k=1}^K w_k} \tag{13}$$

Where  $(\square)^+$  represents  $\max(x, 0)$ , and we define  $P_{\min} = \sum_{k=1}^K p_{k,\min}$ . Without loss of generality, we assume  $P_T > P_{\min}$ .

**Problem 2:** Getting the maximum rate for a given power  $P$ .

For above the problem, we can formulate the optimal problem as following:

$$\max_{p_k \geq 0} \sum_{k=1}^K R_k^* \tag{14a}$$

$$\text{s.t.} \quad \sum_{k=1}^K p_k = P \tag{14b}$$

The optimal power,  $P_k$ , can be got by water-filling equation:

$$p_k = \left( \frac{w_k \left( \sum_{k=1}^K \frac{1}{g_k} + P \right)}{\sum_{k=1}^K w_k} - \frac{1}{g_k} \right)^+ \tag{15}$$

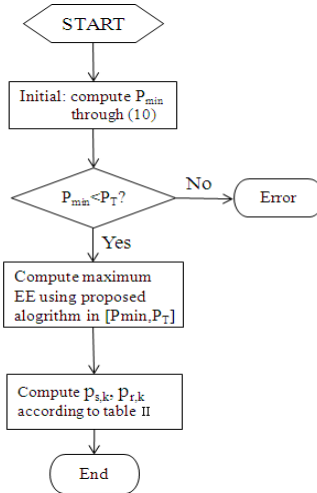
From above the theorem, we can get that the maximum EE,  $\eta(P)$ , is quasiconcave in range  $[P_{\min}, P_T]$ . For this problem that finding optimal value of a concave or quasiconcave function in a certain range, there are some developed algorithms in [16]. Next, we will describe our search algorithm for optimal power allocation in detail. First, we get the initial value  $P_{\min}$  by solving the problem 1 and get the maximum EE in  $P_{\min}$ . At the same time, the maximum EE in  $P_T$  can also be got by water-filling equation (15). Then we can find the optimal power to maximum EE by Golden Section Search (GSS) algorithm, which is illustrated in Table 2. After getting the optimal power in every ECG subcarrier, we can work out the optimal power allocation for every subchannel at resource and relay by Table 1.

Certainly, we can also search optimal value according to theorem (2). Due to the limited space available, we will further discuss this search algorithm in later research.

**Table 2. Golden Section Search Algorithm**

- 
- 1) Initialization:  $[a_0, b_0]$ ,  $\delta > 0$ ,  $\lambda = 0.618$ ,  
 $k = 0$ .
  - 2)  $x_{k+1} = a_0 + \lambda(b_0 - a_0)$ ,  $x_{k+1}^* = a_0 + (1 - \lambda)(b_0 - a_0)$ ,  
calculate  $\varphi(x_{k+1})$ ,  $\varphi(x_{k+1}^*)$ .
  - 3) If  $\varphi(x_{k+1}) \leq \varphi(x_{k+1}^*)$ , go to step 4).  
else go to step 5).
  - 4)  $a_{k+1} = a_k$ ,  $b_{k+1} = x_{k+1}$ ,  
if  $[(b_{k+1} - a_{k+1}) / (b_0 - a_0)] < \delta$ , go to 6).  
else {  $x_{k+2} = x_{k+1}$ ,  
 $x_{k+2}^* = a_{k+1} + (1 - \lambda)(b_{k+1} - a_{k+1})$ ,  
calculate  $\varphi(x_{k+2})$ ,  $\varphi(x_{k+2}^*)$ ,  
 $k = k + 1$ , go to step 3). }
  - 5)  $a_{k+1} = x_{k+1}$ ,  $b_{k+1} = b_k$ ,  
if  $[(b_{k+1} - a_{k+1}) / (b_0 - a_0)] < \delta$ , go to 6).  
else {  $x_{k+2} = x_{k+1}$ ,  
 $x_{k+2} = a_{k+1} + (1 - \lambda)(b_{k+1} - a_{k+1})$ ,  
calculate  $\varphi(x_{k+2})$ ,  $\varphi(x_{k+2}^*)$ ,  
 $k = k + 1$ , go to step 3). }
  - 6) Finish:  $\theta = (b_{k+1} + a_{k+1}) / 2$ , calculate  $\varphi(\theta)$ .
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Based on the above discussion, a detailed flowchart of the proposed algorithm is given in Figure 2.



**Figure 2. Flowchart of the Optimal Power Allocation Algorithm**

Due to the independent fading in the same subcarrier on the two phases, the results cannot be optimal if the signal received by the relay from the source is forwarded on the same subcarrier. It has been shown in [4] that in this case, the ordered subcarrier pairing (OSP) can get further performance under optimal power allocation, which means that the best  $S \rightarrow R$  subcarrier in the first phase with the best  $R \rightarrow D$  subcarrier in the secondary phase, pairing the next best  $S \rightarrow R$  subcarrier in the first phase with next best  $R \rightarrow D$  subcarrier in the secondary phase, and until all the subcarrier are paired. Therefore, we also adopt the OSP to improve the EE under different relaying scenarios.

Now, we analyze the complexity of the proposed GSS algorithm. When we have  $k$  iterations, the interval length at the  $k$ th iteration:

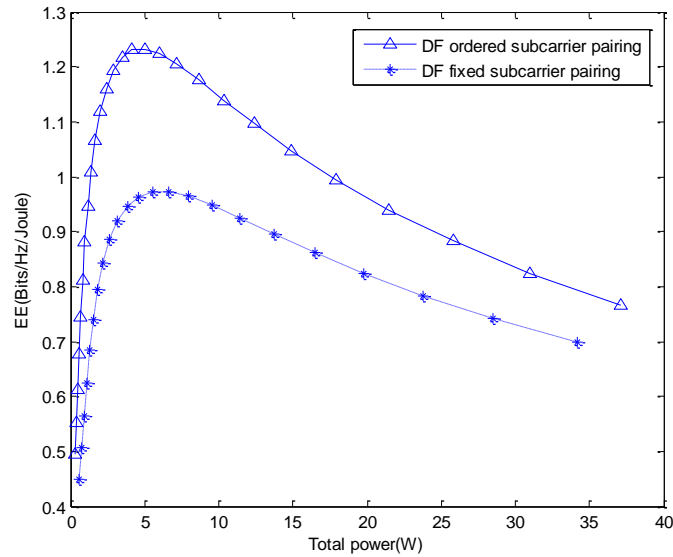
$$(b_k - a_k) = 0.618^k (b_0 - a_0) \quad (16)$$

From  $\frac{(b_k - a_k)}{(b_0 - a_0)} = 0.618^k = \delta$ , we can get that the GSS needs at least  $O(\log_{0.618}(\delta))+1$  times of

water-filling for the accuracy required  $\delta$ . The complexity of the order subcarrier pairing is  $O(2K^2)$ ,  $K$  is the number of subcarriers.

#### 4. Simulation Results

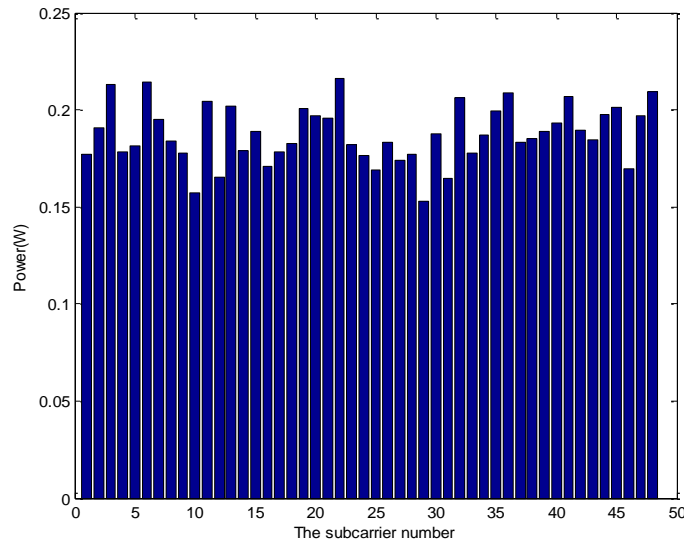
In this section, simulation results are presented to demonstrate the EE performance under different relaying scenarios. In simulation, the total power at the source and the relay is constrained to be 20 Watt, and the circuit power at the source and the relay is 5Watt. The number of the subcarrier,  $K$ , is 48, and subcarrier spacing is 50kHz. For the resource, the minimum rate requirement is 3bit/s/Hz. We set  $\delta_1^2 = \delta_2^2 = 0$ dB for all the simulations. We assume all channels are independent, identically distributed Rayleigh random variable.



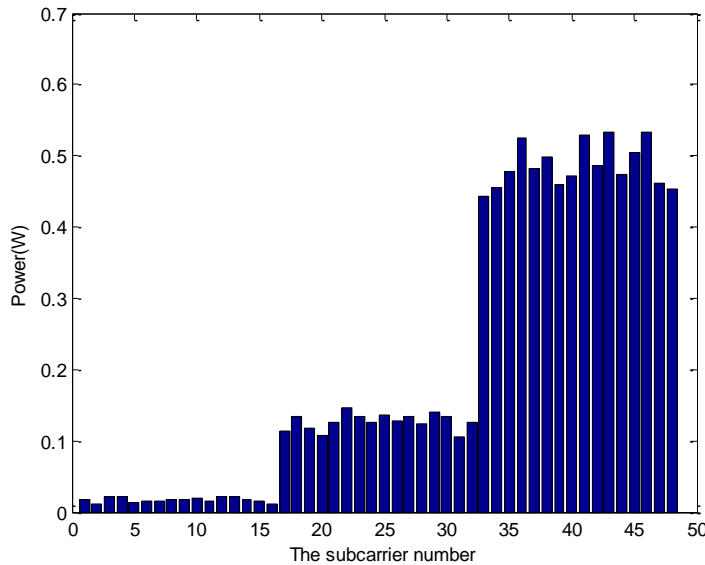
**Figure 3. EE Versus Total Power with  $w_k=1(k=1:K)$**

Figure 3 demonstrates the EE-Power relation in the case that DF with diversity for ordered subcarrier pairing or fixed subcarrier pairing, and  $\delta_0^2 = -8$ dB. Fixed subcarrier pairing is that the signal transmitted by the source on the subcarrier is forwarded on the same subcarrier by relay to the destination. The Figure shows that as the total power increases, the energy-efficiency first increases and then decreases, and gets maximum when the total power is about 5 Watt. It also indicates that energy-efficiency is quasiconcave in total power, and there is always a unique global maximum. From the Figure, we can get that the energy-efficiency is higher with ordered subcarrier pairing than that with fixed ordered subcarrier, and it has been proved in [4] that ordered subcarrier pairing is optimal.



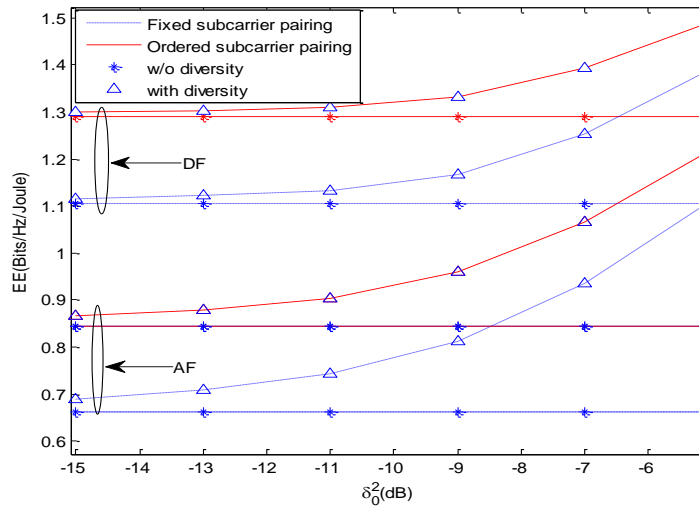


**Figure 4. Every Subcarrier Power with  $w_k=1(k=1:K)$**

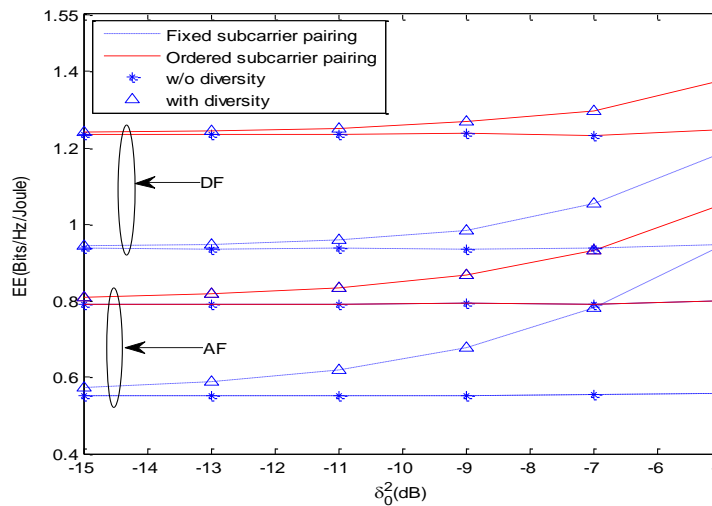


**Figure 5. Every Subcarrier Power with  $w_k(k=1:16)=1$ ,  $w_k(k=17:32)=2$ ,  $w_k(k=33:48)=4$**

In Figures 4 and 5, the power of every subcarrier with different weight  $w_k$  is present. Figure 4 plots the power of every subcarrier when the weighted factor is the same, and the power of every subcarrier is allocated according to their channel gain only. Figure 5 plots the power of every subcarrier when weighted factor is different. We can find when the weighted factor for a given subcarrier is larger, the power assigned to this subcarrier is bigger, and the rate is higher. So, in an actual system, we can allocate a larger weighted factor for a subcarrier when the QoS requirement is higher for this subcarrier.



**Figure 6. EE Versus  $\delta_0^2$  with  $w_k=1(k=1:K)$**



**Figure 7. EE Versus  $\delta_0^2$  with  $w_k(k=1:16)=1, w_k(k=17:32)=2, w_k(k=33:48)=4$**

Figures 6 and 7 compare the energy efficiency achieved by different relaying scenarios when  $w_k$  is different. Due to  $S \rightarrow D$  link is not considered under AF or DF without diversity scenario, the EE remains the same when  $\delta_0^2$  is changed. From these results, we observe that the energy efficiency with ordered subcarrier pairing is higher than that with fixed subcarrier pairing, and the energy efficiency under DF scenario is also better than that under AF scenario for a given  $\delta_0^2$ . It is very easy to understand this phenomenon from their principle. In AF, the signal received by relay is amplified and retransmitted to the destination, and the noise is also amplified at the relay. In DF, the relay attempts to decode the received signal. If successful, it reencodes the information and retransmits signal, so the noise is not amplified. When  $\delta_0^2$  is lower, the gap of energy efficiency is larger between DF and AF. However, as the  $\delta_0^2$  increases, the gap decreases. It indicates that as the distance between the source and the destination

decreases, it is better to use direct communication for every subcarrier. However, we find that the energy efficiency in Figure 6 is higher than that in Figure 7 for the same condition. We can get that the sum energy efficiency is the highest when the weight of every subcarrier is the same. In fact, from equation (15), Figures 4 and 5, we can find that the power of every subcarrier depends on its weight and channel gain. When the weight is the same, the power of every subcarrier only depends on its channel gain, the better the channel gain, the more the power, and the sum rate can be got maximize. However, when the weight is different, the subcarrier may not get more power even its channel gain is bigger, for example its weight is small, and the sum rate may not get maximize. So the energy efficiency is higher in Figure 6 than that in Figure 7.

## 5. Conclusion

In this paper, we study the EE power allocation in OFDM-based relaying networks. In order to simplify the system model, ECG scenario is proposed. We compare the EE under different relaying scenarios, including AF and DF with or without diversity, and subcarrier pairing is also considered. The priorities or QoS requirement of every subcarrier is reflected by weighted factor  $w_k$ , and the factor can be set larger when its QoS is higher, vice versa. Though the analysis of this paper, under satisfying the QoS of the relaying system, we can select the optimal relaying scenario and power allocation by considering the EE performance for the relaying system when the channel variances of S→R, R→D and S→D are known.

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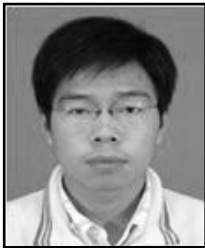
## References

- [1] R. Pabst, B. H. Walke, D. C. Schultz and P. Herhold, "Relay-based deployment concepts for wireless and mobile broadband radio", IEEE Communications magazine, vol. 42, no. 9, (2004), pp. 80-89.
- [2] J. N. Laneman, D. N. C. Tse and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior", IEEE Transactions on Information Theory, vol. 50, no. 12, (2004), pp. 3062-3080.
- [3] Y. Li, W. Wang, J. Kong, W. Hong, X. Zhang and M. Peng, "Power allocation and subcarrier pairing in OFDM-based relaying networks", IEEE International Conference on Communications, (2008), pp. 2602-2606.
- [4] Y. Li, W. Wang, J. Kong and M. Peng, "Subcarrier pairing for amplify-and-forward and decode-and-forward OFDM relay links", IEEE Communications Letters, vol. 13, no. 4, (2009), pp. 209-211.
- [5] M. Alam, J. Mark, and X. Shen, "Relay Selection and Resource Allocation for Multi-User Cooperative OFDMA Networks", IEEE Transactions on wireless communications, vol. 12, no. 5, (2013), pp. 2193-2205.
- [6] F. He, Y. Sun, L. Xiao, C. Chi and S. Zhou, "Capacity region bounds and resource allocation for two-way of dm relay channels", IEEE Transactions on wireless communications, vol. 12, no. 6, (2013), pp. 2904-2217.
- [7] W. D. Lu, Y. Gong, S. H. Ting, X. L. Wu and N. T. Zhang, "Cooperative OFDM relaying for opportunistic spectrum sharing: protocol design and resource allocation", IEEE Transactions on Wireless Communications, vol. 11, no. 6, (2012), pp. 2126-2135.
- [8] B. Da, and C. C. Ko, "Dynamic resource allocation in relay-assisted OFDMA cellular system", Transactions on Emerging Telecommunications Technologies, vol.23, no. 1, (2012), pp. 96-103.
- [9] K. Lahiri, S. Dey, D. Panigrahi and A. Raghunathan, "Battery-driven system design: A new frontier in low power design", Proceedings of the 2002 Asia and South Pacific Design Automation Conference. IEEE

Computer Society, (2002), pp. 261.

- [10] C. Xiong, G. Y. Li, S. Zhang, Y. Chen and S. Xu, "Energy-efficient resource allocation in OFDMA networks", IEEE Global Telecommunications Conference, (2011), pp. 1-5.
- [11] C. Xiong, G. Y. Li, S. Zhang, Y. Chen and S. Xu, "Energy-and spectral-efficiency tradeoff in downlink OFDMA networks", IEEE Transactions on Wireless Communications, vol. 10, no. 11, (2011), pp. 3874-3886.
- [12] D. W. K. Ng, E. S. Lo and R. Schober, "Energy-Efficient Resource Allocation in Multi-Cell OFDMA Systems with Limited Backhaul Capacity". IEEE transactions on wireless communications, vol. 11, no. 10, (2012), pp. 3618-3631.
- [13] J. Zhang, Y. Jiang and X. Li, "Energy-efficient resource allocation in multiuser relay-based OFDMA networks", Concurrency and Computation: Practice and Experience, vol. 25, no. 9, (2012), pp. 1113-1125.
- [14] X. Xiao, X. Tao and J. Lu, "QoS-Aware Energy-Efficient Radio Resource Scheduling in Multi-User OFDMA Systems", IEEE transactions on wireless letters, vol. 17, no. 1, (2013), pp. 75-78.
- [15] H. Yu, H. Qin, Y. Z. Li, Y. Zhao, X. Xu and J. Wang, "Energy-efficient power allocation for non-regenerative OFDM relay links", Science China Information Sciences, vol. 56, no. 2, (2013), pp. 185-192.
- [16] S. Cui, A. J. Goldsmith and A. Bahai, "Energy-constrained modulation optimization. IEEE Transactions on Wireless Communications", vol. 4, no. 5, (2005), pp. 2349-2360.
- [17] S. P. Boyd and L. Vandenberghe, "Convex optimization", Cambridge University press, (2004).

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