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Reliability Model for Warm Standby System under Consideration of Replace Time

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Abstract

There are two effective ways to maintain the system to perform a high or required level of reliability and availability: one is spare units for key units; the other is several maintainers for the system. To solve reliability indexes of complex mechanical system under these assumptions, a system model was established, which consists of two dissimilar subsystems in series and each subsystem is warm standby system, which consists two same units. We divided maintain time into replace time and repair time. In this paper, we arrange two maintainers for the system, and take 'the system in normal states' as maintenance strategy. Taking high efficiency of cooperative maintainers into account, the formula of system availability, system reliability, rate of occurrence of failure (ROCOF) and mean time to first failure (MTTFF) can be derived by operating the state transition probability matrix. This research provides evidence for further studies of complex mechanical system reliability.

Keywords*: system reliability, maintenance strategy, warm standby, series system, Markov process*

1. Introduction

As far as reliability is concerned, it has been researched as a useful tool for risk analysis, production availability studies and design of systems [1-3].In general case, many industrial systems consisted of massive units or subsystems, such as airplane, fleet, power supply systems and so on, were considered as complicated systems[4-6].Availability has been considered as an important measure of performance for these systems which are generally considered as repairable ones [7-11]. These systems undergo during their lifetimes multiple unscheduled failure and repair cycles. System performance depends on reliability and availability of the subsystems/units, operate environment, maintenance efficiency, operation process and technical expertise of operators, *etc*. To improve the system reliability and availability, implementation of appropriate maintenance strategies play an important role. On the other hand availability of the system can be improved by improvement in its reliability and maintainability[12].Lam[13] investigated a two-component parallel repairable system assuming that all work and repair time obeys the exponential distributions and one unit after repair is 'as good as new', while the other is not. The system ROCOF is determined through the Monte Carlo numerical method. Nakagawa and Osaki[14] assumed that both the work life and the repair time of the priority unit are general distributed, but that of the ordinary unit are exponential distributed. Some reliability indexes of the system were obtained by using Markov renewal theory. Recently, Sharma *et al*. [15] model the performance of a urea plant using RAM analysis by applying Markovian approach. They utilized crisp historical data without quantification of involved uncertainties. Srinivasan *et al*. [16] analyzed a three-unit system consisting of a single unit working online and two warm standby units. The failure time and the repair time of the online unit have general distributions while in standby situation the failure rate is a constant. A two-unit cold standby repairable system with one maintainer and use priority is often used. Zhang *et al*. [17] studied a cold standby repairable system consisting of two dissimilar components and one repairman. And the repair time distributions of the two components are both exponential and component 1 is given priority in use. The explicit expression for the long-run average cost per unit time of the system is evaluated.

Rakesh [18] studied that whenever an operative unit fails, a delay occurs in locating the repairman and getting him to the system location due to certain administrative actions. Yang *et al*. [19] established a model for a discrete-time one-unit repairable system with delay repair. It is supposed that the system has three states: normal, repair and waiting repair, and that the system life, repair time and waiting repair time all have general distribution. Yang *et al*. [20] also analyzed the continuous-time model and discrete-time instantaneous availability model of the delay repairable system, established the instantaneous availability models in continuous and discrete time are proposed for the one-unit-delay repairable systems with exponential distribution, and get the numerical solutions for the continuous-time model and the discrete-time model. Zhang *et al*. [21] considered a replacement policy *N* based on the number of failures of the system. They determined the optimal policy *N* such that the long-run expected profit per unit time is maximized and showed that the model for the multistate system forms a general monotone process model which includes the geometric process model as a special case. Lam [22] reported a maintenance model for two-unit redundant system with one repairman. Under this model, he studied two kinds of replacement policy, respectively based on the number of failure and the work age for two units. The long-run average cost per unit time for each kind of replacement policy is derived. Sridharan [23], Mokaddis[24] and Parashar[25] investigated the probabilistic analysis of a two-unit standby system with two types of repairmen and patience time. The first repairman, usually called regular repairman, always remained with the repair facility. An expert repairman was called for the system if and only if the regular repairman is unable to do the job, within some fixed time or on system failure.

Most mechanical systems can be equivalent to series systems. Maintaining the system to perform a high or required level of reliability and availability is often an essential requisite. In practice, the key units are often equipped with spare units. The system can be boiled down to several standby systems in series. During the standby time of the units, because of the aging effect and the environment influences, many units (especially some precise instruments) will break down. So the subsystems are warm standby systems. In this paper, maintenance time divides into two parts. One part is replace time and the other part is maintenance time. In the series system which has two parts, each part is equipped with an identical spare unit as replace unit in fault state. The standby units have risk of failure. In this system model, there are two maintainers, who have the same work efficiency, also can repair one failure unit at the same time. To make sense, cooperative efficiency must be more than one maintainer's efficiency, and less than sum of two single maintainers'. Different maintenance strategy makes the result of reliability and availability difference. We take 'the system in normal states' as the first priority. For example: if one unit of the system is failure, and the spare unit is in normal state at the moment, the two maintainers stop the current work, instead, they cooperate to replace the failure unit by installing the spare part. Another time, if there is only one failure unit, two maintainers can cooperate; If there are two failure units, two maintainers work on separate units. Furthermore, we assume that the work lives distributions and the repair time distributions of the two subsystems are both exponential, and if system states are defined appropriately, the system can be described by Markov process. By using state transition rate matrix and the Laplace transform, not only some important reliability indexes but also some important steady system indexes are derived.

NOTATION

E	system state space
W	system work state space
F	system failure state space
$X(t)$	system is in state i, $i\in E$, at time t a stochastic process
$P_i(t)$	probability of system in state i, $i\in E$ at time t.
$N(t)$	system failure times before time t
$M_i(t)$	mean system failure times in (0, t] where system is in state i at time t=0
$m_i(t)$	rate of occurrence of failure where the system in state i , $i\in E$ at time t=0
$Q_i(t)$	probability of absorbing system in state i at time t
X_i	work live of unit in subsystem i, i=1,2

2. Model Assumption

Now, we present a repairable system with multi-states by making the following assumptions.

Assumption 1. This repairable system is connected of two dissimilar warm standby systems in series; each warm standby system is composed of two same units. They are independent with each others. In the following assumptions, i=1, 2.

Assumption 2. Work lives obey dissimilar exponential distributions with parameters w_i; Warm standby lives obey different exponential distributions with parameters c_i.

Assumption 3. Two maintainers work for this system. They have the same work efficiency, and after maintenance, failure units are as good as new. We should consider replace time in the maintenance process. Maintenance time obeys different exponential distributions with parameters r_{i1}. In order to recover the system quickly, the two maintainers should replace cooperatively. Replace time obeys different exponential distributions with parameters h_i.

Assumption 4. We take the 'first repair to first to failure unit' as the repair rule. Just in the standby state, the failure unit can be repaired, *i.e*. the work unit can not be repaired immediately after failure. Under the replace process, we do not consider standby failure.

Assumption 5. If there is only one failure unit or two failure units in the same subsystem, two maintainers cooperate with failure unit or the first failure unit, and cooperate can improve repair efficiency.Repair time obey different exponential distributions with parameters r_{i2}.

3. Model Analysis

14 mutually exclusive states flow from these assumptions, as follow:

State 0: none of system is failure;

State 1: In subsystem 1, two maintainers begin to cooperate to repair (rush repair) the failure unit, the other unit begins to work. All units are normal in subsystem 2.

State 2: In subsystem 2, two maintainers begin to rush repair the failure unit, the other unit begins to work. All units are normal in subsystem 1.

State 3: In subsystem 1, failure unit begins to be repaired; the other unit begins to work. In subsystem 2, failure unit begins to be repaired; the other unit begins to work.

State 4: In subsystem 1, two units are all failure. In subsystem 2, two units are all normal. Two maintainers begin to rush repair the first failure unit.

State 5: In subsystem 2, two units are all failure. In subsystem 1, two units are all normal. Two maintainers begin to rush repair the first failure unit.

State 6: In subsystem 1, two units are all failure. In subsystem 2, the standby unit is failure. Two maintainers begin to repair the first failure unit in subsystem1.

State 7: In subsystem 2, two units are all failure. In subsystem 1, the standby unit is failure. Two maintainers begin to repair the first failure unit in subsystem2.

State 8: In subsystem 1, two units are all failure. In subsystem 2, the work unit is standby failure. Two maintainers begin to rush repair the first failure unit in subsystem 1.

State 9: In subsystem 2, two units are all failure. In subsystem 1, the work unit is standby failure. Two maintainers begin to rush repair the first failure unit in subsystem 2.

State 10: Four units are all failure. Two maintainers begin to rush repair the first failure unit in subsystem 1.

State 11: Four units are all failure. Two maintainers begin to rush repair the first failure unit in subsystem 2.

State 12: Two maintainers begin to replace the failure unit with the normal standby unit in subsystem 1.

State 13: Two maintainers begin to replace the failure unit with the normal standby unit in subsystem 2.

Obviously, state 4 to state 13 are system failure states. System state space is E={0,1,2,3,4,5,6,7,8,9,10,11,12,13}, system work state space is W={0,1,2,3,4}, system failure state space is F={5,6,7,8,9,10,11,12,13}.

It can be proved that $\{X(t),t\geq 0\}$ is a continuous-time homogeneous Markov process in state space E.

4. Model Solution

4.1. The System State Transition Rate Matrix

This thesis focuses on homogeneous Markov process, so the transition probability matrix is as follow, where $i, j \in E$.

$$\boldsymbol{P}(t) = P_{ij}(t) = P\{X(t+u) = j \mid X(u) = i\}, i, j \in E \tag{1}$$

In Δt, the transition rate matrix is

$$\boldsymbol{A} = q_{ij} = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{P_{ij}(\Delta t)}{\Delta t} & \text{if } \ i \neq j, \\ -\sum\limits_{j \neq i} q_{ij} & \text{if } \ i = j, \end{cases} i, j \in E \tag{2}$$

Let $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{D} & \boldsymbol{E} \end{pmatrix}$, then

$$B = \begin{pmatrix} -c_1 - c_2 - w_1 - w_2 & c_1 & c_2 & 0 \\ r_{12} & -w_1 - w_2 - c_2 - r_{12} & 0 & c_2 \\ r_{22} & 0 & -w_1 - w_2 - c_1 - r_{22} & c_1 \\ 0 & r_{21} & r_{11} & -w_1 - w_2 - r_{11} - r_{21} \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 \\ w_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_2 \\ 0 & w_2 & 0 & 0 & 0 & 0 & 0 & 0 & w_1 & 0 \\ 0 & 0 & w_1 & w_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} & \boldsymbol{O}_{8\times4} & & \\ 0 & h_1 & 0 & h_1 \\ 0 & 0 & h_2 & h_2 \end{pmatrix}$$

$$E = \begin{pmatrix} -r_{12}-2c_2 & 0 & c_2 & 0 & c_2 & 0 & 0 & 0 & r_{12} & 0 \\ 0 & -r_{22}-2c_1 & 0 & c_1 & 0 & c_1 & 0 & 0 & 0 & r_{22} \\ 0 & 0 & -r_{12}-c_2 & 0 & 0 & 0 & c_2 & 0 & r_{12} & 0 \\ 0 & 0 & 0 & -r_{22}-c_1 & 0 & 0 & 0 & c_1 & 0 & r_{22} \\ 0 & 0 & 0 & 0 & -r_{12}-c_2 & 0 & c_2 & 0 & r_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & -r_{22}-c_1 & 0 & c_1 & 0 & r_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_{12} & 0 & r_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_{22} & 0 & r_{22} \\ 0 & 0 & 0 & h_1 & 0 & 0 & 0 & 0 & -4h_1 & h_1 \\ 0 & 0 & h_2 & 0 & 0 & 0 & 0 & 0 & h_2 & -4h_2 \end{pmatrix}$$

4.2. System Availability and Steady Availability

Let $P_i(t)=P\{X(t)=i\}$, $i\in E$, denote the probability that system is in state i at time t. When initial state probability $\{P_0(0), P_1(0), \ldots, P_{14}(0)\}$ is known, then the instantaneous availability can be solved.

$$A(t) = \sum_{j\in W} P_j(t) \tag{3}$$

Where P_i(t), $i\in W$ is the solution of eqn (4)

$$\begin{cases} \left(\boldsymbol{P}_W'(t), \boldsymbol{P}_F'(t)\right) = (\boldsymbol{P}_W(t), \boldsymbol{P}_F(t)) \begin{pmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{D} & \boldsymbol{E} \end{pmatrix} \\ (\boldsymbol{P}_W(0), \boldsymbol{P}_F(0)) = \left((1 \quad 0 \quad 0 \quad 0), \boldsymbol{O}_{1\times8}\right) \end{cases} \tag{4}$$

Applying Laplace transform on eqn (4), we get the equation

$$\boldsymbol{P}^*(s) = \boldsymbol{P}(0)(s\boldsymbol{I}-\boldsymbol{A})^{-1}, s>0$$

Where I is 14 order identity matrix. Let $\boldsymbol{S}$=s$\boldsymbol{I}$-$\boldsymbol{A}$, and partition $\boldsymbol{S}$.

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} & \boldsymbol{S}_{13} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} & \boldsymbol{S}_{23} \\ \boldsymbol{S}_{31} & \boldsymbol{S}_{32} & \boldsymbol{S}_{33} \end{pmatrix} \tag{5}$$

Where S11, S22, and S33 are respectively 4×4, 8×8, and 2×2. S12, S13, S21, S23, S31, S32 are respectively 4×8, 4×2, 8×4, 8×2, 2×4, 2×8.

S_{22} is a diagonal matrix. S_{21} is a zero matrix. S_{11} is an invertible matrix. Let

$$\boldsymbol{S}_k = \boldsymbol{S}_{33} - \boldsymbol{S}_{k1} - \boldsymbol{S}_{k2}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{23} \tag{6}$$

Where

$\boldsymbol{S}_{k1} = \boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13}$; $\boldsymbol{S}_{k2} = \boldsymbol{S}_{32} - \boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}$

If $\boldsymbol{S}_k$ is an invertible matrix, then $\boldsymbol{S}$ is an invertible matrix. Its derivation is deferred to Appendix

$$\boldsymbol{S}^{-1} = \begin{pmatrix} \boldsymbol{S}_{11}^{-1} + \boldsymbol{S}_1\boldsymbol{S}_3^{-1}\boldsymbol{S}_2 & \boldsymbol{S}_1\boldsymbol{S}_3^{-1} \\ \boldsymbol{S}_3^{-1}\boldsymbol{S}_2 & \boldsymbol{S}_3^{-1} \end{pmatrix} \tag{7}$$

Where

$$\boldsymbol{S}_1 = \left(-\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} \quad -\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13}\right), \quad \boldsymbol{S}_2 = \begin{pmatrix} \boldsymbol{O}_{8\times4} \\ -\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1} \end{pmatrix}$$

$$\boldsymbol{S}_3 = \begin{pmatrix} \boldsymbol{S}_{22} & \boldsymbol{S}_{23} \\ \boldsymbol{S}_{32} - \boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} & \boldsymbol{S}_{33} - \boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13} \end{pmatrix}$$

Thus the Laplace transform on availability $A^*(s)$ is

$$A^*(s)=\int_0^{\infty}e^{-st}A(t)\mathrm{d}t=\sum_{i\in W}P_i^*(s) \\ =\boldsymbol{P}_W(0)\left(\boldsymbol{S}_{11}^{-1}+\boldsymbol{S}_1\boldsymbol{S}_3^{-1}\boldsymbol{S}_2\right)\boldsymbol{e}_W \tag{8}$$

Where e_w=(1, 1, 1, 1)$^{\mathrm{T}}$.

4.3. System ROCOF

Let $N(t)$ denote system failure times before time t. Let $M_i(t)$ denote mean system failure times in (0, t], where system is in state i at time t=0.

$$M_i(t)=E\left\{N(t)\mid X(0)=i\right\},i\in E \tag{9}$$

Where M_i(t) is the solution of eqn (10)

$$\begin{cases}\boldsymbol{M}'(t)=\boldsymbol{A}\boldsymbol{M}(t)+\begin{pmatrix}\boldsymbol{C}\boldsymbol{e}_{\mathrm{F}}\\ \mathbf{0}\end{pmatrix}\\ \boldsymbol{M}(t)=\mathbf{0}\end{cases} \tag{10}$$

Where e_{F} is 10 dimensional vector, whose components are all 1.

Let $m_i(t)$ denote the ROCOF where the system in state i at time t=0.

$$m_i(t)=M_i'(t),\ i\in E \tag{11}$$

Applying Laplace transform on eqn (11), then

$$\boldsymbol{m}^*(s)=\left(s\boldsymbol{I}-\boldsymbol{A}\right)^{-1}\begin{pmatrix}\boldsymbol{C}\boldsymbol{e}_{\boldsymbol{F}}\\ \mathbf{0}\end{pmatrix}=\begin{pmatrix}\left(\boldsymbol{S}_{11}^{-1}+\boldsymbol{S}_1\boldsymbol{S}_3^{-1}\boldsymbol{S}_2\right)\boldsymbol{C}\boldsymbol{e}_{\boldsymbol{F}}\\ \boldsymbol{S}_3^{-1}\boldsymbol{S}_2\boldsymbol{C}\boldsymbol{e}_{\boldsymbol{F}}\end{pmatrix} \tag{12}$$

The Laplace transform on system instantaneous ROCOF at time t is calculated as

$$m^*(s)=\sum_{i\in E}P_i(0)m_i^*(s) \tag{13}$$

The system steady ROCOF is calculated as

$$M=\lim_{t\to\infty}m(t)=\lim_{s\to 0}sm^*(s) \tag{14}$$

4.4. The System Reliability and MTTFF

We transform all failure states in original system into absorbing state. Then can get a new Markov process $\{\tilde{X}(t),t\geq 0\}$, D=E=0 in corresponding transition rate matrix.

Let $Q_i(t)=P\{\tilde{X}(t)=i\},i\in E$ denote the probability of absorbing system in state i at time t. If at time t=0, the system is brand new. The initial state probability is

$$(\boldsymbol{Q}_W(0),\boldsymbol{Q}_F(0))=\left(\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & 0 & 0 & 0 & 0\end{pmatrix}\right) \tag{15}$$

The system reliability is obtain by solving (16)

$$\begin{cases}\left(Q_0'(t),Q_1'(t),Q_2'(t),Q_3'(t)\right)=\left(Q_0(t),Q_1(t),Q_2(t),Q_3(t)\right)B\\ \left(Q_0(0),Q_1(0),Q_2(0),Q_3(0)\right)=(1,0,0,0)\end{cases} \tag{16}$$

We apply Laplace transform on eqn (16), then

$$\begin{cases}Q_0^*(s)=\dfrac{a_2a_3a_4-a_2c_1r_{11}-a_3c_2r_{21}}{d}\\ Q_1^*(s)=\dfrac{c_1(a_3a_4-c_1r_{11}+c_2r_{21})}{d}\\ Q_2^*(s)=\dfrac{c_2(c_1r_{11}+a_2a_4-r_{21}c_2)}{d}\\ Q_3^*(s)=\dfrac{c_1c_2(a_2+a_3)}{d}\end{cases} \tag{17}$$

Where

$$d = a_1 a_2 a_3 a_4 + (c_2 r_{21} r_{22} - a_1 a_3 r_{21} - a_2 a_4 r_{22}) c_2 + \left(c_1 r_{11} r_{12} - a_1 a_2 r_{11} - a_3 a_4 r_{12} - (r_{12} r_{21} + r_{11} r_{22}) c_2\right) c_1$$

$$a_1 = s + c_1 + c_2 + w_1 + w_2 \quad a_2 = s + c_2 + r_{12} + w_1 + w_2$$

$$a_3 = s + c_2 + r_{22} + w_1 + w_2 \quad a_4 = s + r_{11} + r_{21} + w_1 + w_2$$

Applying Laplace transform on system reliability

$$R^*(s) = \sum_{i=0}^{3} Q_i^*(s) \tag{18}$$

And MTTFF is also can be calculated.

$$\text{MTTFF} = \int_0^\infty R(t)\text{d}t = R^*(0) \tag{19}$$

5. Illustrative Applications

In this section, we use two examples to illustrate the theoretical results reported in this paper.

5.1. The First Example

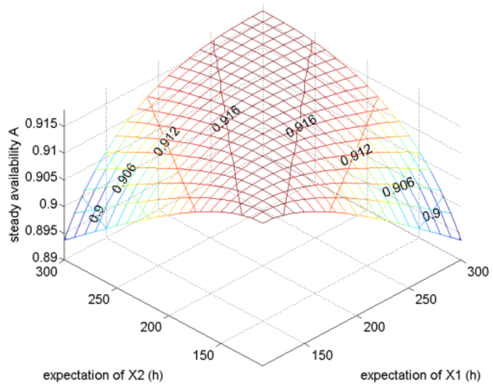
The longer expectation of work life is, the longer expectations of standby life, replace time and repair time are. We named it proportional system.

Expectation of work life X1 in subsystem 1 and expectation of work life X2 in subsystem 2 change from 100 h to 300 h. Expectation of standby life is 10 times of work life. Expectation of work life is 10 times of repair time. Expectation of repair time is 10 times of replace time. Expectation of rush repair time is 2/3 times of repair time. Expressions of steady availability, steady ROCOF and MTTFF are obtained using Eqs. (8), (14) and (19). Computed steady availability steady ROCOF and MTTFF of proportional system have been plotted and shown in Figure 1. The corresponding maximum and minimum values are given in Table 1.

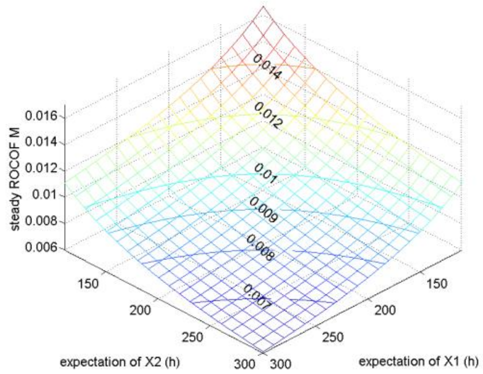
Table 1. Variation in Reliability Indexes of The Proportional System

Reliability indexes	*Maximum*	*Minimum*
Steady availability	0.9166	0.8936
Steady ROCOF	0.01667	0.00611
MTTFF(h)	150	55

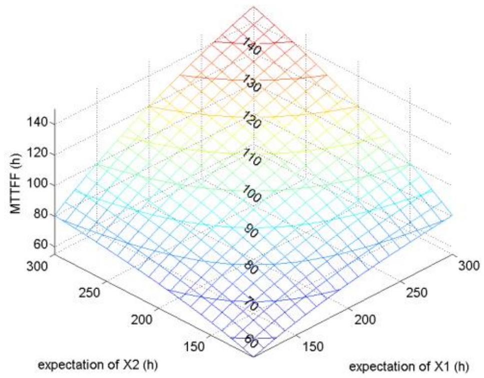
The Figure 1 (a) indicates that steady availability increases with the decrease in the distance between two subsystem work lives. In some range of distance, steady availability gets maximum 0.9166. It means to achieve higher performance of availability, the distance between two subsystem work lives should be minimized. The Figure1 (b) and Figure1 (c) indicate that work lives increase with the decrease in ROCOF and the increase in MTTFF.



(a) Steady availability



(b) Steady ROCOF



(c) MTTFF

Figure 1. Variation In Reliability Indexes for Proportional System by Varying Expectations of Work Lives

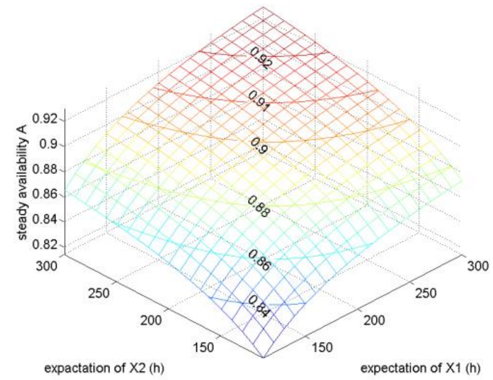
5.2. The Second Example

On the basis of the first example, we fixed the mean repair time at 15 h for subsystem 1, and 35 h for subsystem 2. We named it constant example. Computation have been made and shown graphically by Figure 2 for reliability indexes, respectively. The maximum and minimum values obtained for each reliability indexes noticed and given in Tables 2.

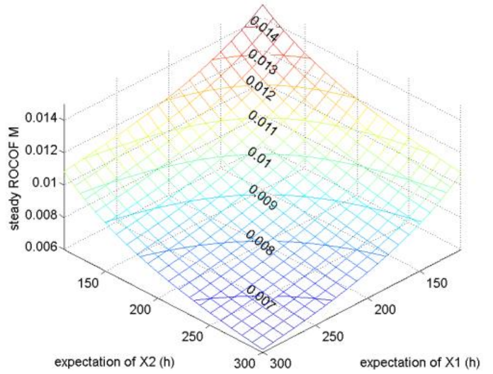
Table 2. Variation In Reliability Indexes of the Constant System

Reliability indexes	*Maximum*	*Minimum*
Steady availability	0.9288	0.8140
Steady ROCOF	0.01480	0.00619
MTTFF(h)	150	55

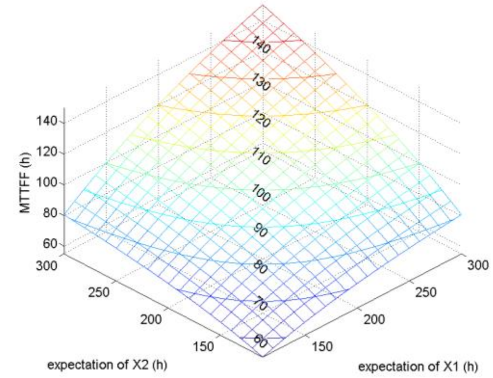
Figure 2 (a) indicates that work lives increase with the increase in steady availability and MTTFF. The indexes get maximums at (300,300). Figure 2 (b) shows that variation in steady ROCOF is almost as same as that in proportional system. Figure 2 (c) indicates that MTTFF in two systems are same, because MTTFF just has relationship with work lives of two subsystems.



(a) Steady availability



(b) Steady ROCOF



(c) MTTFF

Figure 2. Variation in Reliability Indexes for Constant Example By Varying Expectations of Work Lives

On the basis of results curve, it is analyzed that to improve the performance of the proportional system, more attention should be given in minimizing the distance between work lives of the two subsystems. Similarly for constant system more attention should be given in increasing work lives of the two subsystems.

6. Conclusions

1) In the present study, an attempt has been made by the authors to build a repairable system model by utilizing Markov process and system state transition rate matrix. In this process, the authors fully take replace time and cooperative repair of two maintainers into account, and maintenance strategy is making sure the system is in work states.

2) The paper reports the steady availability, steady ROCOF and MTTFF reliability indexes of the system by using the character of block matrix to increases calculation efficiency.

3) The present study has illustrated two examples of proportional system and constant system. Obviously, the examples studied in this paper are reasonable for some systems in the engineering. Computed results will facilitate the designer in arrangement the resources, achieving long run availability of the system.

7. Appendix

According to Eqn (5), we got Block matrix. Where S11, S22, and S33 are respectively m×m, n×n, and s×s. S12, S13, S21, S23, S31, S32 are respectively n×m, n×s, m×n, m×s, s×n, s×m. S11 is an invertible matrix.

$$\begin{pmatrix} \boldsymbol{I}_n & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1} & \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{S}_{23}\boldsymbol{S}_{11}^{-1} & \boldsymbol{0} & \boldsymbol{I}_s \end{pmatrix} \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} & \boldsymbol{S}_{13} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} & \boldsymbol{S}_{23} \\ \boldsymbol{S}_{31} & \boldsymbol{S}_{32} & \boldsymbol{S}_{33} \end{pmatrix} \begin{pmatrix} \boldsymbol{I}_n & -\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} & -\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13} \\ \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_s \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_{22}-\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} & \boldsymbol{S}_{23}-\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13} \\ \boldsymbol{0} & \boldsymbol{S}_{32}-\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} & \boldsymbol{S}_{33}-\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13} \end{pmatrix} = \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_3 \end{pmatrix}$$

So $\boldsymbol{S}$ is invertible $\Leftrightarrow \boldsymbol{S}_{11}$ and $\boldsymbol{S}_3$ are all invertible.Let

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{pmatrix}$$

If $\boldsymbol{P}_{11}$ is invertible, $\boldsymbol{P}$ is invertible $\Leftrightarrow \boldsymbol{P}_{22}-\boldsymbol{P}_{21}\boldsymbol{P}_{11}^{-1}\boldsymbol{P}_{12}$ is invertible.Then $\boldsymbol{S}_3$ is invertible $\Leftrightarrow \boldsymbol{S}_{33}-\boldsymbol{S}_{k1}-\boldsymbol{S}_{k2}\boldsymbol{S}_{k3}^{-1}\boldsymbol{S}_{k4}$ is invertible.

Where

$\boldsymbol{S}_{k1}=\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13}$; $\boldsymbol{S}_{k2}=\boldsymbol{S}_{32}-\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}$

$\boldsymbol{S}_{k3}=\boldsymbol{S}_{22}-\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}$; $\boldsymbol{S}_{k4}=\boldsymbol{S}_{23}-\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13}$

Let

$$\boldsymbol{S}_1 = \left(-\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}, -\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{13}\right)$$

$$\boldsymbol{S}_2 = \begin{pmatrix} -\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1} \\ -\boldsymbol{S}_{31}\boldsymbol{S}_{11}^{-1} \end{pmatrix}$$

Then

$$\boldsymbol{S}^{-1} = \begin{pmatrix} \boldsymbol{I}_n & \boldsymbol{S}_1 \\ \boldsymbol{0} & \boldsymbol{I}_{m+s} \end{pmatrix} \begin{pmatrix} \boldsymbol{S}_{11}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_3^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{S}_2 & \boldsymbol{I}_{m+s} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{S}_{11}^{-1}+\boldsymbol{S}_1\boldsymbol{S}_3^{-1}\boldsymbol{S}_2 & \boldsymbol{S}_1\boldsymbol{S}_3^{-1} \\ \boldsymbol{S}_3^{-1}\boldsymbol{S}_2 & \boldsymbol{S}_3^{-1} \end{pmatrix}$$

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